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MODELS OF DIFFRACTIVE PRODUCTION PROCESSES ON SIMPLE AND COMPOSITE TARGETS

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A unified description of diffractive production processes on nucleons and nuclei based on a model proposed some time ago by Białas, Kotański and the author is discussed.

1. Introduction

In collisions of very high energy particles there exist elastic and inelastic processes in specific channels which, judging from existing experimental data, do not vanish in the limit of very high energies. In these lectures I shall try to outline a possible approach to analyse these processes.

Let us start by giving a few examples of such processes. There are many known elastic processes of this kind. In Quantum Electrodynamics [4]: elastic electron (positron) scattering from a Coulomb field or from some other charged particle, elastic scattering of photons from a Coulomb field (Delbrück scattering) or from an electron (Compton scattering) — just to name the better known processes. Among hadronic elastic process one can list: the hadron-hadron elastic scattering cross-sections ($p-p$, $\pi-p$ *etc.*) which all seem to have energy independent (or very weakely dependent) cross-sections $d\sigma/d\Delta^2 = f(\Delta^2)$, where Δ^2 is the momentum transfer squared (for small angle scattering $\Delta^2 \cong -t$). The same can be said about hadron-nucleus elastic (good energy resolution: the target nucleus remains in its ground state, poor energy resolution: the experiment sums over all nuclear excitations) cross-sections. All these elastic process had been discussed *e. g.* at the X Cracow School of Theoretical Physics (see *Acta Physica Polonica* **B2**, *Fasciculus* 1 (1971)).

This time we shall discuss the diffractive inelastic production processes (one may say that this is a continuation of the discussion which was presented at XI Cracow School of Theoretical Physics, see *Acta Physica Polonica* **B3**, *Fasciculus* 1 (1972)). Starting again with QED we have *e. g.* the well known processes of bremsstrahlung and pair production, less known positronium photoproduction in strong Coulomb field, production of two electron-positron pairs in photon-photon collisions and many others.

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We shall give below a brief discussion of the first three processes treated the way which can be extended to hadronic processes. There are many hadronic diffractive production process known. The present status of the experiment the reader can find in the lectures by drs Beusch and Lubatti given at this school. In the present lectures we shall discuss briefly diffractive dissociation of protons into pions and neutrons off nucleons and nuclei.

Our discussion will be based on the assumption that diffractive production processes in QED and in hadronic interactions have similar mechanism: The process of dissociation of the incident particle into its components is weak compared to the process of their scattering from the target. For instance, in the case of pair production by high energy photons in a strong Coulomb field, the process of photon dissociation into the electron--positron pair is weaker than elastic scattering of electrons and positrons from the Coulomb field (the ratio of the effective coupling constants is of order $\alpha = 1/137$). Similar statements can be made about hadronic diffractive dissociation. For example the cross-section for proton dissociation into pion and neutron is of order 1 mb while the elastic scattering cross-sections of hadrons from hadronic targets are more than an order of magnitude larger. It has to be stressed that such statements of weakness of diffractive dissociation are being made for specific channels, not for the sum of all possible dissociations. Another characteristic feature of the process discussed here is a "passive" role of the target: nothing happens to it. It looks indeed as if the incident particles were dissociating into their components. All these process are very strongly peaked in the forward direction.

In these lectures a model is discussed which may be applied to all these processes with "elementary" targets (nucleons) and "composite" targets (nuclei). It is very likely that nuclear targets will be able to provide us with details on elementary processes which are difficult or even impossible to get from the processes on nucleon targets. In the lectures by dr Gottfried given at this school some recent general results relevant to this point are discussed. Here, we shall limit ourselves to a specific model and attempt to propose a unified description of diffractive dissociation on simple (nucleon) and composite (nuclear) targets.

2. *The model*

The model we shall employ to describe all diffractive processes (both on simple and composite targets) rests on two assumptions:

(*i*) The colliding objects can be decomposed into subunits whose interactions with the target are assumed to be known. Such a decomposition is not always obvious and sometimes may become ambiguous but, in many cases it is self evident: *e. g.* a nucleus is, to a very good approximation, composed of nucleons, an atom of electrons and a nucleus *etc.* The internal structure of colliding objects is described by the wave functions which give distributions of the subunits. Again, in some cases these are well known wave functions in other cases they are some speculative objects. The examples discussed below illustrate the situation.

(*ii*) The process of diffractive dissociation at high energy is caused by elastic scattering of subunits from the target. These subunits scatter independently from each other. Hence the total S-matrix is a product of individual S-matrices. In momentum space

$$S = \prod_i S_j(\vec{k}_{j\perp} - \vec{p}_{j\perp}) = \prod_i [(2\pi)^2 \delta^{(2)}(\vec{k}_{j\perp} - \vec{p}_{j\perp}) - \tilde{\gamma}_j(\vec{k}_{j\perp} - \vec{p}_{j\perp})]. \tag{2.1}$$

In order to compute the transition matrix elements we have to known $\tilde{\gamma}(\vec{k}_{j\perp} - \vec{p}_{j\perp})$ which are the elastic scattering amplitudes. The transition matrix element is

$$\mathfrak{M} = \langle f|\mathcal{M}|i\rangle = \langle f|(\prod_j S_j - 1)|i\rangle \tag{2.2}$$

where the initial and final states are given by the wave functions mentioned in (i).

A few additional comments referring to the second assumption are in order here. Eq. (2.1) is written in momentum representation. Sometimes it is more convenient to use position space representation and we shall do it often below. The next comment is that our S-matrix (2.1) depends only on transverse momentum transfers. This approximation is justified by the observation that in the limit of very high energies and small momentum transfers the longitudinal momentum transfers go to zero like ω^{-1} (where ω is the longitudinal momentum of the incident particle). There are however cases where one has to consider longitudinal momentum transfers. This is so when very small transverse momentum transfers, of the order of ω^{-1}, contribute significantly to the amplitudes, which happens *e. g.* in QED productions processes in forward directions. Then one has to use some different techniques. Some examples are discussed below.

