PRESENT EXPERIMENTAL STATUS OF HELICITY CONSERVATION IN DIFFRACTIVE PROCESSES

By G. Otter

Institut für Hochenergiephysik der Österr. Akademie der Wissenschaften, Wien*

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The experimental situation concerning helicity conservation in diffractive processes is reviewed. As an introduction a short repetition of the necessary formulae of the helicity formalism is carried out. The tests for helicity conservation are then derived and the corresponding experimental data are discussed. The results are the following:

s-channel helicity conservation seems to hold for elastic diffraction scattering (I include here the photoproduction of the ϱ-meson), but nearly all inelastic diffractive processes show small deviations from *t*-chanel helicity conservation.

Introduction

Elastic scattering at high energy seems to tend to a finite cross-section and people have tried to describe it in terms of a diffraction picture. Good and Walker [1] predicted that the elastic diffraction scattering of a particle on a nucleon or nucleus can at high energies produce other kinds of particle, whose production cross-section behaves in a similar way to the elastic cross-section. This process is called diffraction dissociation and has been found experimentally. The hadronic complex of particles produced is expected to have the same quantum numbers as the incident dissociating particle (except for a possible change in spin and parity), to have a narrow diffraction peak in momentum transfer and to be produced mainly at low effective masses. People also tried to apply the Regge picture for these processes and used Pomeron exchange for its description. If this is correct only natural parity is exchanged. Besides the expected properties some others seem also to be observed. One of them is known under the name "helicity conservation". The expressions "*s*-channel helicity conservation" (SCHC) and "*t*-channel helicity conservation" (TCHC) are now often used in high energy physics but the ideas behind these words are not so new. Pokorski and Satz [2] proposed a model for diffraction dissociation in which the Pomeron couples like a particle with spin-parity 0^+. This gives just TCHC (I would like to mention that TCHC is not a new idea, since *e.g.* one-pion-exchange models haveTCHC incorpoiated).

* Address: Institut für Hochenergiephysik der Österr. Akademie der Wissenschaften, Wien, Austria.

SCHC is also not new. The Aachen–Berlin–Bonn–Hamburg–Heidelberg–Munich collaboration [3] studied the photoproduction of the ϱ-meson with unpolarized photons and found that some of the density matrix elements of the ϱ were zero in the helicity frame ($\varrho_{00} = \varrho_{1-1} = \text{Re}\,\varrho_{10} = 0$), independent of the production angle of the ϱ. This is just SCHC.

An experiment done at SLAC [4] brought more attention to the question of helicity conservation. With linearly polarized photon beams at 2.8 and 4.7 GeV/c they studied again the photoproduction of the ϱ-meson. Because of the linearly polarized beam they had more quantities to test the hypothesis of helicity conservation and they found really very good evidence of SCHC. Gilman *et al.* [5] proposed that SCHC might be a feature of all diffractive reactions. There is experimental evidence that SCHC holds for high energy πN elastic scattering [6]. The diffractive reactions, on the other side, show strong violation of SCHC. They are also not in agreement with TCHC but this hypothesis looks to agree much better.

In these lectures I want to talk about the experimental situation concerning the helicity conservation: how has it been tested and what the results are.

There are 3 sections I want to treat

1. short repetition of the helicity formalism
2. helicity conservation for the two-body reactions:
   $ab \to cd$
3. helicity conservation for the general case:
   $ab \to (c_1 c_2 \ldots)\, d$.

1. Repetition of the helicity formalism

This is more or less a collection of all the formulae we need later on. Proofs and phase conventions are found in Ref. [7].

1.1. One-particle states

Consider a particle with mass M and spin S. A convenient basis in the Hilbert space for a relativistic theory is the following set of common eigenvectors

$$|\vec{p}\lambda(M, S)\rangle$$

where $\vec{p}$ is the momentum and λ the helicity ($\vec{j} \cdot \vec{p}/|\vec{p}|$, j being the angular momentum), the projection of the angular momentum along the direction of the momentum. The quantities M and S will be omitted from now on.

$$|\vec{p}\lambda\rangle = |\varphi\vartheta p\lambda\rangle = R(\varphi, \vartheta, 0)L_z(v)|000\lambda\rangle.$$

The parity operation transforms

$$\mathscr{P}|\vec{p}\lambda\rangle = \eta \exp(-i\pi S)|-\vec{p}-\lambda\rangle,$$

where η is the intrinsic parity of the particle. The combination of parity and rotation around the y-axis transforms

$$R(0, \pi, 0)\mathscr{P}|\vec{p}\lambda\rangle = (-)^{S-\lambda}\eta|\vec{p}^{M}-\lambda\rangle,$$

$\vec{p}^M$ is the momentum reflected in the $x-z$-plane. It is sometimes convenient to introduce another basis given by the set of common eigenvectors

$$|jmp\lambda\rangle,$$

j is the angular momentum and m its third component. It is

$$|jmp\lambda\rangle = N_j \int d\cos\vartheta d\varphi D^{j*}_{m\lambda}(\varphi, \vartheta, 0)|\varphi\vartheta p\lambda\rangle$$

with

$$|\varphi\vartheta p\lambda\rangle = \sum_{jm} N_j D^j_{m\lambda}(\varphi, \vartheta, 0)|jmp\lambda\rangle$$

where N_j is a convenient normalization factor and D^j_{lm} is the usual rotation matrix.
The parity operation transforms

$$\mathscr{P}|jmp\lambda\rangle = \eta(-)^{j-S}|jmp-\lambda\rangle.$$

1.2. Two-particle states

Consider two particles with spins S_1, S_2 and masses M_1 and M_2. The Hilbert space of this system is the direct product of the Hilbert spaces of the corresponding single--particle states and the simplest basis is

$$|\vec{p}_1\lambda_1, \vec{p}_2\lambda_2\rangle = |\vec{p}_1\lambda_1\rangle|\vec{p}_2\lambda_2\rangle$$

or, with a different parametrization,

$$|\vec{P}\vec{p}\lambda_1\lambda_2\rangle = |\varphi\theta P, \varphi\vartheta p, \lambda_1\lambda_2\rangle$$

$\vec{P}$ is the momentum of the CM and $\vec{p}$ is the relative momentum in CM. Instead of p (the absolute value of the momentum in CM) we often use the total effective mass M of the 2 particles, which can be calculated from p and the masses of the two particles. The angular momentum eigenstates are

$$|\vec{P}\Lambda(MJ)\lambda_1\lambda_2\rangle = \frac{N_J}{2\pi}\int d\cos\vartheta d\varphi D^{J*}_{\Lambda(\lambda_1-\lambda_2)}(\varphi, \vartheta, 0)\times$$
$$\times|\vec{P}, \varphi\vartheta M, \lambda_1\lambda_2\rangle$$

with

$$|\vec{P}\varphi\vartheta M\lambda_1\lambda_2\rangle = \sum_{J\Lambda} N_J D^J_{\Lambda\lambda_1-\lambda_2}(\varphi, \vartheta, 0)|\vec{P}\Lambda(MJ)\lambda_1\lambda_2\rangle.$$

J and Λ are the total angular momentum and the helicity of the two-particle system respectively.

