COHERENT NUCLEAR PRODUCTION OF MULTI-BODY STATES*

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This is a review of the theory of coherent nuclear production of multi-body states having a broad mass spectrum. Two kinds of phenomena are of central interest: the production of broad enhancements such as A_1 and Q, and inclusive spectra in very high energy hadron-nucleus collisions. The major topics discussed are: 1) the very interesting and informative inclusive spectra that are expected to result from nuclear collisions if diffraction dissociation plays a major role in the underlying hadron-hadron collisions; 2) a theory of coherent production that allows for strong coupling between the elastic and coherent production channels, the longitudinal momentum transfer, and changes of mass resulting from successive diffractive collisions within the nucleus; 3) Van Hove's model for explaining the astonishingly small total cross-sections that have been extracted from multi-boson production experiments, and the questions raised thereby concerning the structure of the amplitudes that describe the scattering of the produced states by a system of nucleons.

1. Introduction

There are at least two motivations for studying the coherent nuclear production of states lying in a broad mass continuum:

1. the astonishingly large mean free-paths of 3π, 5π, and $K\pi\pi$ systems observed in nuclear production experiments [1];
2. the possibility that at very high energies a major portion of all inelastic channels in hadron-hadron collisions are diffractively produced [2], [3].

In connection with the puzzlingly small cross-sections that the multi-boson enhancements appear to have, Van Hove [4] has proposed a theory in which transitions within the mass continuum play an essential role. In the conventional (Glauber) analysis of these production processes, the optical potential is assumed to be diagonal in mass, except, of course, for the single and small element that describes production. In Van Hove's approach, on the other hand, the optical potential is an off-diagonal and continuous matrix in mass space, without any singular element $\delta(m-m')$.

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Turning to the diffraction excitation model, we recall (see Fig. 1) that its basic assumption is that in an NN interaction there are three underlying s asymptotic mechanisms: where the ingoing N in unexcited while the target N is excited (process I); where the target is unexcited and the observed N is a product of a fireball decay (process II); and where

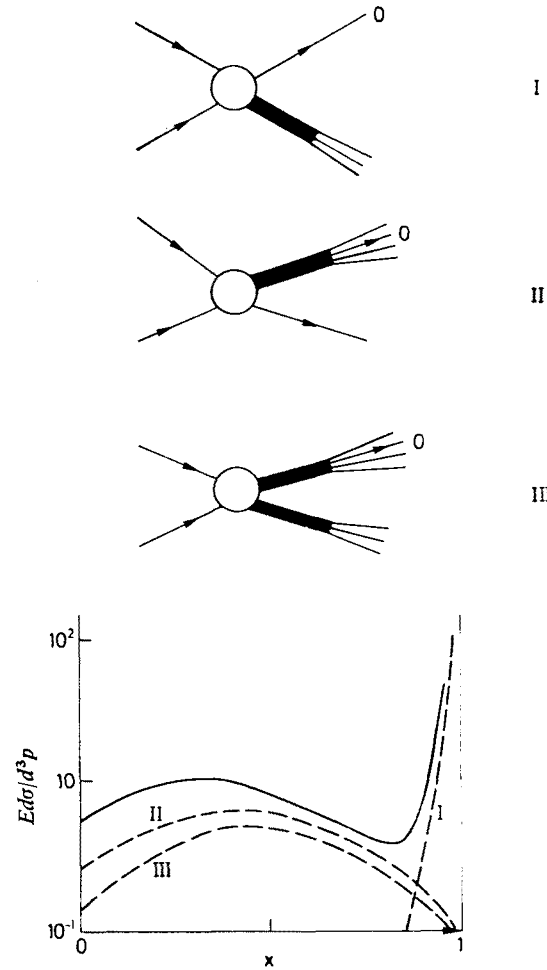

Fig. 1. The various contributions to the inelastic cross-section in the diffraction-excitation model. The observed particle is designated by 0. The invariant cross-section $Ed\sigma/d^3p$ in (mb/GeV^2) is sketched for $pp \to pX$ at fixed angle and as a function of x; the separate contributions of diagrams I, II and III are shown as dashed curves, and the full curve is their sum. For further details, see Ref. [3]

both N's become excited (process III). All these excitation processes are assumed to be diffractive. We recall that process I leads to particles crowded into the immediate neighbourhood of $x = 1$, whereas processes II and III lead to broad spectra in x. If this description of inelastic processes is correct, then of these three portions of all the inelastic channels those, and only those that arise from process II can be coherent in nuclear production, for it is only in this category of collisions that the particle in the nucleus is unexcited [3]. The qualitative aspects of the proton spectrum in nuclear scattering would therefore differ markedly from the ISR observations, and the same would hold true when pion spectra from πp and π-nucleus collisions become available at NAL.

Finally, a remark that could be of practical importance. If diffraction dissociation is really such an important contribution to multiple production, then at very high energy accelerators secondaıy pion beams produced in large A materials will be far more copious than one would infer from nucleon-nucleon collisions.

2. *Inclusive proton spectra from proton-nucleus collisions*

A qualitative understanding of the inclusive spectra that one would expect in nuclear collisions can be easily attained if one bears in mind the following essential points:

(*i*) the Feynman x of a particle of mass m emerging from a fireball of mass M in process II is $x \approx m/M$;

(*ii*) the observed $p_\perp$ distribution in processes II and III results from a convolution of the $p_\perp$ distribution in the initial fireball production step, and the subsequent decay of the fireball;

(*iii*) a nuclear production amplitude can only be coherent over a nucleus of radius R, if $R\sqrt{t_{\min}} \lesssim 1$, *i.e.*, if

$$(M^2 - M_0^2) \lesssim \frac{2p_L}{R}, \tag{2.1}$$

where M_0 and p_L are the mass and lab. momentum of the incident particle.