The final comment is about structure of subunits. When they are themselves composite they may undergo various excitation processes in consecutive collisions — if the target is composed of many subunits. Phenomena of this kind occur presumably in scattering processes from nuclei. In such processes the same final state may be reached through many different "histories" of a multiple collision processes. In order to take into account such phenomena one should somewhat modify the above outlined scheme. We shall come back to this process while discussing diffractive production on nuclear targets and describe two techniques of dealing with such phenomena [1], [2].

First, however, we shall discuss "simple" targets whose internal structure can be neglected. There, such effects as described above play less important role.

3. Elementary diffractive production processes in Quantum Electrodynamics and hadronic physics

It is instructive to discuss first such elementary diffractive processes as bremsstrahlung, pair production and photoproduction of positronium[1], all in strong external Coulomb field. All these processes do not vanish in the high energy limit. They also exhibit the "weakness" characteristic of diffractive processes in a given channel:

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[1] This process has a very small cross-section. Its order of magnitude:

$$\lesssim 10^{-30}\ \text{cm}^2$$

for para-, and somewhat smaller for orthopositronium production for $Z = 100$ [7] and, probably, it is going to be swamped by *e.g.* pair production process. It is however worth analyzing as a model process for the purpose of understanding several subtle aspects of diffractive production of bound composite objects.

the elastic scattering cross-section of "subunits" (which are in these cases electrons and positrons) from the external strong Coulomb field are much larger ($\sim (\alpha Z)^2$) than the production cross-sections ($\sim \alpha(\alpha Z)^2$). So, they may serve as a model for strong diffractive production.

There exist in the literature some very complete calculations of such processes (except for positronium photoproduction) [3] whose transition matrix elements are taken in the form

$$\mathfrak{M} = \int d\tau \psi_2^* \vec{j} \cdot \vec{A} \, \psi_1 \tag{3.1}$$

where the radiation field is taken to first order but the wave functions of the two electrons (bremsstrahlung) or a positron and an electron (pair production) ψ_1, ψ_2 are the exact solutions of Dirac equation in a given (strong) Coulomb field.

This is certainly a correct way of finding cross-sections for bremsstrahlung and pair production in a strong Coulomb field, but the possibilities to apply it to other diffractive production processes (like *e. g.* positronium photoproduction[1]) or to extend it to hadronic processes are very limited.

Recently, however, a different approach to high energy QED processes was proposed ([4], [5] and many others) which gives prescriptions for calculating transition matrix elements of various diffractive production processes equivalent to the ones formulated in Section 2. So, one can cast the matrix elements for the above three processes in a form of an operator $\mathcal{M}$ which describes elastic scattering of subunits sandwiched in between the initial and final state wave functions of the system. This approach is already very flexible and can be extended to hadronic processes: The wave functions can be, with some modifications, constructed as in QED, the operator $\mathcal{M}$ can also be, to a good approximation, expressed in terms of some realistic elastic scattering amplitudes.

Let us start by discussing some details of the pair production cross-section. Let us work in the rest system of the infinitely heavy target where the momenta of e^+e^- pair are very large.

First we specify the subunits of a physical photon by writing out the wave functions of the initial and final states:

$$\text{the photon state } |\gamma\rangle = \sqrt{\mathscr{Z}}\,[|\mathring{\gamma}\rangle + \Sigma\varepsilon|\mathring{e^+e^-}\rangle + \ldots],$$
$$\text{the } e^+e^- \text{ state } \langle e^+e^-| = \sqrt{\mathscr{Z}}\,[\langle \mathring{e^+e^-}| + \langle\mathring{\gamma}|\varepsilon' + \ldots]. \tag{3.2}$$

The coefficients ε, ε' are small and from the orthogonality condition $\langle e^+e^-|\gamma\rangle = \mathscr{Z}(\varepsilon+\varepsilon') = 0$, we get $\varepsilon' = -\varepsilon$. $\mathscr{Z}$ is the renormalization constant which is, to first order in ε, equal 1. Eq. (3.2) presents the physical states as superpositions of the bare states. We can compute ε as the first order perturbation and we get in the limit of very large longitudinal momentum of the incident photon ($\omega \to \infty$):

$$\begin{aligned} \varepsilon(p_1 p_2) &\sim e\bar{u}(p_2)\vec{\gamma}\cdot\vec{\varepsilon}\, v(p_1)[|\vec{P}| - E(p_1) - E(p_2)]^{-1} \\ &\underset{\omega\to\infty}{\to} -\frac{\omega\sqrt{\eta_1\eta_2}}{m_e}\, eU_2^*\vec{\varepsilon}\cdot\vec{\vec{\Phi}}(\vec{q}, \eta_2-\eta_1)U_1, \end{aligned} \tag{3.3}$$

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[1] See footnote on the previous page.

where

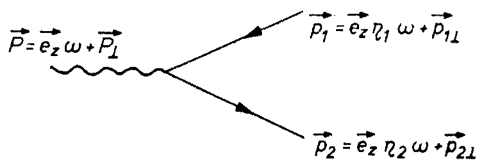
$$\vec{\Phi}(q, \eta_2 - \eta_1) = [\sigma_z(\eta_2 - \eta_1)\vec{q} - i(\vec{q} \times \vec{e}_z) + m_e \vec{\sigma}_\perp](q^2 + m_e^2)^{-1},$$

$$\vec{q} = (\eta_1 \vec{p}_{2\perp} - \eta_2 \vec{p}_{1\perp})(\eta_1 + \eta_2)^{-1}.$$

Eq. (3.3) gives the amplitude of a e^+e^- bare pair in the physical photon. Fig. 1 explains the notation. We have used the conventions of Ref. [6]. $U_{1,2}$ are just the two component spinors which are related to the four spinors as follows

$$u(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} U \\ \dfrac{\vec{\sigma} \cdot \vec{p}}{E+m} U \end{pmatrix}.$$

We shall accept that all particles move in the positive direction of z-axis, hence $0 < \eta_1, \eta_2 < 1$. To meet the requirements of the assumption (*ii*) of Section 2 and construct $\mathcal{M}$ we have to specify the interactions of the subunits (bare photons, electrons and positrons)



From momentum conservation: $\eta_1 + \eta_2 = 1$, $P_\perp = p_{1\perp} + p_{2\perp}$
$\mathbf{e}_z$ *-unit vector in the direction of z-axis*