The parity operation in the CM-system:

$$\mathscr{P}|\vec{P} = 0\Lambda(MJ)\lambda_1\lambda_2\rangle = \eta_1\eta_2(-)^{J-S_1-S_2}|\vec{P} = 0\Lambda(MJ)-\lambda_1-\lambda_2\rangle.$$

1.3. Multi-particle states

There is more than one way to couple three particles with masses M_i and spins S_i. One possibility is to couple particles 1 and 2 to a system (1 2) and couple to it the third particle. The basis in the Hilbert space is

$$|\vec{P}, \varphi\vartheta M, \bar{\varphi}\bar{\vartheta}\overline{M}, \lambda_1, \lambda_2, \lambda_3\rangle$$

P is the CM-momentum, $M\varphi\vartheta$ effective mass and the direction of the momentum of the system (1 2) in the CM-frame, $\overline{M}\bar{\varphi}\bar{\vartheta}$ are the total are the effective mass of the system (1 2) and the direction of the momentum of particle 1 in the (1 2) rest frame.

λ_1 and λ_2 are the helicities of particles 1 and 2 in the (1 2) rest frame and λ_3 the helicity of particle 3 in the total CM-system.

It is also possible to define an angular momentum state which is related to the momentum state

$$|\vec{P}\Lambda(MJ)\overline{M}\bar{J}\lambda_1\lambda_2, \mu\lambda_3\rangle =$$

$$= N_J N_{\bar{J}} \int d\cos\vartheta d\varphi \int d\cos\bar{\vartheta} d\bar{\varphi} D^{J*}_{\Lambda\mu-\lambda_3}(\varphi, \vartheta, 0)\times$$

$$\times D^{\bar{J}*}_{\mu\lambda_1-\lambda_2}(\bar{\varphi}, \bar{\vartheta}, 0)|\vec{P}, \varphi\vartheta M, \bar{\varphi}\bar{\vartheta}\overline{M}, \lambda_1\lambda_2, \lambda_3\rangle$$

and

$$|\vec{P}, \varphi\vartheta M, \bar{\varphi}\bar{\vartheta}\overline{M}, \lambda_1\lambda_2, \lambda_3\rangle =$$

$$= \sum_{J\Lambda\bar{J}\mu} N_J N_{\bar{J}} D^J_{\Lambda\mu-\lambda_3}(\varphi, \vartheta, 0) D^{\bar{J}}_{\mu\lambda_1-\lambda_2}(\bar{\varphi}, \bar{\vartheta}, 0)\times$$

$$\times |\vec{P}\Lambda(MJ)\overline{M}\bar{J}\lambda_1\lambda_2, \mu\lambda_3\rangle.$$

J and Λ are the angular momentum (spin) and helicity of the total system, $\bar{J}$ and μ the angular momentum and helicity of the system (1 2) in the CM-system. The parity operation in the CM-system transforms

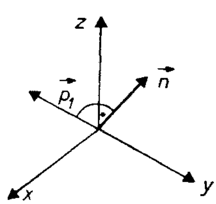
$$\mathscr{P}|\vec{P} = 0\Lambda(MJ)\overline{M}\bar{J}\lambda_1\lambda_2, \mu\lambda_3\rangle =$$

$$= \eta_1\eta_2\eta_3(-)^{J-S_1-S_2-S_3}|\vec{P} = 0\Lambda(MJ)\overline{M}\bar{J}-\lambda_1-\lambda_2, -\mu-\lambda_3\rangle.$$

Another possibility for coupling the three particles is the following basis

$$|\vec{P}, \alpha\beta\gamma, M\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle.$$

$\vec{P}$ is the CM-momentum, α, β, γ are the 3 Euler angles defining the orientation of the plane of the three particles in the CM-system, M is the total effective mass, κ_1 and κ_2 the effective masses of two of the three particles. $\lambda_1, \lambda_2, \lambda_3$ are the helicities of the three particles in the CM-system.



$\vec{n}$ (normal to the three particle plane) and $\vec{p}_i$ define the orientation of the three particle system. They are given by $\alpha\beta\gamma$.

The corresponding angular momentum state is:

$$|\vec{P}, \Lambda(MJ)K\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle = \frac{N_J}{\sqrt{2\pi}} \int d\alpha \cdot d\cos\beta \cdot d\gamma \times$$

$$\times D^{J*}_{\Lambda K}(\alpha, \beta, \gamma)|\vec{P}, \alpha\beta\gamma M\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle,$$

and

$$|\vec{P}, \alpha\beta\gamma M\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle = \frac{1}{\sqrt{2\pi}} \sum_{J\Lambda K} N_J D^J_{\Lambda K}(\alpha\beta\gamma) \times$$

$$\times |\vec{P}\Lambda(MJ)K\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle.$$

J and Λ are the angular momentum (spin) and helicity of the three particle-system. K is the component of the angular momentum along the direction which is specified by the Euler angles (α, β) (for the above definition it is the normal to the three-particle plane).

The parity operation in the CM-system (for the case where K is the angular momentum component along the normal)

$$\mathscr{P}|\vec{P} = 0\Lambda(MJ)K\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3\rangle =$$

$$= (-)^K \prod_{i=1}^{3} \eta i(-)^{S_i}|\vec{P} = 0\Lambda(MJ)K\kappa_1\kappa_2, -\lambda_1-\lambda_2-\lambda_3\rangle.$$

It is of course also possible to couple the three particle state considered with a fourth particle. The momentum basis is

$$|\vec{P}, \varphi\vartheta M, \alpha\beta\gamma M_{123}\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3, \lambda_4\rangle.$$

φ and ϑ define the direction of the system (1 2 3) with mass φ_{123} in the CM-frame. M is the total effective mass and κ_1, κ_2 the effective masses of two of the first three particles. α, β, γ are the three Euler angles defined in the same way as before. λ_1, λ_2, λ_3 are the helicities of the particles 1, 2, 3 in the M_{123} rest frame and λ_4 the helicity of the fourth particle in the overall CM-system.