Consider first an experiment where the target nucleus in known to be in its ground state after the collision, as in the streamer chamber experiment planned at NAL [5]. There is then no incoherent background, and processes I and III are completely suppressed. On the other hand, II is coherent, and therefore

$$\left(E\frac{d\sigma}{d^3p}\right)_{\text{Nucleus}} \gg A\left(E\frac{d\sigma_{\text{II}}}{d^3p}\right)_{\text{Hydrogen}}.$$

In view of the observations listed above, the $(x, p_\perp)$ distribution in such a nuclear experiment will have further characteristic features. Because of (*iii*), the range of masses M that can be coherently produced is limited, and from (*i*) it then follows that only the values of x that exceed

$$x_0 \approx m\sqrt{\frac{R}{2p_L}} \tag{2.2}$$

will show the enhancement. (Here we assumed $(2p_L/R)^{\frac{1}{2}} \gg M_0$.) In the reaction $pA \to pA +$ anything, the $p_\perp$ distribution will narrow appreciably because of (*ii*), which implies that the initial fireball production step in a coherent nuclear process is restricted to transverse momenta that are far smaller than in a collision with a free proton. In such a nuclear experiment, one should therefore see the considerably narrower $p_\perp$ distribution implied by the fireball decay alone. These qualitative arguments are summarized in Fig. 2.

Experiments on heavy nuclei, where one cannot ensure that the nucleus remains in its ground state, are also of interest. Here the coherent enhancement of II *vs* I and III can be amplified enormously by going to large A. Furthermore, the larger range of radii that

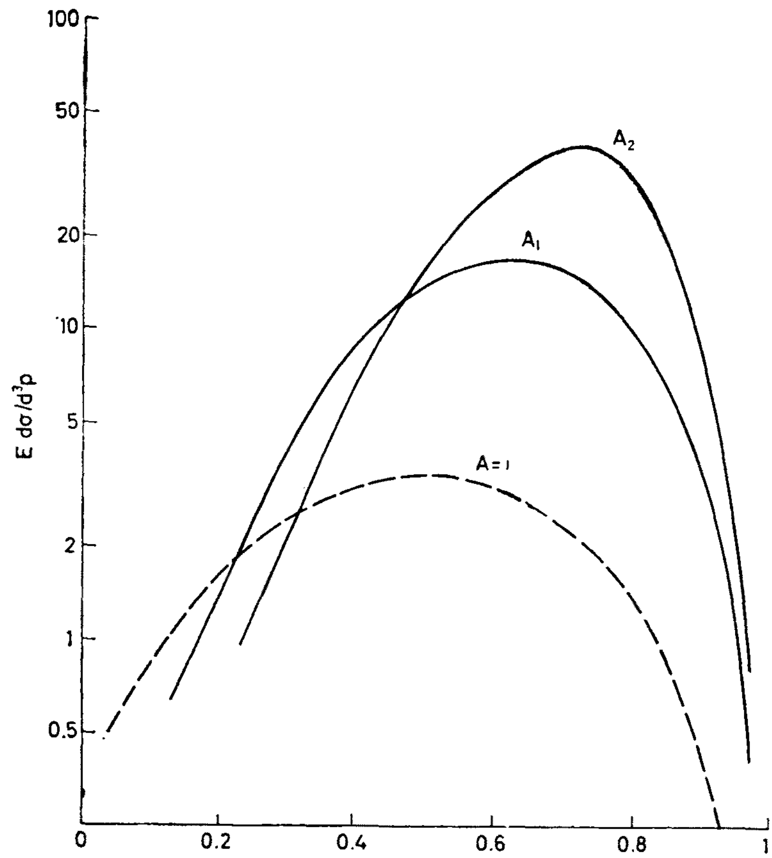

Fig. 2. The inclusive cross-section in the reaction $pA \to pAX$, where A is a nucleus which is known to remain in its ground state. The dashed curve is process II in hydrogen; the solid curves show two nuclei with $A_2 > A_1$. It should be stressed that the curves for $A \neq 1$ are not based on calculations; they are merely intended to illustrate the arguments in the text

then become available should, according to (2.2), lead to an A dependence of the shape of the inclusive x distribution. Offsetting these attractive possibilities is the danger that experiments in heavy nuclei may be difficult to interpret precisely. There are at least two reasons for this: 1) the incoherent background could be large due to cascading inside the nucleus; and 2) the coherent rescattering of the diffractively produced states is determined by amplitudes concerning which we have no knowledge at present. This last point shall be discussed at length in Section 5.

These reservations need not mean that the inclusive spectra from scattering by complex nuclei will never teach us anything concerning the basic processes. Rather, what will be required is a well-planned experimental program. Presumably measurements on heavy nuclei, being the easiest to execute, would come first. For large A, and medium x, the incoherent background should be tolerable, and it should therefore be possible to verify whether the qualitative features as sketched above are correct. Really precise interpretation will require measurements of inclusive spectra from deuterium in the single and double scattering region, with the final deuteron detected [6], and similar measurements on light nuclei such as He [5].

To summarize, such an experimental program would provide the following information.

1. The experiments in heavy nuclei would show whether the $x \simeq 1$ spike observed at the ISR is diffractive, because if it is not, its counterpart in the $x < 0$ hemisphere, process II, need not be coherent over the nucleus; such experiments would also provide a rough check on the mass spectrum assumed in the diffraction excitation models.

2. The small $p_\perp$ experiments on the lightest nuclei, where single scattering dominates, would permit one to disentangle the contribution to the $(x, p_\perp)$ distribution arising from fireball production, and from fireball decay.

3. Coherent scattering from deuterium in the double scattering region would determine the amplitudes for rescattering of diffractively produced states by nucleons, just as ϱ^0 photoproduction in the same region provided invaluable information concerning the $\varrho^0 N$ amplitude [6].

3. Propagation of a mass continuum in nuclear matter

From the foregoing discussion, it is clear that a detailed investigation of the ideas just sketched requires a theory of coherent production which:

1) predicts the production amplitude for an exceedingly broad mass continuum, and allows for mass changes within this continuum as the wave propagates through the nucleus;

2) accounts for the minimum momentum transfer given to the nucleus; and

3) treats the coupling between the discrete incoming state, and the mass continuum non-perturbatively.

Van Hove [4] has already developed an elegant formalism that handles point 1); however, he was able to ignore points 2) and 3) because he was not concerned with copious production of states of exceedingly high mass. The formalism described below is therefore a fairly straightforward generalization of Van Hove's designed to include the minimum momentum transfer and the possibly strong over-all coupling between the nucleon and the N^* continuum [7]. Unfortunately no explicit calculations have as yet been done with this more general theory, and the only concrete results are those of Van Hove.

