PARTIAL WAVE ANALYSIS OF DIFFRACTIVELY PRODUCED SYSTEMS

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The Ascoli method for the partial wave analysis of the 3π system produced diffractively is discussed. The tests of the factorization assumption are proposed. It seems that the most efficient test is to compare the density matrix elements for events in various regions of the Dalitz plot.

1. *Introduction*

Recently some experimental groups performed the partial wave analysis of diffractively produced systems. To discuss the problems encountered in such an analysis I shall consider the method proposed by Ascoli and described in the Ph. D. thesis by D. V. Brockway (Univ. of Illinois, 1970). It was applied, for instance, to analyse the process $\pi^-p \to \pi^+\pi^-\pi^-p$ at 5 and 7.5 GeV/c. This is a more complicated process than *e. g.* $pp \to n\pi p$, as the symmetrization over the identical pions in the final state is necessary. In the following, I shall ignore this symmetrization as it is irrelevant for further discussion.

It was assumed that the process $\pi^-p \to \pi^+\pi^-\pi^-p$ goes mainly through $\varrho\pi$ with some admixture of $\varepsilon\pi$. This simplifies the analysis considerably. Then the main purpose is to determine the spin-parity contents of the object A (see Fig. 1).

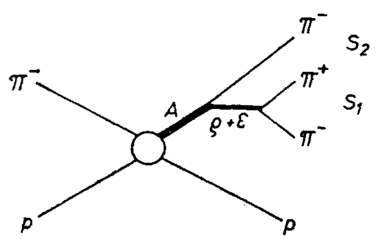

Fig. 1. Schematic representation of the process

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There are three sets of variables used to describe such a process:

(*i*) production process variables — CM energy squared W^2, momentum transfer squared t and the three pion effective mass $M_{3\pi}$. The dependence on these variables is in the density matrix $\varrho_{MM'}(A)$ of the object A. This matrix describes the mixture of several objects with various spins and parities, including their interference. It carries the subscripts which are the projections of the total angular momentum of A on the Gottfried-Jackson axis.

(*ii*) the Dalitz plot variables s_1, s_2. The Dalitz plot amplitude G depends on these variables.

(*iii*) the three-pion decay angles α, β, γ which are in the rotation functions $\mathscr{D}(\alpha, \beta, \gamma)$.

2. *Decay amplitude*

As I already mentioned, the production process is described by the density matrix $\varrho_{MM'}(A)$. This is a fairly complicated object and is unknown in the analysis.

Still more complicated is the decay amplitude $\mathfrak{M}$. It may be represented as the product of three functions (some normalization constants have been dropped here)

$$\mathfrak{M}_M^{JP} \sim \sum_{lS} C_{lS}^{JP} \sum_{\nu=-J}^{J} G_\nu^{JPl}(s_1, s_2) \mathscr{D}_{M\nu}^J(\alpha, \beta, \gamma)^*. \tag{1}$$

Here J and P is the spin and parity of the components of A, and C_{lS}^{JP} is a coefficient which determines the $\varepsilon\pi/\varrho\pi$ percentage and depends also on the relative orbital momentum l of the dipion and the third pion, and on the spin s of the dipion. Further, $G(s_1, s_2)$ is the Dalitz plot amplitude. It depends on ν, the projection of J on an axis internal to the 3π system, for instance on the momentum $\vec{p}_{\pi_3}$ of π^+. Another possible choice is the normal to the 3π decay plane. Finally, the rotation matrix $\mathscr{D}$ connects the 3π internal reference frame (the z-axis being $\vec{p}_{\pi_3}$ and the J projection on it ν) with an external reference frame (the z-axis being in this case the Gottfried-Jackson axis with J projection M).

Now it is necessary to specify the Dalitz plot amplitude. Its s_1 and s_2 dependence is easy to write down — it is just a combination of Breit-Wigner amplitudes with threshold behaviour added. More difficult to work out is the dependence on spin variables. It is given by the formula (again some unimportant factors are dropped)

$$G_\nu^{JPl} \sim \sum_\lambda d_{\nu\lambda}^J(\vartheta_{13}) d_{\lambda 0}^S(\chi_1) A_{S\lambda}^{JPl}(s_1). \tag{2}$$

Here A is the dipion wave function, essentially Breit-Wigner with threshold behaviour added. The rotation functions d are necessary to get the dependence on ν, the J projection on $\vec{p}_{\pi_3}$.