The relation between this state and the corresponding angular momentum state is:

$$|\vec{P}, \varphi\vartheta M, \alpha\beta\gamma M_{123}\kappa_1\kappa_2\lambda_1\lambda_2\lambda_3, \lambda_4\rangle =$$

$$= \frac{1}{\sqrt{2\pi}} \sum_{J\Lambda} \sum_{J_{123}\mu K} N_J N_{J_{123}} D^J_{\Lambda\mu-\lambda_4}(\varphi, \vartheta, 0) D^{J_{123}}_{\mu K}(\alpha, \beta, \gamma) \times$$

$$\times |\vec{P}\Lambda(MJ)M_{123}J_{123}K\kappa_1\kappa_2, \lambda_1\lambda_2\lambda_3, \mu\lambda_4\rangle$$

J and Λ are the angular momentum and helicity of the total system, J_{123}, μ and K the angular momentum, helicity and component of the angular momentum along the direction (α, β) of the system (1 2 3). Expanding only one state in angular momentum states we obtain

$$|\vec{P}, \varphi\vartheta M, \alpha\beta\gamma M_{123}\kappa_1\kappa_2\lambda_1\lambda_2\lambda_3, \lambda_4\rangle = \frac{1}{\sqrt{2\pi}} \sum_{\substack{J_{123} \\ \mu K}} N_{J_{123}} \times$$

$$\times D^{J_{123}}_{\mu K}(\alpha, \beta, \gamma)|\vec{P}, \varphi\vartheta M, J_{123}\mu K M_{123}\kappa_1\kappa_2\lambda_1\lambda_2\lambda_3, \lambda_4\rangle.$$

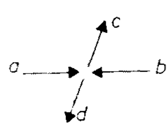
The meaning of the symbols used is the same as just explained.

2. *Helicity conservation for two-body reactions*

I repeat shortly the formulae for scattering of two particles with spin into two other particles with spin.

2.1. The reaction *ab* → *cd*

We consider this reaction in the CM-system and take the coordinate frame such that the *z*-axis points in the direction of particle *a* and *y*-axis is perpendicular to the production plane.



The initial state is

$$|\vec{P} = 0, 00M\lambda_a\lambda_b\rangle = \sum_{J\Lambda} N_J \delta_{\Lambda\lambda_a-\lambda_b}|\vec{P} = 0\Lambda(MJ)\lambda_a\lambda_b\rangle$$

the final state

$$|\vec{P} = 0, \varphi\vartheta M\lambda_c\lambda_d\rangle = \sum_{J'\Lambda'} N_{J'} D^{J'}_{\Lambda'\lambda_c-\lambda_d}(\varphi, \vartheta, 0)|\vec{P} = 0\Lambda'(MJ')\lambda_c\lambda_d\rangle$$

the transition matrix-element

$$f_{\lambda_c\lambda_d\lambda_a\lambda_b}(s,t) = \langle \vec{P} = 0, 0\vartheta M\lambda_c\lambda_d | T | \vec{P} = 0, 00M\lambda_a\lambda_b\rangle =$$

$$= \sum_{JJ'\Lambda'} N_J N_{J'} D^{J'}_{\Lambda'\lambda_c-\lambda_d}(0,\vartheta,0)\langle \vec{P} = 0\Lambda'(MJ')\lambda_c\lambda_d | T | \lambda_a - \lambda_b(MJ)\lambda_a\lambda_b\rangle =$$

$$= \sum_J d^J_{\lambda_a-\lambda_b\lambda_c-\lambda_d}(\vartheta) A^J_{\lambda_c\lambda_d\lambda_a\lambda_b}.$$

Parity conservation ($T = P^{+}TP$) restricts the number of independent amplitudes:

$$A^J_{\lambda_c\lambda_d\lambda_a\lambda_b} = \frac{\eta_a\eta_b}{\eta_c\eta_d}(-)^{S_c+S_d-S_a-S_b} A^J_{-\lambda_c-\lambda_d-\lambda_a-\lambda_b}.$$

Assuming the initial state to be a mixed state described by a density matrix

$$\varrho^i_{\lambda_a\lambda_b,\lambda_a'\lambda_b'},$$

the joint density matrix in the final state is

$$\varrho^f_{\lambda_c\lambda_d,\lambda_c'\lambda_d'} = \sum_{\lambda_a\lambda_a'}\sum_{\lambda_b\lambda_b'} f_{\lambda_c\lambda_d\lambda_a\lambda_b}\varrho^i_{\lambda_a\lambda_b,\lambda_a'\lambda_b'} f^*_{\lambda_c'\lambda_d'\lambda_a'\lambda_b'}.$$

The density matrix of the particle c (in the rest frame of c with quantization axis along $-\vec{p}_{d'}$ the y-axis perpendicular to the production plane and the x-axis such that we obtain a right-handed coordinate frame: the helicity system) is

$$\varrho_{\lambda_c\lambda_c'} = \sum_{\lambda_d}\varrho^f_{\lambda_c\lambda_d,\lambda_c'\lambda_d'}$$

It should be noted that ϱ^f is not yet normalised. ϱ^f has therefore to be divided by the trace of ϱ^f. For unpolarized incident particles, the joint density matrix in the final state fulfils, due to parity conservation, the condition [8]

$$\varrho^f_{-\lambda_c-\lambda_d,-\lambda_c'-\lambda_d'} = (-)^{\lambda_c-\lambda_c'+\lambda_d-\lambda_d'}\varrho^f_{\lambda_c\lambda_d,\lambda_c'\lambda_d'}$$

I would like to treat also the more general case where for particle c not only the spin component is unknown (therefore described by a density matrix) but also the spin-parity value. We describe this situation with the more geneial density matrix

$$\varrho^f_{S\eta\lambda_c,S'\eta'\lambda_c'} = \sum_{\lambda_a\lambda_a'\lambda_b\lambda_b'}\sum_{\lambda_d} f_{S\eta\lambda_c\lambda_d,\lambda_a\lambda_b}\times$$

$$\times\varrho^i_{\lambda_a\lambda_b\lambda_a'\lambda_b'} f^*_{S'\eta'\lambda_c'\lambda_d\lambda_a'\lambda_b'}.$$

Parity conservation gives for an unpolarized initial state the following condition

$$\varrho_{S\eta-\lambda_c,S'\eta'-\lambda_c'} = \frac{\eta_c}{\eta'_{c'}}(-)^{S-\lambda_c+S'-\lambda_c'}\varrho_{S\eta\lambda_c,S'\eta'\lambda_c'}.$$

2.2. *s*-channel helicity conservation (SCHC)

Helicity conservation in the *s*-channel means that the scattering amplitude vanishes when the helicities of the initial and final particles are not the same (helicity flip amplitudes are zero)

$$f_{\lambda_c\lambda_d\lambda_a\lambda_b} = \bar{f}_{\lambda_c\lambda_d\lambda_a\lambda_b}\delta_{\lambda_c\lambda_a}\delta_{\lambda_d\lambda_b}.$$

Examples:

A) $$K^-p \to Q^-p \text{ or } \pi^+p \to A_1^\pm p \;(0^-1/2^+ \to 1^+1/2^+)$$

Assuming an unpolarized proton target and no measurement of the final proton polarization, the density matrix of the pseudovector particle is (in the helicity frame)

$$\varrho_{\lambda_c\lambda_c'}(t) = \sum_{\lambda_b\lambda_d} f_{\lambda_c\lambda_d\lambda_a=0\lambda_b}\delta_{\lambda_a\lambda_c}\delta_{\lambda_b\lambda_d}\times$$

$$\times f^*_{\lambda_c'\lambda_d\lambda_a'=0\lambda_b}\delta_{\lambda_c'\lambda_a'}\delta_{\lambda_b\lambda_d} =$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The density matrix has to have only one non-zero element, independent of the four--momentum transfer t. It should be noted that this prediction also has to be true when SCHC holds only for the mesonic system ($\lambda_c = \lambda_a = 0$ but $\lambda_b \neq \lambda_d$).