The incoming particle, and the states into which it may dissociate, are described by the wave function Ψ. In general the dissociated system will have a number of internal variables, but we assume that the evolution of the state can be adequately described solely in terms of the mass. Ψ is then a column vector in mass space, having a discrete component at m_0, the nucleon say, and a continuum for $m > m_1$. Let ν be the total energy, M the diagonal mass matrix, and $-2i\nu W$ the optical potential, which need be neither Hermitian nor anti-Hermitian. For simplicity, we shall, however, assume that W is a real, symmetric, positive-definite, and ν independent operator. This corresponds to a purely absorptive medium where all eigen-mean-free-paths are energy independent when $\nu \to \infty$.

In nuclear matter the wave equation then reads

$$(\nabla^2+\nu^2-M^2-2i\nu\, W)\,\Psi = 0. \tag{3.1}$$

We reduce this to a one-dimensional problem with the usual high energy ansatz

$$\Psi = e^{i\nu z}\Phi,$$

where Φ is a slowly varying function of z, the co-ordinate along the incident direction. Then (3.1) becomes

$$i\frac{d\Phi}{dz} = H\Phi; \tag{3.2}$$

that is to say, it looks like a Schrödinger equation with the non-Hermitian Hamiltonian

$$H = \frac{M^2}{2\nu} - iW. \tag{3.3}$$

As we see, M plays the role of the momentum, ν that of mass, and z that of time.

Equation (3.2) only applies inside the nucleus, which for our purpose is the slab $0 < z < L$, where $L = 2\sqrt{R^2-b^2}$, R being the nuclear radius and b the impact parameter. For $z < 0$, and $z > L$, $W = 0$. What we therefore seek is a solution of (3.2) for $z > 0$ that conforms to the initial condition that at $z = 0$ the amplitude for mass m_0 be one, and that for all $m \geqslant m_1$ be zero:

$$\begin{aligned}\Phi(mz) \to \Phi(m0) &= 1 \;\; (m = m_0).\\ &= 0 \;\; (m \geqslant m_1)\end{aligned} \tag{3.4}$$

The imposition of such an initial condition is most conveniently done in terms of Green's function belonging to the "stationary state" equation associated with the "time" (actually $z-$) dependent Schrödinger equation, (3.2). This Green's function satisfies the equation

$$(\omega-H)\,\mathcal{G}(\omega) = 1, \tag{3.5}$$

where ω is a complex variable.

The notation of (3.5), although convenient, is deceptively compact. It is best to separate explicitly the discrete proton state at $m = m_0$ from the continuum $m \geqslant m_1$. That is, we write $\mathscr{G}$ and W in matrix form

$$\mathscr{G}(\omega) = \begin{pmatrix} \mathscr{G}_{00}(\omega) & \mathscr{G}_{0m'}(\omega) \\ \mathscr{G}_{m0}(\omega) & \mathscr{G}_{mm'}(\omega) \end{pmatrix}, \tag{3.6}$$

$$W = \begin{pmatrix} \alpha & w(m') \\ w(m) & U(m, m') \end{pmatrix}. \tag{3.7}$$

The subscript 0 in (3.6) refers to the discrete state, while in (3.7) $1/2\alpha$ is the proton's mean--free-path in the absence of coherent coupling (*i.e.*, $2\alpha = n\,\sigma_T$), and $w(m)$ is proportional to the amplitude for producing a state of mass m when a proton strikes a free nucleon.

We now demonstrate that the desired solution can be written conveniently in terms of the functions defined in (3.6). Define

$$\Xi(m_0 z) = \frac{i}{2\pi}\int_C d\omega e^{-i\omega z}\mathscr{G}_{00}(\omega), \tag{3.8}$$

$$\Xi(mz) = \frac{i}{2\pi}\int_C d\omega e^{-i\omega z}\mathscr{G}_{m0}(\omega), \tag{3.9}$$

where C lies just above the real axis of the ω plane. Using (3.5) we immediately find that

$$\left(i\frac{d}{dz} - H\right)\Xi(m_0 z) = i\delta(z), \tag{3.10}$$

$$\left(i\frac{d}{dz} - H\right)\Xi(mz) = 0. \tag{3.11}$$

Thus Ξ is certainly a solution for $z > 0$. Furthermore, it vanishes for $z < 0$ because in the right-hand side of (3.8) and (3.9) C can then be closed in the upper half-plane, for $\mathscr{G}(\omega)$ cannot have any singularities there as they would correspond to exponentially growing solutions (more on this below). Thus $\Xi(mz)$, for $m \geqslant m_1$, is the desired solution $\Phi(mz)$, whereas $\Xi(m_0 z)$ is related to the solution by

$$\Xi(m_0 z) = \theta(z)\Phi(m_0 z). \tag{3.12}$$

That $\Phi(m_0 z)$ has the required unit amplitude at $z = 0$ is assured by the coefficient of $\delta(z)$ in (3.10).

Having established the relationship between Ξ and the sought-after wave function, we shall henceforth replace Ξ by Φ as we are only concerned with values of z inside the nucleus where $\Xi \equiv \Phi$.

Next we show that a solution of the continuum problem ($m \geqslant m_1$) by itself provides a complete solution of the problem including the discrete state. For this purpose we note that the "differential" equation (3.5) is equivalent to the integral form

$$\mathscr{G} = G - iG\tilde{W}\mathscr{G}. \tag{3.13}$$

Here G is the diagonal matrix

$$G(\omega) = \begin{pmatrix} \dfrac{1}{\omega-\omega_0} & 0 \\ 0 & \dfrac{\delta(m-m')}{\omega-\omega(m)} \end{pmatrix}, \tag{3.14}$$

and

$$\omega_0 = \frac{m_0^2}{2v} - i\alpha,$$

$$\omega(m) = \frac{m^2}{2v}, \tag{3.15}$$

while $\tilde{W}$ is obtained from (3.7) by putting $\alpha = 0$. Observe that $v-\omega_0$ is just the complex momentum that would describe the elastic wave if there were no coherent coupling to other channels. The $0-0$ component of (3.13) is then

$$\mathscr{G}_{00} = \frac{1}{\omega-\omega_0} - \frac{i}{\omega-\omega_0}\int_{m_1}^{\infty} w(m)\mathscr{G}_{m0}dm. \tag{3.16}$$

Next we write the $m-0$ element of (3.5):

$$[\omega-\omega(m)]\mathscr{G}_{m0} + i\int_{m_1}^{\infty} U(m, m')\mathscr{G}_{m'0}dm' = -iw(m)\mathscr{G}_{00}. \tag{3.17}$$