Fig. 1

of (3.2) with the strong Coulomb field of a heavy nucleus (Z large). A reasonable assumption is to neglect the interaction of the bare photon with Coulomb field, and to take for positron (electron) amplitudes the amplitudes one gets by solving Dirac equation with Coulomb potential:

$$\tilde{\gamma}_\pm(\vec{k}_\perp - \vec{p}_\perp) = \int d^2 b e^{i(\vec{k}_\perp - \vec{p}_\perp)\cdot\vec{b}}(1 - e^{\pm i\chi_c(b)}),$$

where we neglected the spin flip part of the amplitude because we shall consider only small momentum transfers, hence, in the limit $\omega \to \infty$, this part goes to zero. χ_c is the phase shift of a screened Coulomb field (to avoid lengthy discussions of handling infinities caused by the infinite range of an unscreened Coulomb field). From (2.2) we get

$$\mathcal{M} = \tilde{\gamma}_+(\vec{k}_{1\perp} - \vec{p}_{1\perp}) + \tilde{\gamma}_-(\vec{k}_{2\perp} - \vec{p}_{2\perp}) - \tilde{\gamma}_+(\vec{k}_{1\perp} - \vec{p}_{1\perp})\tilde{\gamma}_-(\vec{k}_{2\perp} - \vec{p}_{2\perp}). \tag{3.4}$$

By using such amplitudes we accept two different coupling constants in the process: "small" e and "large" eZ. The pair production process is considered here to lowest order in e and to all orders in eZ. Note that $\mathcal{M}$ does not create any new subunits: it just scatters them elastically. The first two terms in (3.4) give scatteiing of just one bare lepton, the third term represents scattering of both bare leptons. In the two dimensional space spanned by a bare photon and a bare pair states, $\mathcal{M}$ is represented by a matrix

$$\begin{pmatrix} \langle \mathring{\gamma}|\mathcal{M}|\mathring{\gamma}\rangle & \langle \mathring{\gamma}|\mathcal{M}|e^{\mathring{+}}e^{-}\rangle \\ \langle e^{\mathring{+}}e^{-}|\mathcal{M}|\mathring{\gamma}\rangle & \langle e^{\mathring{+}}e^{-}|\mathcal{M}|e^{\mathring{+}}e^{-}\rangle \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \langle e^{+}\mathring{e}^{-}|\mathcal{M}|e^{+}\mathring{e}^{-}\rangle \end{pmatrix}.$$

The transition matrix element is therefore

$$\langle f|\mathcal{M}|i\rangle = \langle e^{\mathring{+}}e^{-}|\sum \mathcal{M}\varepsilon|e^{+}\mathring{e}^{-}\rangle,$$

where the sum is over all intermediate momenta (note that the longitudinal components do not change during the scattering process — in the present approximation). The three

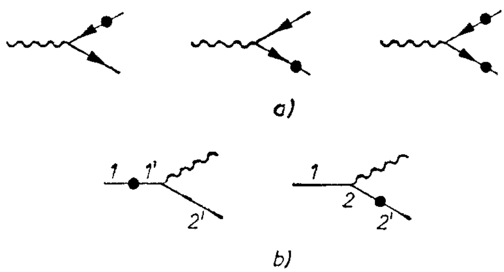


Fig. 2

contributions coming from the three terms of (3.4) can be represented graphically as shown in Fig. 2a), where each dot corresponds to one factor γ.

One can use the same technique to compute the bremsstrahlung matrix elements:

$$|i\rangle = |e^{-}\rangle = \sqrt{\mathscr{Z}}\,[|\mathring{e}_1^{-}\rangle + \varepsilon(1)|e_2^{\mathring{-}}\gamma\rangle + \ldots]$$

$$\langle f| = \langle e^{-}\gamma| = \sqrt{\mathscr{Z}}\,[\langle e_{2'}^{\mathring{-}}\gamma| - \langle \mathring{e}_{1'}^{-}|\varepsilon(1') + \ldots]$$

with

$$\varepsilon = e\bar{u}(p_2)\vec{\gamma}\cdot\vec{\varepsilon}\,u(p_1)\,[E(p_1) - |\vec{K}| - E(p_2)]^{-1}.$$

Assuming again that the photon interaction is negligible we have $\mathcal{M} = \gamma-(1)+\gamma-(2)$ but in the two dimensional space spanned by $|\mathring{e}^{-}\rangle$ and $|e^{\mathring{-}}\gamma\rangle$ this operator looks schematically as follows

$$(-\varepsilon(1'), 1)\begin{pmatrix} \langle \mathring{e}_{1'}^{-}|\mathcal{M}|\mathring{e}_1^{-}\rangle & 0 \\ 0 & \langle e_{2'}^{\mathring{-}}\gamma|\mathcal{M}|e_2^{\mathring{-}}\gamma\rangle \end{pmatrix}\begin{pmatrix} 1 \\ \varepsilon(1) \end{pmatrix} =$$

$$= -\varepsilon(1')\,\langle \mathring{e}_{1'}^{-}|\mathcal{M}|\mathring{e}_1^{-}\rangle + \langle e_{2'}^{\mathring{-}}\gamma|\mathcal{M}|e_2^{\mathring{-}}\gamma\rangle\varepsilon(1) = \langle f|\mathcal{M}|i\rangle.$$

The two corresponding processes are depicted in Fig. 2b).

The results of such calculations have been, to some extent, compared with the "classical" results for bremsstrahlung, pair production and Delbrück scattering all in the high energy limit. The general conclusion (to the best of this author understanding) is that agreement exists for all "physically relevant" results. In other words the differences exist only (we hope) in the results which depend crucially on the infinite range of Coulomb interaction (*e.g.* in forward Delbrück scattering). When one neglects screening one cannot neglect the longitudinal momentum transfers at the scattering verticles (as it is done in (2.2)), because contributions from very small transverse momentum transfers may cause divergences which do not occur if the longitudinal transfers are kept (even though they go to zero like $\mathcal{O}(\omega^{-1})$). These problems disappear when there is a natural cut-off radius like a screening radius.