Let us assume that instead of a pure 1^+ in the final state, we have a mixture of 0^- and 1^+ (this can be easily generalized to all possible spin-parity states), the predicted density matrix for this is

$$\varrho_{S\eta\lambda_c,S'\eta'\lambda_c'} = \begin{pmatrix} \varrho_{0-0,0-0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varrho_{1+0,1+0} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

B) $$\gamma p \to \varrho^0 p\; (1^-1/2^+ \to 1^-1/2^+)$$

We assume an unpolarized p target and linearly polarized photons in the initial state and do not measure any polarization of the proton in the final state. The density matrix for ϱ^0 is now calculated. For linearly polarized photons the density matrix is

$$\varrho_{\lambda_a\lambda_a'} = \tfrac{1}{2}\begin{pmatrix} 1 & -Pe^{-2i\Phi} \\ -Pe^{2i\Phi} & 1 \end{pmatrix} = \tfrac{1}{2}\left(1 + \sum_{i=1}^{3} P_i\sigma_i\right).$$

P is the absolute value of the polarization and Φ is the angle between the photon polarization vector and the production plane [9]. If the density matrix is decomposed in terms of the Pauli matrices σ_i we obtain

$$P_1 = -P\cos 2\Phi$$
$$P_2 = -P\sin 2\Phi$$
$$P_3 = 0.$$

Inserting the density matrix in the formula above we obtain the following density matrix for the ϱ-meson in the helicity frame

$$\varrho_{\lambda_c\lambda_{c'}} = \sum_{\lambda_a\lambda_{a'}} \sum_{\lambda_b\lambda_d} f_{\lambda_c\lambda_d\lambda_a\lambda_b} \varrho_{\lambda_a\lambda_{a'}} f^*_{\lambda_{c'}\lambda_d\lambda_{a'}\lambda_b} = \\ = \varrho^{(0)}_{\lambda_c\lambda_{c'}} + P_1 \varrho^{(1)}_{\lambda_c\lambda_{c'}} + P_2 \varrho^{(2)}_{\lambda_c\lambda_{c'}} .$$

SCHC means

$$\varrho^{(0)}_{\lambda_c\lambda_{c'}} = \begin{pmatrix} \varrho^{(0)}_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varrho^{(0)}_{-1-1} \end{pmatrix} \qquad \varrho^{(1)}_{\lambda_c\lambda_{c'}} = \begin{pmatrix} 0 & 0 & \varrho^{(1)}_{1-1} \\ 0 & 0 & 0 \\ \varrho^{(1)*}_{1-1} & 0 & 0 \end{pmatrix}$$

$$\varrho^{(2)}_{\lambda_c\lambda_{c'}} = \begin{pmatrix} 0 & 0 & -i\varrho^{(1)}_{1-1} \\ 0 & 0 & 0 \\ i\varrho^{(1)*}_{1-1} & 0 & 0 \end{pmatrix} .$$

When parity conservation and only natural parity exchange are taken into account [10], there are further restrictions

$$\varrho^{(0)}_{11} = \varrho^{(0)}_{-1-1} = \mathrm{Re}\, \varrho^{(1)}_{1-1} = -\mathrm{Im}\, \varrho^{(2)}_{1-1} = \tfrac{1}{2} .$$

These results should be valid independently of the four-momentum transfer t from p to p.

2.3. t-channel helicity conservation (TCHC)

The two-body reaction can also be described by the amplitudes in the t-channel, where these amplitudes are related to the s-channel amplitudes *via* the crossing matrices. The helicity axis for the t-channel reaction

$$\bar{d}b \to c\bar{a}$$

is along $-\vec{p}_{\bar{a}}$. Helicity conservation in the t-channel means that the scattering amplitude vanishes when the helicity of the corresponding particles are not the same for the t-channel amplitudes (t-channel helicity flip amplitudes vanish). The consequences for the density matrix for c are the same as for the SCHC but now with another quantization axis, namely along the direction of the particle a. This is just the Gottfried-Jackson system [11] (rest frame of c with z-axis along $\vec{p}_a$ and y-axis perpendicular to the production plane). It should be noted that the angle between Gottfried-Jackson and helicity frames is just the crossing angle χ and that both frames coincide in the forward direction of the two-body process.

The density matrix of particle c with spin S in the helicity frame: ϱ^H and in the Gottfried-Jackson frame: ϱ^{GJ} are related by

$$\varrho^H_{nn'} = \sum_{mm'} d^S_{mn}(\chi) \varrho^{GJ}_{mm'} d^S_{m'n'}(\chi)$$

(χ is taken as positive for y-axis defined as

$$Y = \frac{\vec{p}_b \times \vec{p}_d}{|\vec{p}_b \times \vec{p}_d|}\Bigg) .$$

2.4. Determination of the density matrix elements

In this part I would like to repeat the procedure whereby the density matrix elements can be determined from the decay distribution of this particle in 2 or 3 particles. Only decays *via* strong interaction are considered.