Finally we introduce Green's function $D(mm'; \omega)$ of the continuum alone:

$$[\omega-\omega(m)]D(m, m') + i\int_{m_1}^{\infty} U(m, m'')D(m'', m')dm'' = \delta(m-m'),$$

where, of course, $m, m' > m_1$. Then the solution of (3.17) is just

$$\mathscr{G}_{m0} = -i\mathscr{G}_{00}\int D(m, m')w(m')dm', \tag{3.18}$$

and from (3.16)

$$\mathscr{G}_{00} = \frac{1}{\omega-\omega_0-\int w(m)D(m, m')w(m')dmdm'}. \tag{3.19}$$

These last two expressions give the desired quantities for elastic scattering and production in terms of the solution of the continuum problem. Equation (3.19) describes the modification of the elastic wave propagation due to virtual excitation of continuum states [8]; this elastic mode is no longer described by the single pole at $\omega = \omega_0$, but has a much more complicated structure as a consequence of which the wave function is not just a single exponential. For large depths z the propagation will be described by the singularity closest to Im $\omega = 0$.

4. *Eigenfunctions of the effective Hamiltonian*

We now turn to the effective Hamiltonian (3.3):

$$H = \frac{M^2}{2\nu} - iW, \tag{4.1}$$

where W, it will be recalled, is real, symmetric, and positive definite.

It is of course not clear that such a non-Hermitian operator possesses a complete set of eigenfunctions, *etc.*, and no general investigation of this question has been attempted. Instead we shall assume the existence of eigenfunctions, and then see what properties they must possess. Then we shall examine a special W where an explicit and complete set can be constructed. Finally we shall see that from this explicit solution one can infer the existence of a complete set under conditions that, for all practical purposes, are really very general.

Let ε_n and $\varphi_n(m)$ be the eigenvalues and eigenfunctions of H. As W is symmetric, the right and left eigenfunctions of H are identical, and the orthogonality condition is [1]

$$\int dm \varphi_n(m)\varphi_l(m) = 0 \tag{4.2}$$

for eigenfunctions belonging to different eigenvalues. As we shall see, in general H has both a discrete and continuous spectrum; for wave functions in the discrete spectrum we shall therefore use the norm

$$\int [\varphi_n(m)]^2 dm = 1. \tag{4.3}$$

Observe that there are no complex conjugates in these equations — this is not a misprint.

It is now a simple exercise to show that

$$\mathrm{Re}\, \varepsilon_n = \left\langle \frac{M^2}{2\nu} \right\rangle_n, \qquad \mathrm{Im}\, \varepsilon_n = -\langle W \rangle_n, \tag{4.4}$$

where for any operator O the expectation value is defined as

$$\langle O \rangle_n \equiv \frac{\int \varphi_n^*(m) O(m, m')\varphi_n(m')dm dm'}{\int \varphi_n^*(m)\varphi_n(m)dm}. \tag{4.5}$$

Once more, there is no misprint concerning the complex conjugates. Equation (4.4) shows that the eigenvalues all lie in the lower half plane; we shall actually find that for the continuous spectrum $\mathrm{Im}\ \varepsilon_n = 0$, but this is quite consistent with (4.4) and (4.5) because the denominator is then infinite.

Green's functions of the preceding section can be written in terms of the φ_n provided they form a complete set; for example,

$$\mathscr{G}_{m0}(\omega) = \sum_n \frac{\varphi_n(m)\varphi_n(m_0)}{\omega - \varepsilon_n}. \tag{4.6}$$

Observe that $\mathscr{G}(\omega)$ is analytic for $\mathrm{Im}\ \omega > 0$ because of (4.4).

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[1] Note that our integrals over m are a shorthand for a single term at $m = m_0$, and an integral over $m > m_1$.

As we saw in the argument leading to (3.18) and (3.19), we actually only need to concern ourselves with the continuum subspace $m \geqslant m_1$. That is to say, it is not really the eigenfunctions of H that concern us, but of its projection onto the continuum subspace. Calling these $\psi_n(m)$, and their eigenvalues μ_n, we are therefore interested in the equation

$$[\mu_n - \omega(m)]\psi_n(m) + i \int_{m_1}^{\infty} U(m, m')\psi_n(m')dm' = 0, \tag{4.7}$$

where $m, m' \geqslant m_1$ throughout. Green's function D introduced above is then

$$D(m, m'; \omega) = \sum_n \frac{\psi_n(m)\psi_n(m')}{\omega - \mu_n}, \tag{4.8}$$

provided the set ψ_n is complete. The real and imaginary parts of μ_n are given by equations analogous to (4.4).

It is very instructive to have at least one explicit solution of a problem of type (4.7). For that reason we consider the case of a separable kernel

$$U(m, m') = gu(m)u(m'). \tag{4.9}$$

Aside from rendering (4.7) soluble, this optical potential automatically obeys the threshold conditions if $u(m) = 0$ for $m < m_1$.

We begin by showing that despite the anti-Hermitian interaction iU, there is a continuum of solutions with real eigenvalues $\varepsilon \equiv p^2/2v$, where $p \geqslant m_1$. Write (4.7) as follows:

$$(p^2 - m^2)\psi_p(m) + 2ivgu(m) \int u(m')\psi_p(m')dm' = 0. \tag{4.10}$$

Then the solutions with real p are

$$\psi_p^{\pm}(m) = \delta(m-p) - \frac{2ivgu(p)u(m)}{p^2 - m^2 \pm i\delta} \cdot \frac{1}{1 + 2ivgJ(p^2 \pm i\delta)}, \tag{4.11}$$

where

$$J(\omega) = \int \frac{[u(m)]^2}{\omega - m^2} dm. \tag{4.12}$$

These are like the familiar "in" and "out" states of scattering theory. There are also discrete states if

$$1 + 2ivg\, J(\omega) = 0 \tag{4.13}$$

has roots that do not lie on the real axis $m_1 \leqslant \operatorname{Re} \omega < \infty$. In virtue of (4.4) these roots must lie in the lower half plane (at ε_b, say) because they correspond to the normalizable eigenfunctions

$$\psi_b(m) = \frac{Nu(m)}{\dfrac{m^2}{2v} - \varepsilon_b}. \tag{4.14}$$

The orthogonality relation for the continuum states is

$$\int dm \psi_p^+(m)\psi_{p'}^-(m) = \delta(p-p'), \tag{4.15}$$

and one can show explicitly that the continuum states, together with the bound states, form a complete set:

$$\delta(m-m') = \sum_b \psi_b(m)\psi_b(m') + \int_{m_1}^{\infty} dp \psi_p^+(m)\psi_p^-(m'). \tag{4.16}$$

Equations (4.15) and (4.16) have the same appearance as in the familiar Hermitian problem, but it must be remembered that this is somewhat deceptive because $\psi^- \neq (\psi^+)^*$.