To explain this formula, let us start with s_1, the CMS of the pions π_2^- and π_3^+ (of Fig. 2). In this system, we know the projection of the dipion total angular momentum s on the direction of relative momentum, say on $\vec{p}_{\pi_3}$. It is zero, because the total spin of the dipion is zero and its internal orbital momentum has zero projection on the relative momentum. Starting from this piece of information we work out the dependence on ν in a few steps. First step: knowing the projection of S (always in $\pi_3^+ - \pi_2^-$ CMS) on $\vec{p}_{\pi_3}$ we calculate its projection λ on $-\vec{p}_{\pi_1}$ using the rotation matrix $d_{\lambda 0}^S(\chi_1)$. Here χ_1 is the angle between $-\vec{p}_{\pi_1}$ and $\vec{p}_{\pi_3}$. Next step is the Lorentz transformation from the dipion CMS s_1 to the 3π CMS. This transformation does not change λ, as this is the projection on the direction of transformation. Finally, we rotate through ϑ_{13}, the angle between

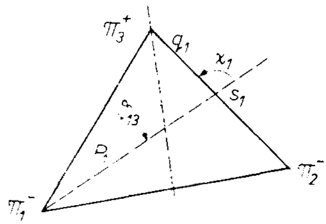

Fig. 2. Kinematics of the 3π system in the velocity space

$\vec{p}_{\pi_3}$ and $\vec{p}_{\pi_1}$ in the 3π CMS to obtain ν, the projection on $\vec{p}_{\pi_3}$. This step is done by the rotation function $d^J_{\nu\lambda}(\vartheta_{13})$.

The last thing to explain now in Eq. (2) is the dipion wave function $A^J_{S\lambda}(s_1)$. It is given by the formula

$$A^{JPl}_{S\lambda}(s_1) \sim C(l, 0; S, \lambda | J, \lambda) \frac{P_1^l q_1^S}{s_1 - m_S^2 - i m_S \Gamma(s_1)}. \tag{3}$$

Here P_1 is the relative momentum of the dipion and π_1^- in 3π CMS and q_1 is the relative momentum of π_3^+ and π_2^- in their CMS (of Fig. 2).

The Clebsch-Gordan coefficient is used to convert from the helicity basis to the $L-S$ basis, where the threshold behaviour is simpler. The width Γ had also the threshold behaviour.

3. *Experimental procedure*

The procedure applied by Ascoli *et al.* may be summarized in the following points:

1. Cut off all $N^*(1236)$. Check for bias introduced by this cutoff. N^* cannot be produced diffractively. It influences mainly the decay distribution of the object A.

2. Fit for the A density matrix ϱ and coefficients C^{JP}_{lS} using small intervals in $M_{3\pi}$ and a large range in t. This is to determine the dependence on $M_{3\pi}$ and find possible structures. Fitting is done by maximum likelihood method.

3. Fit for ϱ and C in two mass intervals with small t steps. The two mass intervals, 1.0–1.2 and 1.2–1.4 GeV are called the A_1 and A_2 mass regions.

4. Vary ϱ meson and ε meson masses and widths looking at the likelihood function. Choose best values for ϱ and ε parameters.

5. Check whether the fit for ϱ and C is good. This is done by calculating one dimensional distributions with fitted parameters, *e.g.* mass distributions along bands on the Dalitz plot, angular distributions in various mass regions.

As a result, after the parameters ϱ and C are determined, also the distributions not directly accessible from experimental data are calculated, like the spin-parity content as a function of $M_{3\pi}$ or the density matrix of the 1^+ component.

4. *Comments on assumptions*

Eqs (1)–(3) are not the most general formulae one can imagine for this process. Let us discuss now the assumptions that were made while writing it down. These may be divided in three groups.

(*i*) Dalitz plot amplitude is just a combination of $\varrho\pi$ and $\varepsilon\pi$.

(*ii*) Spin variables in the decay factorize from the spin variables in the production (*e.g.* the Dalitz plot amplitude G does not depend on the Gottfried-Jackson projection M).

(*iii*) Continuous variables in decay factorize from the continuous variables in production. This means that the density matrix elements $\varrho_{MM'}$ should not depend on 3π internal variables (*e.g.* the Dalitz plot variables) and, on the other hand, the decay amplitudes should not depend on total energy W^2 or momentum transfer t.

Assumption (*i*) can be checked experimentally by studying the Dalitz plot distribution or even by performing the partial wave analysis. If $\varrho\pi+\varepsilon\pi$ is not enough, more states must be added or something else substituted for G. This "something else" may depend on: J^P — spin and parity of A, l — relative orbital angular momentum of π^+ and the dipion, S — total angular momentum of the dipion, and λ — its projection on $\vec{p}_{\pi_3}$.

Assumptions of group (*ii*) seem to be difficult to verify. First of all, it is known that t-channel helicity is conserved. This means that $M = 0$ and detecting the dependence on M is hopeless. Another possibility is that for a pure J^P state functions $G_\nu^{J^Pl}$ satisfy certain parity relations. This is true, however, only if there is no interference of states with the same J but opposite P. Usually all possible J^P (up to some value of J) are assumed, so again this feature is difficult to check.

Most promising looks assumption (*iii*). It is easy to check whether $\varrho_{MM'}$ depends on s_1 or s_2 (it should not). In fact, to verify this it is enough to calculate the density matrix elements ϱ for various regions on the Dalitz plot and check whether they are equal. If at the same time coefficients C_{lS}^{JP} depend on s_1, s_2 one should also get suspicious.

Finally, C_{lS}^{JP} should not depend on the production variables W^2 or t. This is also not difficult to check.

To summarize, the easiest to perform seem to be the tests on factorization of continuous variables, in particular the independence of C_{lS}^{JP} on W^2, t and especially of $\varrho_{MM'}(A)$ on s_1, s_2.

When preparing these lectures I profitted from the discussion with Dr G. Otter.