2.4.1. Decay into two particles ($c \to c_1 c_2$).This decay has been treated *e.g.* by Gottfried-Jackson [11]. The decay amplitude for the decay of a particle with spin S and third component m along the z-axis into 2 particles with S_1 and S_2 is in the two particle rest system

$$A_{m\lambda_1\lambda_2}(\vartheta, \varphi) = \langle \vec{P} = 0, \varphi\vartheta p\lambda_1\lambda_2 | T | Sm \rangle =$$

$$= D^{S*}_{m\lambda_1-\lambda_2}(\varphi, \vartheta, 0) T_{\lambda_1\lambda_2}.$$

The joint density matrix for c_1 and c_2 is, assuming a density matrix for describing the spin content of particle c,

$$\varrho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} = \sum_{mm'} A_{m\lambda_1\lambda_2}\varrho^{(c)}_{mm'}A^*_{m'\lambda_1'\lambda_2'} =$$

$$= T_{\lambda_1\lambda_2}T^*_{\lambda_1'\lambda_2'} \sum_{mm'} D^{S*}_{m\lambda_1-\lambda_2}(\varphi, \vartheta, 0)\varrho^{(c)}_{mm'}D^{S}_{m'\lambda_1'-\lambda_2'}(\varphi, \vartheta, 0).$$

The angular distribution of particle 1 in the c rest frame

$$W(\vartheta, \varphi) \propto \sum_{\lambda_1\lambda_2} \varrho_{\lambda_1\lambda_2,\lambda_1\lambda_2}(\varphi, \vartheta).$$

Parity conservation for the decay process gives the relation

$$T_{\lambda_1\lambda_2} = \frac{\eta_1\eta_2}{\eta}(-)^{S-S_1-S_2}T_{-\lambda_1-\lambda_2},$$

where η, η_1 and η_2 are the intrinsic parities. This gives the well-known decay property

$$W(\pi-\vartheta, \pi+\varphi) = W(\vartheta, \varphi).$$

As an example let us consider the decay distribution for $1^- \to 0^-0^-$ ($\varrho \to \pi\pi$)

$$W(\vartheta, \varphi) = \frac{3}{8\pi}\{2\cos^2\vartheta \cdot \varrho_{00} + \sin^2\vartheta(1-\varrho_{00}) - 2\sqrt{2}\sin\vartheta\cos\vartheta \times$$

$$\times [(\mathrm{Re}\,\varrho_{10} - \mathrm{Re}\,\varrho_{-10})\cos\varphi - (\mathrm{Im}\,\varrho_{10} + \mathrm{Im}\,\varrho_{-10})\sin\varphi] -$$

$$-2\sin^2\vartheta(\mathrm{Re}\,\varrho_{1-1}\cdot\cos 2\varphi - \mathrm{Im}\,\varrho_{1-1}\sin 2\varphi)\}.$$

Therefore for the case considered $\gamma p \to \varrho^0 p$ (linearly polarized photons and unpolarized proton target)

$$W(\vartheta, \varphi, \Phi) = \frac{3}{4\pi}\{\tfrac{1}{2}(1-\varrho^{(0)}_{00}) + \tfrac{1}{2}(3\varrho^{(0)}_{00}-1)\cos^2\vartheta -$$

$$-\sqrt{2}\,\mathrm{Re}\,\varrho^{(0)}_{10}\sin 2\vartheta\cos\varphi - \varrho^{(0)}_{1-1}\sin^2\vartheta\cos 2\varphi -$$

$$-P\cos 2\Phi[\varrho_{11}^{(1)}\sin^2\vartheta+p_{00}^{(1)}\cos^2\vartheta-\sqrt{2}\,\mathrm{Re}\,\varrho_{10}^{(1)}\sin 2\vartheta\cos\varphi-$$
$$-\varrho_{1-1}^{(1)}\sin^2\vartheta\cos 2\varphi]-P\sin 2\Phi[\sqrt{2}\,\mathrm{Im}\,\varrho_{10}^{(2)}\times$$
$$\times\sin 2\vartheta\sin\varphi+\mathrm{Im}\,\varrho_{1-1}^{(2)}\sin^2\vartheta\sin 2\varphi]\}.$$

The density matrix elements are calculated by fitting the experimental decay distribution.

2.4.2. Decay into three particles ($c\to c_1c_2c_3$). This decay has been treated by Berman and Jacob [12]. The decay amplitude for the decay of a particle with spin S and mass M into 3 particles with spins S_i in the three particle rest frame is

$$A_{m\lambda_1\lambda_2\lambda_3}(\alpha\beta\gamma\kappa_1\kappa_2) = \langle\vec{P}=0,\alpha\beta\gamma M\kappa_1\kappa_2\lambda_i|T|S_m\rangle =$$
$$=\sum_K D^{S*}_{mK}(\alpha,\beta,\gamma)T_{K,\lambda_1\lambda_2\lambda_3}(\kappa_1\kappa_2),$$

where the variables are the same as explained in section 1.3. (α,β) are the Euler angles specifying the normal to the three particle plane and γ is the third Euler angle which describes the direction of particle 1. Assuming a density matrix for particle c the angular distribution of the normal and of particle 1 is

$$W(\alpha\beta\gamma) = N\sum_{mm'}\varrho_{mm'}\sum_{\lambda_1\lambda_2\lambda_3}\sum_{KK'}D^{S*}_{mK}(\alpha\beta\gamma)\times$$
$$\times D^{S}_{mK'}(\alpha\beta\gamma)\int_{\text{Dalitz plot}}d\kappa_1 d\kappa_2 T_{K\lambda_1\lambda_2\lambda_3}\cdot T^*_{K'\lambda_1\lambda_2\lambda_3} =$$
$$=N\sum_{\lambda_1\lambda_2\lambda_3}\sum_{mm'KK'}(-)^{m'-K'}\varrho_{mm'}F^{KK'}_{\lambda_1\lambda_2\lambda_3}\sum_{j=0}^{2S}\langle S-m,S'm'|jm'-m\rangle\times$$
$$\times\langle S-K,SK'|jK'-K\rangle D^{j*}_{m-m'K-K'}(\alpha\beta\gamma)$$

with:

$$F^{KK'}_{\lambda_1\lambda_2\lambda_3} = \int_{\text{Dalitz plot}}d\kappa_1 d\kappa_2 T_{K\lambda_1\lambda_2\lambda_3}(\kappa_1\kappa_2)T^*_{K'\lambda_1\lambda_2\lambda_3}(\kappa_1\kappa_2).$$

Parity conservation in the decay process yields

$$T_{K\lambda_1\lambda_2\lambda_3}(\kappa_1\kappa_2) = \frac{\eta_1\eta_2\eta_3}{\eta}(-)^{K+S_1+S_2+S_3}T_{K-\lambda_1-\lambda_2-\lambda_3}(\kappa_1\kappa_2)$$

η and η_i are the intrinsic parities of the particles considered. This gives the well-known decay property

$$W(\alpha,\beta,\pi+\gamma) = W(\alpha,\beta,\gamma).$$

The meaning of this formula is, that by a parity operation the particle momenta reverse direction whereas the normal remains the same ($\vec{p}_i\to -\vec{p}_i, \vec{n}\to\vec{n}$). From this formula one sees immediately that the distribution of the normal is the same for the total sample of experimental data or for the half of the particle-1-distribution.

$$\overline{W}(\alpha,\beta) = \int_{\Omega}^{\Omega+2\pi}W(\alpha\beta\gamma)d\gamma = 2\int_{\Omega}^{\Omega+\pi}W(\alpha\beta\gamma)d\gamma.$$

I want to stress that this is true only for one spin state decaying.