Before going over to extensions of the outlined technique to hadronic processes let us discuss the structure of the "wave functions" of the fluctuations like *e.g.* e^+e^- fluctuation given by (3.3) (compare Ref. [5]). The variable $\vec{q}$ can be interpreted as a relative momentum of the pair (η_1, η_2 play the roles of masses). If one introduces the transverse position vectors $\vec{s}_1$, and $\vec{s}_2$ for the two particles, and interprets $\vec{q}$ as being conjugate to the relative distance $\vec{s} = \vec{s}_1 - \vec{s}_2$, we get through the Fourier transform

$$\vec{\Phi}(\vec{s}, \eta_2-\eta_1) = \frac{1}{(2\pi)^2}\int d^2q e^{i\vec{q}\cdot\vec{s}}\vec{\tilde{\Phi}}(\vec{q}, \eta_2-\eta_1)$$

the following expression for the transition matrix element in the position representation

$$\langle f|\mathcal{M}|i\rangle \sim \int d^2R d^2s\, e^{i(\vec{P}-\vec{k}_1-\vec{k}_2)\cdot\vec{R}} e^{-i\eta_2\vec{k}_1\cdot\vec{s}+i\eta_1\vec{k}_2\cdot\vec{s}} \times$$
$$\times [\gamma_+(\vec{R}+\eta_2\vec{s})+\gamma_-(\vec{R}-\eta_1\vec{s})-\gamma_+(\vec{R}+\eta_2\vec{s})\gamma_-(\vec{R}-\eta_1\vec{s})] \times$$
$$\times U_2^*\vec{\varepsilon}\cdot\vec{\Phi}(\vec{s}, \eta_2-\eta_1)U_1, \tag{3.5}$$

where $\vec{R} = \dfrac{\eta_1\vec{s}_1+\eta_2\vec{s}_2}{\eta_1+\eta_2}$ should be interpreted as the CM coordinate, and $\gamma_\pm$ are two dimensional Fourier transforms of $\tilde{\gamma}_\pm$'s. The initial state wave fuction is $\vec{\Phi}(\vec{s}, \eta_2-\eta_1)$, the final state wave functions is $e^{i\eta_2\vec{k}_1\cdot\vec{s}}\, e^{-i\eta_1\vec{k}_2\cdot\vec{s}}$. The expression in the bracket is usually called profile operator. This formula is an analog of the well known Glauber formula for a diffraction of a nonrelativistic system composed of two subunits. When their masses are equal it reads

$$\langle f|\mathcal{M}|i\rangle \sim \int d^2b e^{i\vec{\Delta}\cdot\vec{b}}\varphi_f^*(\vec{s})\,[\gamma_1(\vec{b}+\tfrac{1}{2}\vec{s})+\gamma_2(\vec{b}-\tfrac{1}{2}\vec{s})-$$
$$-\gamma_1(\vec{b}+\tfrac{1}{2}\vec{s})\gamma_2(\vec{b}-\tfrac{1}{2}\vec{s})]\varphi_i(\vec{s}),$$

where $\vec{\Delta}$ is the momentum transfer. As we can see the only essential difference between the two formulas are the "effective masses" $\eta_1\eta_2$. Similar considerations can also be applied to the bremsstrahlung process (the role of the photon mass plays its factor η).

Now let us analyse the $\vec{s}$ dependence of $\vec{\Phi}(\vec{s}, \eta_2-\eta_1)$. Having the explicit form of $\vec{\tilde{\Phi}}(\vec{q}, \eta_2-\eta_1)$ given by (3.3) we can perform the Fourier transform and get

$$\vec{\Phi}(\vec{s}, \eta_2-\eta_1) = \frac{1}{2\pi}[-i\sigma_z(\eta_2-\eta_1)\vec{\nabla}+(\vec{\nabla}\times\vec{e}_z)+m_e\vec{\sigma}_\perp]K_0(|\vec{s}|m_e) \tag{3.6}$$

where $\vec{\nabla} = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial y}$, and K_0 is the modified Bessel function. This wave function behaves as $|\vec{s}|^{-1}$ for small s. Though this does not make the expression (3.5) for the amplitude to diverge (there are factors which cancel it: *e.g.* $d^2s = sdsd\varphi$) it favours the configuration where the two subunits are at small transverse distances. A way to introduce phenomenologically some different transverse correlations in the wave function is to use some transverse formfactors $F_\perp(q)$ which cause faster decrease of $\vec{\tilde{\Phi}}(q)$ at large q, hence make $\vec{\Phi}$ more regular at small s. For instance, if we multiply $\tilde{\Phi}$ by $F_\perp(q) = m_e(q^2+m_e^2)^{1/2}$ we get

$$\vec{\Phi}'(\vec{s}, \eta_2-\eta_1) = \frac{1}{2\pi}[-i\sigma_z(\eta_2-\eta_1)\vec{\nabla}+(\vec{\nabla}\times\vec{e}_z)+m_e\vec{\sigma}_\perp]e^{-m_e|\vec{s}|}$$

which is regular.

In our wave functions of hadronic fluctuations discussed below we shall use such form factors hence modify these wave functions at small distances between the subunits. One should stress however the point that since (3.6) seems to be in good agreement with experiment, there is no evidence in QED for any "short distance transverse correlations" between the subunits of electron-positron fluctuation. This is not so in hadronic processes (see below).

The dependence of $\vec{\tilde{\Phi}}$ on $\eta_2-\eta_1$ is the next point of our discussion. We have been describing the production process in the laboratory frame, *i.e.* in the frame where the target is at rest. How to describe it in the rest system of the projectile (when the projectile has mass zero we shall understand by the "projectile system" — a system where the projectile energy is small)?