The separable kernel (4.9) certainly provides a wholly inadequate description of the underlying scattering process. Nevertheless, it is a very useful beginning, because it can easily be generalized to

$$U(m, m') = \sum_i g_i u_i(m) u_i(m'). \tag{4.17}$$

With the help of a simple matrix notation, H with this kernel can be diagonalized as before, and completeness established. Thus the problem is again reduced to quadratures, which is very convenient for numerical computations. Furthermore, if all the $u_i(m)$ vanish for $m < m_1$, the threshold condition is satisfied, which is not the case with most kernels that may, for other reasons, be convenient (see Eq. (5.4), *et seq.*). Finally, by a judicious choice of the u_i one can favour transitions with m close to m'. For all these reasons it is likely that the form (4.17) will provide the most useful kernel in the applications sketched in Section 2.

5. *Van Hove's model; the mass dependence of the optical potential*

Van Hove's model [4] describes the production of a continuum state with a broad but reasonably well-confined mass spread, such as A_1 or Q. In the language of Eq. (3.7), the production amplitude $w(m)$ is restricted to this mass range, and the optical potential $U(m, m')$ permits transitions only within this range. Concerning the potential U, Van Hove argues that it is a smooth function of m and m'; he then shows that this leads to an apparent increase of the over-all absorption length of the produced state in nuclear matter, in accordance with the observations cited in Ref. [1]. This smoothness assumption is rather controversial, and involves some interesting physical concepts. The major purpose of this section is an examination of precisely this point. But before doing so, we must dispense with a few technical details; in particular, we must show how Van Hove's results emerge from our formalism, and describe his solution of the coupled-mode propagation problem.

By hypothesis, the produced states that we are now concerned with are only a minor portion of all inelastic channels. Therefore w can be treated to lowest order, and $\mathscr{G}_{00}$ in (3.18) may be replaced by $(\omega-\omega_0)^{-1}$. If we then use the eigenfunction expansion (4.8) of Green's function D, and carry out the integration as specified in (3.9), we readily find that the wave function of the produced state is

$$\Phi(mz) = \sum_n c_n \frac{e^{-i\omega_0 z} - e^{-i\mu_n z}}{\omega_0 - \mu_n} \psi_n(m), \tag{5.1}$$

with

$$c_n = \int_{m_1}^{\infty} w(m)\psi_n(m)dm. \tag{5.2}$$

This is Van Hove's result except that his eigenvalues and eigenfunctions are not our μ_n and ψ_n because he goes to the $\nu \to \infty$ limit, in which case $\omega(m) \equiv m^2/2\nu$ disappears from (4.7). That is to say, (5.1) does take the minimum momentum transfer into account. For the problems of primary concern to Van Hove, such as $\pi \to A_1$ production, the $\nu \to \infty$ approximation is probably not essential, because even at present energies $2\nu/(m_{A_1}^2 - m_\pi^2)$ is comparable to nuclear radii. On the other hand, one should note that the $\nu \to \infty$ limit changes the nature of the eigenfunction problem in a rather fundamental way. The finite ν equation, (4.7), looks like a Schrödinger equation with kinetic and potential energy terms, whereas the $\nu \to \infty$ equation describes, so to say, an infinitely heavy particle which only has potential energy.

In the $\nu \to \infty$ limit $\omega_0 \to i\alpha$ [*cf.*, (3.15)], and the eigenvalues μ_n become pure imaginary. For that reason we replace them by $-i\lambda_n$, the latter being defined by the $\nu \to \infty$ limit of (4.7):

$$\lambda_n \psi_n(m) = \int_{m_1}^{\infty} U(m, m')\psi_n(m')dm'. \tag{5.3}$$

As Van Hove is only interested in an exploratory investigation, and not with detailed fits to the data, he chooses a form of U which permits an analytic solution of (5.3), namely

$$U(m, m') = \Lambda(|m - m'|). \tag{5.4}$$

Equation (5.3) is then solvable by Fourier inversion provided we relax the threshold constraint $m \geqslant m_1$, and replace m_1 by $-\infty$. This is also not a crucial approximation, because in processes such as $\pi \to A_1$, one can suppress low masses by making the production amplitude $w(m)$ small (or even vanishing) for $m \lesssim m_1 + \mu$, where μ is the width of the mass mixing matrix Λ. There are, however, more fundamental questions concerning (5.4) to which we shall come presently.

It is now a simple matter to solve (5.3). We introduce a continuous variable η conjugate to m, $-\infty < \eta < \infty$. Recall that m plays the role of a momentum in the Schrödinger-like equation, and therefore η is the conjugate co-ordinate. As U (*cf.*, (5.4)) depends only on the difference of "momenta", it is a local interaction in η space, and the eigenfunctions, being those of an infinitely heavy particle, are δ functions in co-ordinate space, and therefore purely oscillating exponentials in the conjugate mass space. Their eigenvalues are the potential at the "point" η where the state is localized.

To put this into formulae, we introduce the eigenfunctions

$$\psi_\eta(m) = \frac{1}{2\pi} e^{i\eta m}, \tag{5.5}$$

and then their eigenvalues are

$$\lambda_\eta = \int_{-\infty}^{\infty} dm e^{-i\eta m} \Lambda(m). \tag{5.6}$$

Recall that λ_η is positive definite If Λ is a smooth function, we see that $\lambda_\eta \to 0$ as $\eta \to \infty$. This gives the astonishing result that eigenmodes (5.5) that oscillate rapidly in mass are less attenuated than those that are slowly varying. To see this more clearly, observe that if we write

$$\lambda_\eta = \tfrac{1}{2} n\sigma_{\eta N}, \tag{5.7}$$

where n is the nuclear density, then $\sigma_{\eta N}$ is the total cross-section for the scattering of mode ψ_η by nucleons. The general aspect that one would expect for λ_η is sketched in Fig. 3a, where μ is the characteristic width of the kernel Λ. Needless to say, by choosing a wiggly Λ one can have oscillations in λ_η as well.