As an example we calculate $\overline{W}(\alpha, \beta)$ for the decay $Q^- \to K^-\pi^+\pi^-$ $(1^+ \to 0^-0^-0^-)$, assuming that the Q is produced by an unpolarized initial state (that means: $\varrho_{mm'} = (-)^{m-m'} \varrho_{-m-m'}$). There are two decay amplitudes $T_1(\kappa_1\kappa_2)$ and $T_{-1}(\kappa_1\kappa_2)$ for this process. Using the formulae above we obtain

$$\overline{W}^{1^+}(\alpha, \beta) = \frac{3}{8\pi}\{\varrho_{00}\sin^2\beta + \varrho_{11}(1+\cos^2\beta) + \varrho_{1-1}\sin^2\beta\cos 2\alpha +$$

$$+2\sqrt{2}\,(\mathrm{Re}\,\varrho_{10}\sin\beta\cos\beta\cos\alpha - X\,\mathrm{Im}\,\varrho_{10}\sin\beta\sin\alpha)\}$$

where X is an unknown quantity which is a function of the two amplitudes T_1 and T_{-1} integrated over the Dalitz plot ($X = (F^{11}-F^{-1-1})/(F^{11}+F^{-1-1})$).

Defining the moments of a distribution $\overline{W}(\alpha\beta)$ by

$$\langle a(\alpha, \beta)\rangle = \int a(\alpha, \beta)\overline{W}(\alpha, \beta)d\cos\beta\, d\alpha$$

we obtain

$$\varrho_{00} = 2-5\,\langle\cos^2\beta\rangle$$

$$\mathrm{Re}\,\varrho_{10} = 5/2\sqrt{2}\,\langle\sin 2\beta\cos\alpha\rangle$$

$$\varrho_{1-1} = 5/2\,\langle\sin^2\beta\cos 2\alpha\rangle$$

$$X\cdot\mathrm{Im}\,\varrho_{10} = -4\sqrt{2}/3\pi\,\langle\sin\alpha\rangle$$

SCHC or TCHC mean that the last three moments have to vanish in the helicity or Gottfried-Jackson systems respectively. I would like to mention a property of this distribution. Assume that the K^- and π^- particles cannot be distinguished experimentally (20% of the events in the Q-region for the experiment $K^-p \to Q^-p \to (K^-\pi^+\pi^-)p$ at 10 GeV/c have this defect). The normal, which can be defined as $p_{K^-}\times p_{\pi^-}$ reverses the direction ($\beta \to \pi-\beta$, $\alpha \to \pi+\alpha$). An admixture of such ambiguous events therefore changes only the moment $X\cdot\mathrm{Im}\,\varrho_{10}$, but leaves the first three moments unchanged.

The whole formalism of this section can easily be generalized to the case where a generalized density matrix (various spin-parity values) is used for the decay into three particles [13]. The distribution of the normal for a $(1^+, 0^-)$ initial state is

$$\overline{W}^{0^-1^+}(\alpha, \beta) = C\{\varrho_{00,00}X_0 + X_1[\varrho_{10,10}\sin^2\beta +$$

$$+\varrho_{11,11}(1+\cos^2\beta) + 2\sqrt{2}\,\mathrm{Re}\,\varrho_{11,10}\sin\beta\cos\beta\cos\alpha +$$

$$+\varrho_{11,1-1}\sin^2\beta\cos 2\alpha] - X_2 2\sqrt{2}\,\mathrm{Im}\,\varrho_{11,10}\sin\beta\sin\alpha\} =$$

$$= \frac{3}{8\pi}\{\tfrac{2}{3} + A[(\varrho_{11,11}-\varrho_{10,10})(\cos^2\beta - \tfrac{1}{3}) +$$

$$+2\sqrt{2}\,\mathrm{Re}\,\varrho_{11,10}\sin\beta\cos\beta\cos\alpha + \varrho_{11,1-1}\times$$

$$\times\sin^2\beta\cos 2\alpha] - A\frac{X_2}{X_1}2\sqrt{2}\,\mathrm{Im}\,\varrho_{11,10}\sin\beta\sin\alpha\}$$

with:

$$C = 3/\{8\pi[\tfrac{3}{2}\,\varrho_{00,00}X_0+(1-\varrho_{00,00})X_1]\}$$

$$A = X_1/\{\tfrac{3}{2}\,\varrho_{00,00}X_0+(1-\varrho_{00,00})X_1\}.$$

X_0, X_1 and X_2 are functions of the three decay amplitudes (they are therefore unknown). We obtain (compare this with the corresponding moments discussed above)

$$A\,\mathrm{Re}\,\varrho_{11,00} = \frac{5}{2\sqrt{2}}\langle\sin 2\beta\cdot\cos\alpha\rangle$$

$$A\varrho_{11,1-1} = \tfrac{5}{2}\langle\sin^2\beta\cdot\cos 2\alpha\rangle$$

$$A\cdot\frac{X_2}{X_1}\,\mathrm{Im}\,\varrho_{11,00} = -\frac{4\sqrt{2}}{3\pi}\langle\sin\alpha\rangle$$

$$A\varrho_{10,10}+\frac{1+A(\varrho_{00,00}-1)}{3} = 2-5\langle\cos^2\beta\rangle.$$

For a vanishing 0^- contribution, $A = 1$ and the formulae are the same as discussed before. We see that even for a 0^- admixture, SCHC or TCHC predict that the first three moments vanish in helicity or Gottfried-Jackson systems.

2.5. Experimental results

In this part we discuss the experimental results concerning helicity conservation in two-body reactions which have been published recently.

2.5.1. SCHC in $\pi N\to\pi N$. A discussion of this process in terms of helicity amplitudes was given by Białas and Svensson [14].

Applying the formulae of section 2.1 we obtain for this process (unpolarized initial states assumed)

$$f_{\lambda_c\lambda_a}(s,t) = \sum_J d^J_{\lambda_a\lambda_c}(\vartheta)A^J_{\lambda_c\lambda_a}$$

where λ_a and λ_c have the values $\pm 1/2$ (for short $\pm$). Parity conservation gives

$$f_{++}(s,t) = f_{--}(s,t)$$

$$f_{+-}(s,t) = -f_{-+}(s,t).$$

With a convenient normalization we obtain

$$\frac{d\sigma}{dt} = |f_{++}|^2+|f_{+-}|^2$$

$$2\cdot\varrho^f_{\lambda_c\lambda_c'}\cdot\frac{d\sigma}{dt} = \begin{pmatrix}|f_{++}|^2+|f_{+-}|^2 & 2i\,\mathrm{Im}\,(f_{++}\cdot f^*_{+-})\\ -2i\,\mathrm{Im}\,(f_{++}\cdot f^*_{+-}) & |f_{++}|^2+|f_{+-}|^2\end{pmatrix} =$$

$$= \frac{d\sigma}{dt}(1+\vec{P}\vec{\sigma}).$$

$\vec{P}$ is the polarization of the final proton.