The criterium for correctness of a procedure to go from one system to another is an identity of the physical contents of the calculational algorithms in both systems. This criterium is met when the following prescription is applied:

a. The wave functions $\vec{\tilde{\Phi}}$ are assumed form-invariant (hence *e.g.* (3.3) has the same form in the projectile rest system).

b. The longitudinal components p_z of the momenta (the only objects which should be transformed) are identified in the laboratory frame with the "light cone" variables (compare [5]):

$$p_+ = \tfrac{1}{2}(E+p_z) \underset{p_z\to\infty}{\to} p_z.$$

We shall not give the proof that our criterium is satisfied in QED processes (the reader is referred to Ref. [4]). Neither shall we argue that there is only one solution to this problem. We shall only discuss the consequences of this prescription which has been tested so far on some very simple production processes of QED with the wave functions similar to (3.3). The functions describe noninteracting pairs of particles. Perhaps the same prescription could be applied to some very loosely bound systems *e.g.* positronium whose binding energy is a very small fraction of the electron mass: it is of order $\alpha^2 m$, where $\alpha = \frac{1}{137}$. Then we would have a method for "boosting" the nonrelativistic wave functions of such systems to the frames where they move with relativistic velocities. The procedure would be as follows: (*i*) Take the wave function $\tilde{\varphi}(k)$ of the system in its rest frame and in momentum representation, where $\vec{k}$ is the relative momentum: $\vec{k} = \vec{q}_\perp + (k_{1z} - k_{2z})\vec{e}_z$. (*ii*) Write the longitudinal momentum in terms of η's:

$$\tilde{\varphi}(\vec{k}) = \tilde{\varphi}(\vec{q}_\perp, k_{1z} - k_{2z}) = \tilde{\varphi}(\vec{q}_\perp, \mathrm{M}(\eta_1 - \eta_2))$$

as $\eta_1 = (m_e + k_{1z})\,\mathrm{M}^{-1}$, $\eta_2 = (m_e - k_{2z})\,\mathrm{M}^{-1}$ in the projectile rest system, and M is the rest mass of the positronium. (*iii*) Identify $\tilde{\varphi}(\vec{q}_\perp, \mathrm{M}(\eta_1 - \eta_2))$ with the relativistic wave function. Note that in the rest system of the positronium the scale for relative momentum is $r_0 \approx \dfrac{2}{\alpha m_e}$ in the fast moving frame, however, the scale for z components is $r \approx \dfrac{2}{\alpha m_e}\,\dfrac{M}{\omega}$ which goes to zero as ω increases. We do have therefore the Lorentz contraction built into $\tilde{\varphi}(\vec{q}_\perp, \mathrm{M}(\eta_1 - \eta_2))$.

If this "boosting procedure" is correct one can calculate photoproduction of para- and orthopositronium in a strong Coulomb field [7]. In fact the calculations are simple because the radius of the positronium is much larger than the radius of the electron-positron fluctuation $\left(\dfrac{2}{\alpha m_e} \text{ compared to } \dfrac{1}{m_e}\right)$ hence, in the momentum space the positronium wave function acts like a Dirac δ-function. Incidentally, it is impossible to test the results of such calculations against anything known (except a Weizsacker-Williams method which does not permit production of orthopositronium) because the present status of QED does not permit any reliable calculations of *e.g.* photoproduction of orthopositronium.

The process of photoproduction of positronium gives some extra insight into the physical interpretation of the formfactors. The following example illustrates this point [7]. One can compute the amplitude for positronium photoproduction in a strong Coulomb field employing a field theoretic interaction Hamiltonian which couples positronium field, treated as an elementary pointlike particle with spin zero for parapositronium and spin one for orthopositronium, to the electron-positron field. For instance in the case of parapositronium we would have

$$H' = iG\int d^3x \bar{\psi}(x)\gamma_5\psi(x)\varphi(x),$$

with $G = 16\pi\sqrt{\dfrac{B}{m_e}}$, where B is the binding energy of parapositronium. Then, one can compute the amplitude in the same way as one does in the case of Delbrück scattering [4] except the outgoing photon is replaced by positronium. The three contributions to such process are depicted in Fig. 3.

Although the amplitude obtained in such a way differs from the one obtained by boosting the nonrelativistic positronium wave function as described above one can make them identical [7] by multiplying the vertex of a pointlike positronium by the transverse formfactor:

$$F_\perp(q) = \frac{Bm_e}{Bm_e + q^2}.$$

So, such a form factor does introduce a structure to a pointlike particle. Note that if the incident particle does not interact with the target the two descriptions are identical provided we allow for formfactors. They are different, however, if the incident particle does interact with the target.

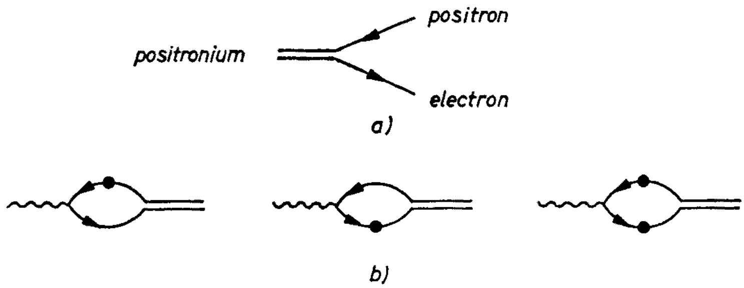


Fig. 3

Now we know enough (hopefully) about QED diffractive processes to discuss extrapolations to hadronic diffractive production processes. Let us discuss a typical process of proton dissociation:

$$p \to \pi^+ + n.$$

We can proceed as in QED. First we expand the physical initial and final states in terms of bare states:

$$|p\rangle = \sqrt{\mathscr{Z}}\,[|\mathring{p}\rangle + \Sigma\varepsilon|\mathring{n\pi}\rangle + \dots]$$

$$|n\pi\rangle = \sqrt{\mathscr{Z}}\,[-\varepsilon|\mathring{p}\rangle + |\mathring{n\pi}\rangle + \dots],$$

where the amplitude of neutron-pion fluctuation should be small to satisfy the well established experimental fact that diffractive production is much (~ 10 times) weaker than elastic scattering. (Since we do not have small coupling constant at our disposal as in QED we have to check in the end that the amplitudes ε we are working with satisfy this condition.