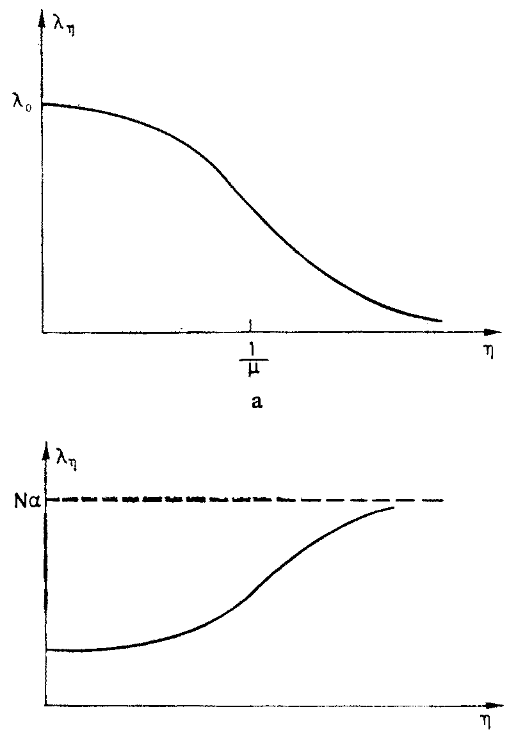

Fig. 3. The spectrum of eigenabsorptivities λ_η. Curve a is Van Hove's spectrum [4], and follows from (5.6) if Λ is a Gaussian. Curve b results from (5.21), with a Gaussian V

Van Hove has carried out a quite complete set of calculations by choosing a Gaussian form for the mass mixing kernel Λ and the production amplitude w. His conclusions can be summarized as follows:

1. because the absorption is maximal for the least rapidly varying eigenmodes ψ_η, the shape of the mass peak changes with increasing depth. As a consequence heavy nuclei should show a narrower peak than light nuclei — a most surprising conclusion;

2. if λ_0 (which is the maximum absorption coefficient) is kept fixed, and the width μ of the mass mixing matrix Λ is increased, the total coherent production cross-section increases. Hence if a wide Λ represents the true situation found in nature, a naïve single channel analysis would erroneously conclude that the nucleonic cross-section of the produced system is smaller than $\lambda_0{}^2$.

As Van Hove points out, his conclusions stem from the assumption that the optical potential is a smooth function of $m-m'$, in contrast to the forward scattering amplitude by a free nucleon which contains a term proportional to $\delta(m-m')$. This δ function is a consequence of energy conservation, and therefore only appears in the amplitude after a time interval t_0 that grows with ν, whereas the time between successive collisions within the nucleus, t_c, is ν independent[3]. On the basis of this observation Van Hove argues that the matrix $U(m, m')$, which describes transitions within the nucleus on a time scale short compared to t_0, need not have a singularity at $m = m'$.

Bell [9] has raised serious doubts concerning the smoothness of U. As a model for diffractive dissociation Bell studies the coherent nuclear scattering of a composite non-relativistic system [10] which possesses internal excitations. In actual fact, Bell only considers the special case of a non-interacting "composite", and shows that U would then have the form

$$U(m, m') = \delta(m-m')\,\tfrac{1}{2}\,n \sum_c \sigma_{cN} - V(m, m'), \tag{5.8}$$

where σ_{cN} is the constituent-nucleon total cross-section, and V is a smooth function of its arguments. (Were U to be fully described by the first term of (5.8), the composite would propagate without change of state, and with the mean-free-path appropriate to the sum of the constituent cross-sections.)

We shall now examine the arguments alluded to in the preceding two paragraphs. As interactions within the composite undergoing dissociation may well be important [4], we shall generalize Bell's model, and take the composite to be an interacting two-particle system. The internal motion of the composite will be treated non-relativistically, but the motion of its centre-of-mass is described by a relativistic wave equation. Let $\chi_\alpha(\vec{\varrho}\zeta)$ be the internal wave functions of the composite system, ζ and $\vec{\varrho}$ being the relative coordinates parallel and perpendicular to the incident direction. The argument is easier to present if one assumes that all the χ_α are bound states with a discrete excitation spectrum, but at the

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[2] What remains somewhat unclear is whether λ_0 is to be identified with our preconceived and naïve notion of the absorption coefficient.

[3] Although this statement is only correct as it stands in the nuclear rest frame, the ratio t_0/t_c is proportional to ν in any frame.

end we shall see that this is inessential and that a spectrum, that is closer to the physical problem of interest can be handled in the same manner.

Consider first the scattered wave Ψ_α that results when the state χ_α is incident on a single nucleon. If the centre-of-mass co-ordinate of the composite, relative to the nucleon, is $(\vec{b}, z)$, the eikonal approximation for ψ_α is

$$\Psi_\alpha(\vec{b}z;\vec{\varrho}\zeta) = -e^{ivz}\sum_\beta \chi_\beta(\vec{\varrho}\zeta)\,[\Gamma^{(1)}_{\beta\alpha}(\vec{b})+\Gamma^{(2)}_{\beta\alpha}(\vec{b})], \tag{5.9}$$

where the profiles $\Gamma^{(i)}$ are

$$\Gamma^{(1)}_{\beta\alpha}(\vec{b}) = \int \chi^*_\beta(\vec{\varrho}\zeta)\,[\gamma_{c_1}(\vec{b}+\tfrac{1}{2}\vec{\varrho})+\gamma_{c_2}(\vec{b}-\tfrac{1}{2}\vec{\varrho})]\chi_\alpha(\vec{\varrho}\zeta)d^2\varrho d\zeta, \tag{5.10}$$

$$\Gamma^{(2)}_{\beta\alpha}(\vec{b}) = -\int \chi^*_\beta(\vec{\varrho}\zeta)\gamma_{c_1}(\vec{b}+\tfrac{1}{2}\vec{\varrho})\gamma_{c_2}(\vec{b}-\tfrac{1}{2}\vec{\varrho})\chi_\alpha(\vec{\varrho}\zeta)d^2\varrho d\zeta, \tag{5.11}$$

and γ_c is the constituent-nucleon profile function. $\Gamma^{(1)}$ describes events where only one of the constituents scatters, whereas those where both scatter are described by $\Gamma^{(2)}$.