We find

$$P_x = P_z = 0$$

$$P_y = \frac{2\,\mathrm{Im}\,(f_{++}\cdot f^*_{+-})}{|f_{++}|^2+|f_{+-}|^2}$$

(the polarization is therefore perpendicular to the production plane).

The polarization and angular distribution do not determine the scattering amplitude. For this we need two further quantities S and T which are defined in the following way (S and T are related to the Wolfenstein parameters A and R, see *e.g.* Barger and Halzen [6]).

$$\frac{d\sigma}{dt} = |f_{++}|^2+|f_{+-}|^2$$

$$P\frac{d\sigma}{dt} = 2\,\mathrm{Im}\,(f_{++}f^*_{+-})$$

$$S\frac{d\sigma}{dt} = |f_{++}|^2-|f_{+-}|^2$$

$$T\frac{d\sigma}{dt} = 2\,\mathrm{Re}\,(f_{++}f^*_{+-}).$$

Expressing the $f_{\lambda_c\lambda_a}$ by the t-channel amplitudes $g_{\lambda_c\lambda_a}$ we obtain

$$\frac{d\sigma}{dt} = |g_{++}|^2+|g_{+-}|^2$$

$$P\frac{d\sigma}{dt} = -2\,\mathrm{Im}\,(g_{++}g^*_{+-})$$

$$S\frac{d\sigma}{dt} = -\cos 2\chi(|g_{++}|^2-|g_{+-}|^2)-\sin 2\chi\cdot 2\,\mathrm{Re}\,(g_{++}g^*_{+-})$$

$$T\frac{d\sigma}{dt} = -\sin 2\chi(|g_{++}|^2-|g_{+-}|^2)+\cos 2\chi\cdot 2\,\mathrm{Re}\,(g_{++}g^*_{+-})$$

χ is the crossing angle. SCHC means $f_{+-} = 0$ and TCHC means $g_{+-} = 0$. The polarization is therefore the same for SCHC and TCHC, namely zero, and we need another quantity to test helicity conservation. For SCHC $T = 0$ and for TCHC $T = -\sin 2\chi$ and this condition has to be checked.

For $\pi^-p \to \pi^-p$ data at 6 GeV/c the parameter T has been determined as a function of the four-momentum transfer [6] and the results, seen in Fig. 1, are well compatible

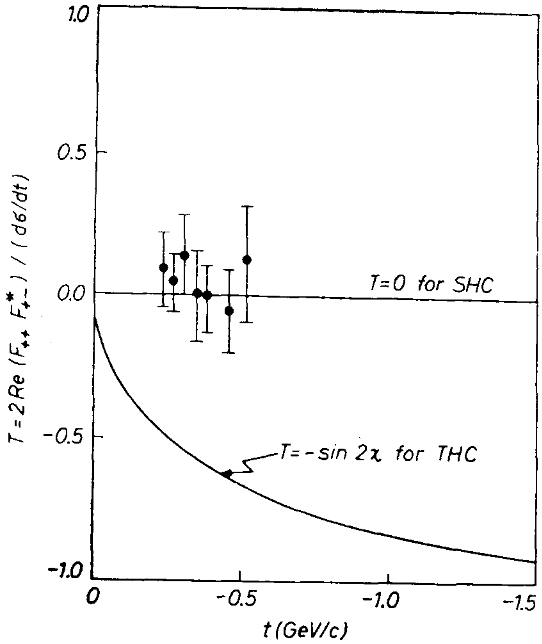


Fig. 1. The spin correlation parameter T (defined in the text) as a function of the four-momentum transfer t for $\pi^- p \to \pi^- p$ at 6 GeV/c, together with the predictions for SCHC ($T = 0$) and TCHC ($T = -\sin 2\chi$) (taken from Ref. [6])

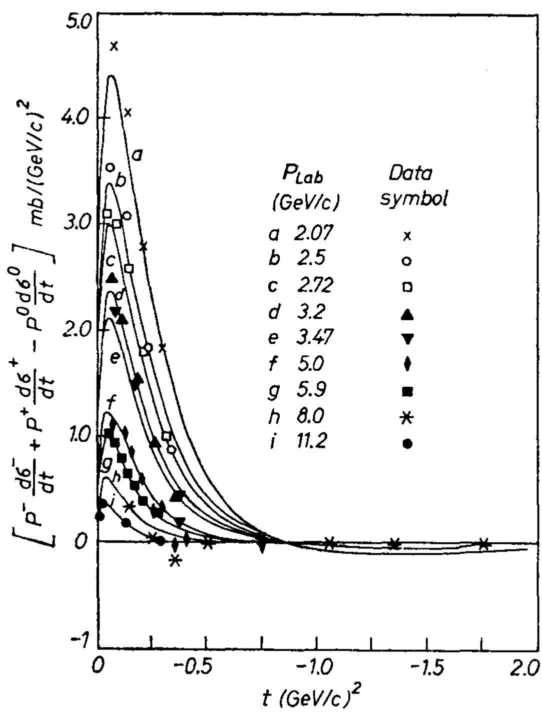


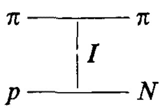
Fig. 2. Polarization data for the $I = 0$ t-channel part of $\pi N \to \pi N$ for various energies as a function of t (taken from Ref. [6])

with SCHC[1]. The polarization data from 2 to 11 GeV/c in the t-range from 0 to 2 (GeV/c)^2 for all three charge states have been used to calculate the isospin $I = 0$ t-channel part of the polarization. This is seen in Fig. 2. The polarization goes to zero with increasing energy with p_{lab}^{-2}.

The t-channel isospin decomposition

$$\pi^{\mp}p \to \pi^{\mp}p : f^0 \pm f^1$$

$$\pi^- p \to \pi^0 n : -\sqrt{2} f^1$$



Therefore the isospin $I = 0$ part is $\sigma^- + \sigma^+ - \sigma^0$.

2.5.2. $\gamma p \to \varrho^0 p \to (\pi^+\pi^-)p$. As already mentioned in the introduction, SLAC constructed a 92–94% linearly polarized photon beam by Compton backward scattering a low energy linearly polarized ruby laser beam on high energy electrons [16]. This was done at 2.8 and 4.7 GeV/c incident photon momentum to study the reaction $\gamma p \to \varrho^0 p \to (\pi^+\pi^-)p$ (the cross-sections for the reaction considered are 16.4 ± 1.0 μb and 14.4 ± 0.7 μb respectively). The density matrix elements of the ϱ^0 can easily be determined by the decay angular distribution of the ϱ^0 into two pions. This was done in the helicity and Gottfried-Jackson frames to test SCHC and TCHC. The density matrix elements are determined as functions of the four-momentum transfer, up to $|t| < 0.4$ GeV^2. As seen in Fig. 3 the data are well compatible with SCHC [4], [17].