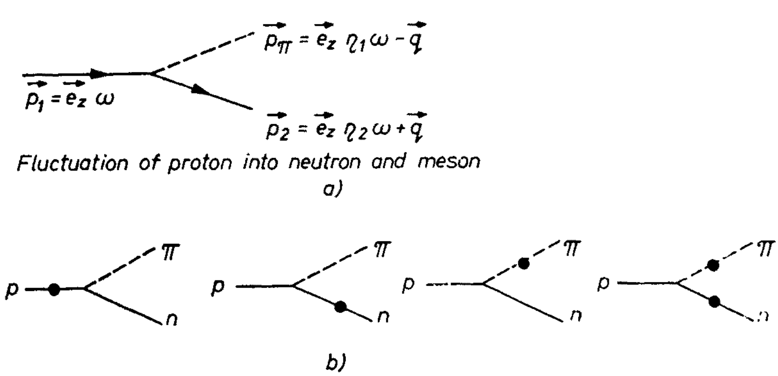


Fig. 4

More precisely: that the renormalization constant $\mathscr{Z} \approx 1$.) We compute ε again from perturbation expansion. For example for pseudoscalar coupling we have (the notation is explained in Fig. 4a)

$$\varepsilon(p_1 p_2) \sim \frac{g\bar{u}(p_2)\gamma_5 u(p_1)}{E(p_1)-E(p_2)-E(p_\pi)} \underset{\omega\to\infty}{\rightarrow} -$$

$$-\frac{\omega \frac{g}{M}\eta_1 \sqrt{\frac{1}{2}(1+\eta_2-\eta_1)}\, U_2^*[(\vec{\sigma}_\perp \cdot \vec{q})+\frac{1}{2}(\eta_1-\eta_2+1)M\sigma_z]U_1}{q^2+\frac{1}{2}(1+\eta_2-\eta_1)\mu^2+\frac{1}{4}(1+\eta_1-\eta_2)^2M^2} =$$

$$= -\omega\frac{g}{M}\eta_1\sqrt{\tfrac{1}{2}(1+\eta_2-\eta_1)}\, U_2^*\tilde{\Phi}(\vec{q},\eta_2-\eta_1)U_1, \tag{3.7}$$

where

$$\tilde{\Phi}(\vec{q},\eta_2-\eta_1) = [\vec{\sigma}_\perp\cdot\vec{q}+\tfrac{1}{2}(\eta_1-\eta_2+1)M\sigma_z]\,[q^2+\tfrac{1}{2}(1+\eta_2-\eta_1)\mu^2+$$
$$+\tfrac{1}{4}(1+\eta_1-\eta_2)^2M^2]^{-1},$$

M is the nucleon mass, μ — meson mass. Its Fourier transform:

$$\Phi(\vec{s},\eta_2-\eta_1) = \frac{1}{2\pi}[i\vec{\sigma}_\perp\cdot\vec{\nabla}+\tfrac{1}{2}(\eta_1-\eta_2+1)M\sigma_z]\times$$
$$\times K_0(|\vec{s}|\sqrt{\tfrac{1}{2}(1+\eta_2-\eta_1)\mu^2+\tfrac{1}{4}(1+\eta_1-\eta_2)^2M^2}).$$

It should be stressed very strongly that we use here a model of the physical states of proton and neutron-pion in close analogy to QED expressions discussed earlier. In contrast to QED, we do not have here a reliable theory to check them. Hence, right from beginning we do phenomenological analysis in which the expressions analogous to QED expressions are the zeroth approximation and we expect some deviations in first approximation. We shall introduce these deviations by introducing transverse and longitudinal correlations into the wave function of the $n\pi$ system. *E.g.* a transverse formfactor $F_\perp(q)$ corrects Φ at small $|\vec{s}|$. From our discussion of $\eta_2-\eta_1$ dependence of Φ one can expect that some corrective factor, a longitudinal formfactor $F(|\eta_2-\eta_1|)$, may also be needed. Such a formfactor changes the longitudinal correlations between the fluctuation subunits.

The operator of diffractive scattering $\mathscr{M}$ is in this case

$$\mathscr{M} = \tilde{\gamma}_p(\vec{p}_\perp-\vec{p}'_\perp)+\tilde{\gamma}_n(\vec{p}_{n\perp}-\vec{k}_{n\perp})+\tilde{\gamma}_\pi(\vec{p}_{\pi\perp}-\vec{k}_{\pi\perp})-$$
$$-\tilde{\gamma}_n(\vec{p}_{n\perp}-\vec{k}_{n\perp})\tilde{\gamma}_\pi(\vec{p}_{\pi\perp}-\vec{k}_{\pi\perp}) \tag{3.8}$$

and the transition matrix element (compare Fig. 4b)

$$\langle f|\mathscr{M}|i\rangle = -\langle\mathring{p}|\mathscr{M}|\mathring{p}\rangle\varepsilon_{p'}+\sum\varepsilon_p\langle\mathring{n\pi}|\mathscr{M}|\mathring{n\pi}\rangle. \tag{3.9}$$

Our calculations are being performed to first order in ε, hence, to this order, the amplitudes $\hat{\gamma}$ of (3.8) can be taken as realistic amplitudes of elastic scattering of protons, neutrons and pions on a given target. This is so because the difference between physical and bare particles is taken to first order in ε.

In order to test this model we have computed [8] the total and two differential cross-sections $\left(\frac{d\sigma}{dt}, \frac{d\sigma}{dM_{\pi n}}\right)$ for $p+p \to \pi+n+p$. The results when compared with the existing experimental data look very encouraging (compare Figs 3 and 4 and Tables I and II of Ref. [8]). Some transverse and longitudinal correlations had to be introduced there in form of formfactors.

These first results suggest that if we follow the technique of QED outlined above we have to correct quite definitely for correlations between the subunits in the transverse direction. It is in fact very amusing that the diffractive production cross-sections are probing the hadronic wave functions at small distances. It is not clear whether we have to introduce correlations in the longitudinal direction. The problem of existence of correlations of the longitudinal degrees of freedom of bare $n\pi$ in the fluctuation is complicated because it overlaps with the problem of the off shell (off-mass-shell, off-energy-shell) propagation of the fluctuation subunits [8]. The degree of such overlap is not yet well understood and the results of correcting transition matrix elements for such two physically different effects are similar, hence difficult to distinguish experimentally. This problem demands further careful analysis. It may become of nonnegligible importance in the case of diffractive production on nuclei (see the next Section).