Over what time interval does the eikonal wave function (5.9) provide an adequate description of the state? As we are using stationary state collision theory, the answer to this question must be phrased in terms of z, *i. e.*, it is the distance beyond the scatterer over which the form (5.9) is maintained. To ascertain this distance, one merely computes the propagation of the centre-of-mass co-ordinate with the free wave equation, and one finds [11] that (5.9) is valid for $z \ll va^2$, where a is a characteristic transverse dimension of the profiles $\Gamma^{(i)}$. Once z reaches $\sim va^2$, diffraction effects set in, and for $z \gg va^2$, (5.9) evolves into a spherical outgoing wave whose scattering amplitudes are given by the familiar Fourier transform

$$F_{\beta\alpha}(q) = \frac{iv}{2\pi}\int d^2b e^{i\vec{q}\cdot\vec{b}}[\Gamma^{(1)}_{\beta\alpha}(\vec{b})+\Gamma^{(2)}_{\beta\alpha}(\vec{b})]. \tag{5.12}$$

We now fix our attention on the single scattering portion of Ψ_α, and examine its composition as we move downstream at a fixed value of $\vec{b}$. Over distances of order va^2 — which grow with energy[4] — the amplitude for finding the internal state χ_β is given by $\Gamma^{(1)}_{\beta\alpha}(b)$. As we see from (5.10), there is nothing that provides a drastic distinction between the elastic ($\beta = \alpha$) and inelastic elements, and so this short-time portion of the downstream amplitude does not distinguish dramatically between transitions that do or do not alter the internal state of the dissociating system. At long times ($z \gg va^2$), on the other hand, the scattered wave at a fixed value of $\vec{b}$ is proportional to $F_{\beta\alpha}$ evaluated at $\vec{q} = 0$, and from (5.10) we immediately see that

$$\frac{iv}{2\pi}\int d^2b\Gamma^{(1)}_{\beta\alpha}(b) = \delta_{\alpha\beta}\sum_c f_{cN}(0), \tag{5.13}$$

[4] This time-dilation effect is contained in our stationary state description because the propagation is described by a wave equation.

where $f_{cN}(0)$ is the forward constituent-nucleon scattering amplitude. There is thus a term in the on-shell amplitude that singles out transitions wherein the internal state does not change. As one sees from (5.11), the contribution from double scattering does not undergo such a profound change between the short-time and the asymptotic regimes.

To summarize, for short times (or distances) the amplitude for finding χ_β in the scattered state is a "smooth function" of β, whereas for long times the amplitude in the forward direction is

$$F_{\beta\alpha}(0) = \delta_{\alpha\beta} \sum_c f_{cN}(0) + F^{(2)}_{\beta\alpha}(0) \tag{5.14}$$

where $F^{(2)}_{\beta\alpha}$ is a "smooth function" of α and β:

$$F^{(2)}_{\beta\alpha}(0) = -\frac{iv}{2\pi}\int d^2b \int d^2\varrho d\zeta \chi^*_\beta(\vec{\varrho}\zeta)\gamma_{c_1}(\vec{b}+\tfrac{1}{2}\vec{\varrho})\gamma_{c_2}(\vec{b}-\tfrac{1}{2}\vec{\varrho})\chi_\alpha(\vec{\varrho}\zeta). \tag{5.15}$$

The quotation marks in the preceding sentence serve to remind us that our model has a discrete excitation spectrum. To make contact with our earlier discussion, one must then suppose that in a model with a continuous spectrum the same characteristic behaviour would emerge, and that the $\delta_{\alpha\beta}$ are merely replaced by $\delta(m-m')$, while $F^{(2)}_{\beta\alpha}$ becomes a smooth function of m and m'.

Having disposed of the scattering by one nucleon, we finally come to the problem of real interest: coherent nuclear scattering. We recall [12], [13] that when the incident energy vastly exceeds the characteristic excitation energies of the collision partners, the wave function in the near zone (*i. e.*, before diffraction becomes important) can be evaluated as if all internal motion were frozen during the collision. If the transverse co-ordinates of the nucleons with respect to the nuclear centre-of-mass are $\vec{y}_i$, the wave function of the dissociating system immediately after a coherent collision is

$$e^{ivz}\langle \prod_{i=1}^{A} [1-\gamma_{c_1}(\vec{b}+\vec{y}_i+\tfrac{1}{2}\vec{\varrho})][1-\gamma_{c_2}(\vec{b}+\vec{y}_i-\tfrac{1}{2}\vec{\varrho})]\rangle_0 \chi_\alpha(\vec{\varrho}\zeta) \tag{5.16}$$

where $\langle\ldots\rangle_0$ is an expectation value in the nuclear ground state. The optical potential U is defined in the usual manner by casting (5.16) into the form

$$e^{ivz}e^{-\int_{-\infty}^{\infty} U(\vec{b}z;\vec{\varrho})dz}\chi_\alpha(\vec{\varrho}\zeta). \tag{5.17}$$

Because of its dependence on $\vec{\varrho}$, U is an operator affecting the internal motion. If we ignore inter-nucleon correlations, a comparison of (5.16) and (5.17) yields [12]

$$U(\vec{b}z;\vec{\varrho}) = U^{(1)}(\vec{b}z;\vec{\varrho}) + U^{(2)}(\vec{b}z;\vec{\varrho}),$$

$$U^{(1)}(\vec{b}z;\vec{\varrho}) = \int d^2y n(\vec{y}z)[\gamma_{c_1}(\vec{b}+\vec{y}+\tfrac{1}{2}\vec{\varrho})+\gamma_{c_2}(\vec{b}+\vec{y}-\tfrac{1}{2}\vec{\varrho})], \tag{5.18}$$

$$U^{(2)}(\vec{b}z;\vec{\varrho}) = -\int d^2y n(\vec{y}z)\gamma_{c_1}(\vec{b}+\vec{y}+\tfrac{1}{2}\vec{\varrho})\gamma_{c_2}(\vec{b}+\vec{y}-\tfrac{1}{2}\vec{\varrho}), \tag{5.19}$$

where $n(\vec{y}z)$ is the nuclear density.

Bell's point is then the following: in a large slab of nuclear matter n is effectively constant, and then the average (5.18) over the positions of the scatterers gives

$$U^{(1)}(\vec{b}z; \vec{\varrho}) = n(\vec{b}z) \sum_c \int d^2y\gamma_c(y) =$$

$$= \tfrac{1}{2}\, n(\vec{b}z) \sum_c \sigma_{cN} \qquad (5.20)$$

which is not a function of $\vec{\varrho}$, and therefore unable to induce transitions. This is then the first term of (5.8). What Bell has therefore shown is that the average of the short--time wave function over the transverse positions of many scatterers gives the same result as the forward amplitude at asymptotic times in scattering from one scatterer[5] and therefore contains the singular piece $\delta(m-m')$, with the coefficient as given in (5.8).