As we have already said our choice of the subunits of the proton was based on the assumption of analogy between hadronic and QED diffractive processes. But one can make some other choices. For instance recently a different suggestion was made by Drell and Lee [9]. They construct the wave function of a physical nucleon from just two subunits

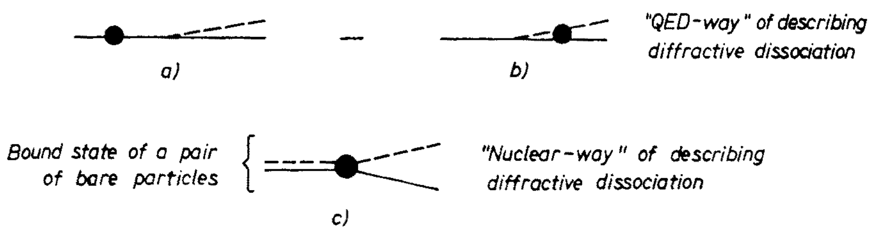


Fig. 5

(in first approximation): a bare nucleon and a bare meson. Without going into the other differences between the two models, just this first assumption differs substantially from our description which (in first approximation) assumes three subunits: two bare nucleons and a bare meson (see the expansion of the physical proton state, given above). This difference introduces some essential differences in the structure of the transition matrix element: In our model there are important destructive interferences between the contributions a) and b) (see Fig. 5), instead, if we use the wave function as proposed in Ref. [9], we have only one process contributing to the diffractive production (see Fig. 5c)) — just as in *e.g.* deuteron dissociation on a nucleus. One might say that our way of ascribing structure to a nucleon is a "QED-way", the way of Ref. [9] one might call the "nuclear (or atomic) way".

Note that if we could neglect the process a) shown in Fig. 5) we would not, in general, be able to distinguish between b) and c) since a formfactor in b) can imitate the wave function of c).

4. *Diffractive dissociation on composite targets*

There exist many measurements of diffractive dissociation on nuclear targets. Beusch and Lubatti are going to discuss them in their lectures. Theory of such processes has not been very developed, however. There were very few attempts to relate such processes to the "elementary act" of diffractive production on one nucleon. In this lecture I shall outline one such possibility without giving, however, any numerical results because they are not ready yet.

One can imagine many processes to happen during the diffractive dissociation of the incident hadron on a nucleus but we shall limit ourselves to a combination of just two

a. multiple re-scattering of a fluctuation

b. a possible multiple excitations and deexcitations of the fluctuation subunits during their travelling through the nuclear matter.

These two processes are depicted in Fig. 6 and 7.

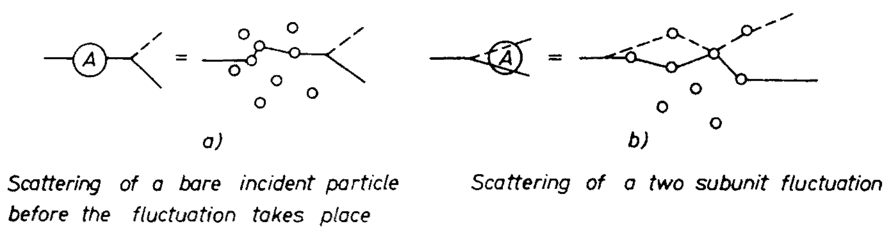


Fig. 6

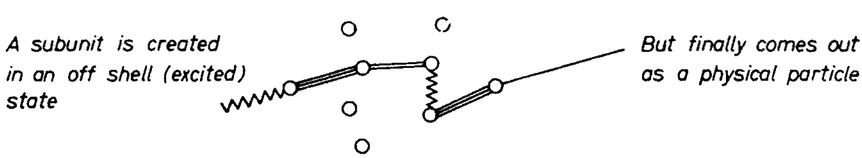


Fig. 7

The process shown in Fig. 7 one may call "dressing-up" a subunit which is born "bare" in a fluctuation. This process may not influence appreciably the process but it would seem to be reasonable to at least keep in mind a process which has to supplement the physical picture of our model which is based on the concept of bare subunits which eventually become real particles.

Let us discuss first the process *a*. It is convenient to describe the structure of the target nucleus in terms of its ground state wave function (we shall not consider any excitations of the target nucleus) which is usually given in the position space. Hence we shall use the fluctuation wave functions also in the position space representation — like in formula (3.5) which is a good starting point for constructing nuclear amplitudes. For simplicity sake let us limit ourselves to the two subunit fluctuations (as in the process considered in the previous Section). We proceed as Eq. (3.5) tells us to: we construct a profile, sandwich it between the initial and final state wave functions and take a two dimensional Fourier transform over the CM coordinate. The profile is constructed as in Ref. [10]. We shall limit ourselves to the simplest possible form of the ground state nuclear wave function:

$$\Psi(\vec{r}_1 \dots \vec{r}_A) = \prod_{j=1}^{A} \varphi_j(\vec{r}_j)$$

and then construct from it a two-dimensional density distribution of nuclear matter $\prod_{j=1}^{A} \varrho_f(\vec{s}_j)$, where

$$\varrho_j(\vec{s}_j) = \int_{-\infty}^{+\infty} dz_j |\varphi_j(\vec{s}_j, z_j)|^2.$$

Let $\vec{s}_1, \vec{s}_2$ be the transverse position vectors of the two subunits of the fluctuation. The total profile operator [10] with respect to the fluctuation subunits is:

$$\Gamma(\vec{R}, \vec{s}_1, \vec{s}_2) = 1 - \prod_{j=1}^{A} \{1 - \int d^2 s_j \varrho_j(s_j) [\gamma_{j1}(\vec{R}-\vec{s}_1+\vec{s}_j) + \\ + \gamma_{j2}(\vec{R}-\vec{s}_2+\vec{s}_j) - \gamma_{j1}(\vec{R}-\vec{s}_1+\vec{s}_j)\gamma_{j2}(\vec{R}-\vec{s}_2+\vec{s}_j)]\} \tag{4.1}$$

where $\vec{R}$ is the transverse distance between the CM's of the two colliding objects and γ_{jk} is the profile of j-th nucleon — k-th subunit elastic scattering (in position space). Since $\vec{s}_1 = \eta_2 \vec{s}$, $\vec{s}_2 = \eta_1 \vec{s}$, we have the following expression for the amplitude on a nuclear target:

$$\langle f|\mathscr{M}|i\rangle_{(b)} \sim \int d^2 R e^{i\vec{\Delta}\cdot\vec{R}} \int d^2 s e^{-i\eta_2 \vec{k}_1 \cdot \vec{s} + i\eta_1 \vec{k}_2 \cdot \vec{s}} \times \\ \times \Gamma(\vec{R}, -\eta_2 \vec{s}, +\eta_1 \vec{s}) U'^* \Phi(\vec{s}, \eta_2 - \eta_1) U, \tag{4.2}$$

where, by inserting spinors, we have symbolically marked a possible spin dependence of the fluctuation. The multiple scattering from nucleons in nucleus we took as spin independent — just for simplicity sake. There is also a contribution from the second process shown in Fig. 6a)

$$\langle f|\mathscr{M}|i\rangle_{(a)} \sim \int d^2 s e^{-i\eta_2 \vec{k}_1 \cdot \vec{s} + i\eta_1 \vec{k}_2 \cdot \vec{s}} e^{-i\vec{\Delta}(\eta_2 - \eta_1)\cdot \vec{s}} U'^* \Phi(\vec{s}, \eta_2 - \eta_1) U \times \\ \times \int d^2 R e^{i\vec{\Delta}\cdot\vec{R}} \Gamma(R) \tag{4.3}$$

where $\Gamma(R) = 1 - \prod_{j=1}^{A} (1 - \int d^2 s_j \varrho_j(s_j) \gamma_j(\vec{R} - \vec{s}_j))$, γ_j's are the profiles of j-th nucleon — incident particle. The complete amplitude is the difference of (4.2) and (4.3) (compare *e.g.* (3.9))

$$\langle f|\mathcal{M}|i\rangle = -\langle f|\mathcal{M}|i\rangle_{(a)} + \langle f|\mathcal{M}|i\rangle_{(b)}.$$

It is interesting to see how the diffractively produced system gets attenuated in nuclear targets. The experiment shows (compare talks given by Beusch and Lubatti) that such systems show much lower attenuation that the one one would get by just adding the attenuations of the components. From our formulae (4.2)–(4.4) one can see qualitatively the effect of "diffusion" of the fluctuation components as depicted in Fig. 6b). The relative spatial extensions of the fluctuation wave functions are probably the most important factors in determining the size of the attenuation effect. Let us consider two limiting cases

(*i*) the wave function Φ is a pointlike object

(*ii*) the wave function Φ is spatially much more extended than the target nucleus.

In the first case we get appreciable attenuation: each nucleon is hit by both components and the cross-section of the diffractively produced object on one nucleon in approximately given by a sum of individual cross-section, reduced slightly by "screening" (the last term of (4.1)). In the second case however there will be virtually no overlap between $\gamma_{j1}(\vec{R} - \vec{s}_1 + \vec{s}_j)$ and $\gamma_{j2}(\vec{R} - \vec{s}_2 + \vec{s}_j)$ because $\langle |\vec{s}| \rangle_{av} \gg \langle |\vec{s}_j| \rangle_{av}$. So, we have an approximate equality

$$1 - [1 - (\bar{\gamma}_1(\vec{R} - \vec{s}_1) + \bar{\gamma}_2(\vec{R} - \vec{s}_2) - \overline{\gamma_1 \gamma_2})]^A \approx$$
$$\approx 1 - (1 - \bar{\gamma}_1(\vec{R} - \vec{s}_1))^A + 1 - (1 - \bar{\gamma}_2(\vec{R} - \vec{s}_2))^A$$

where

$$\bar{\gamma}_{1,2}(\vec{R} - \vec{s}_{1,2}) = \int d^2 s_j \varrho_j(s_j) \gamma_j(\vec{R} - \vec{s}_{1,2} + \vec{s}_j)$$

(which we assume to be j-independent). So, we finally get

$$\langle f|\mathcal{M}|i\rangle \sim \int d^2 R d^2 s e^{i\vec{\Delta}\vec{R}} \{-\Gamma(R) + \Gamma_1(R) + \Gamma_2(R)\} \times$$
$$\times e^{-i\eta_2 \vec{k}_1 \cdot \vec{s} + i\eta_1 \vec{k}_2 \cdot \vec{s}} U'^* \Phi(\vec{s}, \eta_2 - \eta_1) U,$$

where $\Gamma, \Gamma_1, \Gamma_2$ are single particle profiles (as defined below (4.3)) of the incoming and outgoing particles. Now, the single particle profiles are, typically, of the form $1 - e^{-\frac{1}{2}\sigma T(R)}$ and a two particle profile of the same type is $1 - e^{-\frac{1}{2}(\sigma_1 + \sigma_2) T(R)}$, where $T(R)$ is the density of the nuclear target at the transverse distance R. The single particle profiles produce weaker attenuation than multi-particle profiles. Hence the case (*ii*) represents weaker attenuation than the case (*i*). So, when a diffractively produced system diffuses through the target nucleus as shown in Fig. 6b) its absorption is weaker than if all the subunits travel in one close pack.

To illustrate how dramatically the attenuation is diminished in the case (*ii*) let us assume that all three single particle profiles in (4.4) are the same

$$\Gamma(R) = \Gamma_1(R) = \Gamma_2(R) = 1 - e^{-\frac{1}{2}\sigma T(R)}.$$

Then, the total profile of the process is

$$-\Gamma(R)+\Gamma_1(R)+\Gamma_2(R) = 1-e^{-\frac{1}{2}\sigma T(R)},$$

and the total cross-section of the diffractively produced object on one nucleon of the target is zero. This is so because, by definition (see *e.g.* [11]), it is extracted from the experiment by the following parametrization of the profile:

$$\Gamma'(R) = e^{-\frac{1}{2}\sigma_2 T(R)}-e^{-\frac{1}{2}\sigma T(R)}$$

where σ_2 is interpreted as such a total cross-section.

Another factor increasing transparency of a nucleus is the process depicted in Fig. 7. This process should further lower the single particle cross-sections. One can describe it as it was (in a different context) proposed by Van Hove [2]. There, one can find general arguments showing that a particle jumping in subsequent collisions between various excited state increases its penetrability in comparison with a particle which does not. Such processes should be more important in diffractive production than in elastic scattering because in the former the particles are likely to be born in an "off-shell state". Although this process does not seem, at first, to fit to our scheme because our assumption (*ii*) of Section 2 said that the subunit scatter only elastically, one can remedy the situation by employing the technique suggested by Van Hove: one chooses such combinations of the excited states which do scatter back into themselves, hence the technique of handling multiple elastic scattering can be used also in this case.

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