Taking these considerations seriously would then lead us to choose the form (5.8), where V is smooth and, because of (5.19), positive at $m = m'$. If N is the number of constituents, and $\alpha = \frac{1}{2}n\,\sigma_{cN}$, we would therefore replace (5.3) by:

$$(\lambda_n - \alpha N)\psi_n(m) = -\int_{m_1}^{\infty} V(m, m')\psi_n(m')dm'.$$

Van Hove's simplifying assumption that V is merely a function of $|m-m'|$ then yields the spectrum

$$\lambda_\eta = N\alpha - \int_{-\infty}^{\infty} e^{i\eta m} V(m)dm, \qquad (5.21)$$

and one would expect $\lambda \to N\alpha$ for $\eta \to \infty$.

Instead of the spectrum shown in Fig. 3a, one would therefore have the situation depicted in Fig. 3b. Thus the modes that have the more rapid mass variation suffer greater absorption, and instead of the narrowing of the mass distributions found by Van Hove, one would find a broadening, as in normal collision broadening. Even with a spectrum of eigenabsorptivities as shown in Fig. 3b there will be a depression of the apparent nuclear cross-section, because the mean absorptivity is lower than $N\alpha$ — *e. g.*, lower than $\sigma_{\pi N}+\sigma_{\varrho N}$ if one thinks of the A_1 as a $\pi\varrho$ state[6].

The foregoing discussion only applies to a slab of nuclear matter, for $U^{(1)}$, Eq. (5.18), is only independent of the internal co-ordinates of the composite if the density is a constant.

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[5] What is being said here is, of course, "well known": the forward amplitude appears because of the spatial average. What is perhaps not so "well known" is the role of the time scales, especially when the projectile has an internal structure that can be excited.

[6] The (π, ϱ) model, treated as above, is just the one of Goldhaber *et al.* [10] in the first investigation of this problem. It will be recalled that they showed that σ_{A_1N} is then only ~15% smaller than $(\sigma_{\pi N}+\sigma_{\varrho N})$. Our investigation of such models has a different purpose, however: not to compute σ_{A_1N}, but to understand the mass structure of the optical potential.

In a real nucleus the situation is therefore less simple, but more interesting. In the interior, where the density varies slowly in comparison to the elementary profiles γ, the δ function will appear in U, but in the surface region this will not be so because the surface thickness is not large compared to elementary interaction ranges. Furthermore, in a production process the surface is emphasized because the wave functions of the incident and produced particles are strongly attenuated. As a consequence, the singular δ term may not play so prominent a role.

As we have just seen, in these simple models the averaging over the configuration of the scatterers is of paramount importance in determining the structure of re-scattering amplitudes. In a process where only a small subspace of all these configurations is significant, the short-time transition matrix will determine the nuclear amplitude. This is the case in the important example of coherent production from deuterium in the angular range where double scattering dominates [6]. For this process the two nucleons must be roughly aligned with the incident direction, and there will therefore be no δ term in the scattering amplitude of the produced state from the second nucleon. This should serve as a warning that such deuterium measurements are not trivially related to production in complex nuclei.

Our discussion has been marked by illogical jumps from a model with a discrete spectrum of internal excitations to the continuous mass spectrum seen in nature. The development from (5.16) onwards shows that this is quite unnecessary, because there the nature of the internal state χ_α plays a totally passive role. All that is necessary is that the collision be impulsive, *i. e.*, that the configurations of the two systems are momentarily frozen. Indeed, we can see that a more realistic model of a ($\pi \to A_1$)-type process can be

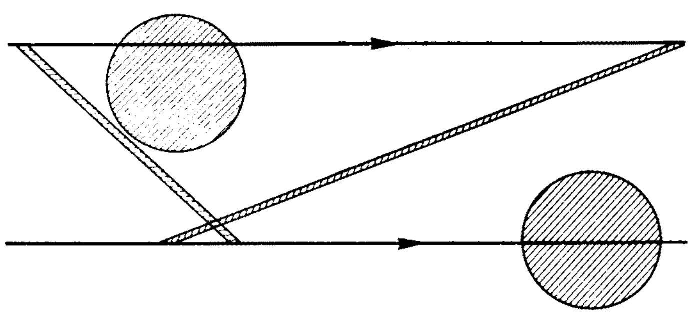

Fig. 4. Graphs that have no counterpart in the non-relativistic models, or their naïve relativistic generalizations. The lines with arrows are constituents of the dissociating systems, and the shaded circles represent an absorptive interaction with nucleons

constructed if one chooses an interaction such that the composite has one bound state χ_π, and a broad resonance not too far above the dissociation threshold. If χ_α is then replaced by χ_π in (5.16), that equation will describe both elastic "pion" scattering, and "A_1" production.

What still remains unclear is whether such naïve models can do justice to the real situation, where the internal dynamics of the dissociating system is also relativistics. What is clear is that nothing essentially different will emerge if the composite is treated by standard parton theory methods [14]. But it could be that diagrams that lack a non-relativistic analogue are of crucial importance (see Fig. 4), as is supposedly the case in diffraction scattering [15]. If that is so our naïve models will not have done justice to the situation, and Van Hove's original argument for a smooth optical potential matrix could well be vindicated.

6. *Conclusions*

Despite all the confusion that reigns at present, it is clear that by studying diffractive production in nuclei and in deuterium, we may obtain some insight into the short-time behaviour of hadronic systems, an insight that can only be gleaned by highly indirect considerations from the S matrix elements measured in conventional two-body collisions. Whether this information will actually be interesting, or merely complicated like the reactions between organic molecules, is of course another matter.

It is also evident that nuclear production experiments will be of considerable importance if diffraction dissociation is a quantitatively significant feature of high energy multiple production.

This article could not have been prepared without the collaboration of several of my colleagues at CERN. I have benefited from a number of discussions with J.S. Bell and L. Van Hove that greatly clarified my understanding of the problems treated in Section 5. Some of the derivations of Sections 2–4 were developed in conversations with O. Kofoed-Hansen, and those of Section 5 in collaboration with C. Schmid and W. Wetzel.

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