Actually the density matrix elements were also determined in the Adair frame (ϱ rest frame with quantization axis along the incident photon direction, but with this axis defined in the total CM-system, and y-axis perpendicular to the production plane). This was done to test whether the reaction is spin-independent, that means conservation of the z-component of the spin. I do not want to talk about this.

2.5.3. $\gamma p \to \omega p \to (\pi^+\pi^-\pi^0)p$. It is interesting to test whether this reaction also shows SCHC. For the following reasons it is expected to be much harder to get a conclusive answer for this process

a) it is a 0-constraint fit in the bubble chamber (it is a 3 prong with π^0 unobserved)
b) besides Pomeron exchange, π-exchange may play an important role
c) the cross-section is much smaller than for ϱ photoproduction (~ 3 μb at 4 GeV for unpolarized photons).

The cross-section for unpolarized photons is seen in Fig. 4 (taken from Erbe *et al.* [3]). It is not flat as would be expected for a purely diffractive process. Using the linearly polarized photon beam, Ballam *et al.* [18] determined the density matrix elements of the ω *via* the decay into $\pi^+\pi^-\pi^0$ (same as for $\varrho \to \pi\pi$, now with the normal to the (3π) decay plane as analyser). For high energies the contributions from natural parity exchange

---

[1] See also Halzen and Michael [15] where the helicity amplitudes for the two t-channel isospin values — $f^1_{++}, f^1_{+-}, f^0_{++}, f^0_{+-}$ — have been calculated from $d\sigma^+/dt$, $d\sigma^-/dt$, $d\sigma^0/dt$, P^+, P^-, P^0 and the Wolfenstein parameter R^-. The signs +, −, 0 refer to $\pi^\pm p \to \pi^\pm p$ and $\pi^- p \to \pi^0 n$. It has been found $|f^0_{+-}|/|f^0_{++}| \sim 0.1$.

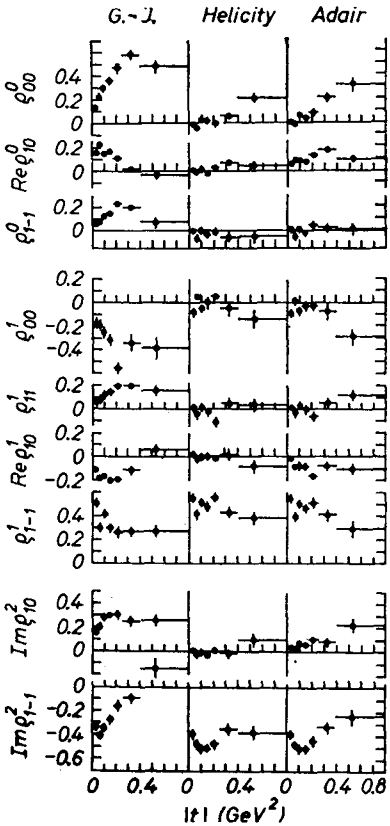


Fig. 3

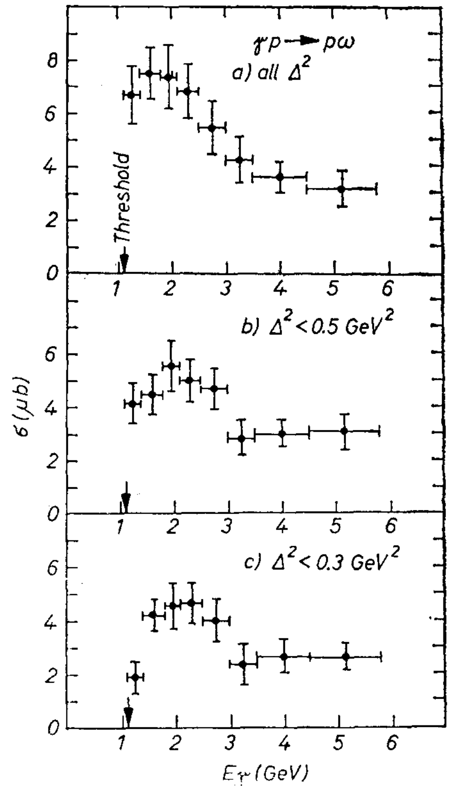


Fig. 4

Fig. 3. The spin-density matrix elements of the ϱ^0 for the reaction $\gamma p \to \varrho^0 p$ at 4.7 GeV as a function of t in the Gottfried-Jackson, helicity and Adair systems (taken from Ref. [17])

Fig. 4. Cross-section for $\gamma p \to \omega p$ as a function of the photon energy. a) for all t (four momentum transfer between incoming and outgoing proton), b) for $t < 0.5\ \mathrm{GeV}^2$, c) for $t < 0.3\ \mathrm{GeV}^2$ (taken from Ref. [3])

(probably Pomeron) and unnatural parity exchange (probably π) can be separated. It can be shown [9] that

$$\frac{\sigma_{\mathrm{nat}}-\sigma_{\mathrm{un}}}{\sigma_{\mathrm{nat}}+\sigma_{\mathrm{un}}} = 2\varrho^{(1)}_{1-1}-\varrho^{(1)}_{00}$$

σ_{nat} is the cross-section for natural parity exchange, whereas σ_{un} is the cross-section for unnatural parity exchange.

For the two energies considered, the cross-section for natural parity exchange is compatible with being constant and it could be produced by Pomeron exchange.

$$2.8\ \mathrm{GeV} \quad \sigma = 5.8 \pm 0.5\ \mu\mathrm{b} \quad \sigma_{\mathrm{nat}} = 2.50 \pm 0.37\ \mu\mathrm{b}$$

$$4.7\ \mathrm{GeV} \quad \sigma = 3.2 \pm 0.3\ \mu\mathrm{b} \quad \sigma_{\mathrm{nat}} = 1.84 \pm 0.28\ \mu\mathrm{b}$$

Assuming a mixture between s-channel helicity-conserving Pomeron exchange and π-exchange amplitudes, the predictions for the density matrix elements are

$$\varrho_{ik} = \frac{d\sigma_{\text{nat}}/dt \cdot \varrho_{ik}^{\text{nat}} + d\sigma_{\text{un}}/dt \cdot \varrho_{ik}^{\text{un}}}{d\sigma_{\text{nat}}/dt + d\sigma_{\text{un}}/dt}$$

σ_{ik}^{nat} is the s-channel helicity conserving density matrix (diagonal in the helicity frame) and σ_{ik}^{un} is the density matrix for π-exchange, which is diagonal in the Gottfried-Jackson frame. The predictions and the results are seen in Fig. 5 and are in reasonable agreement.

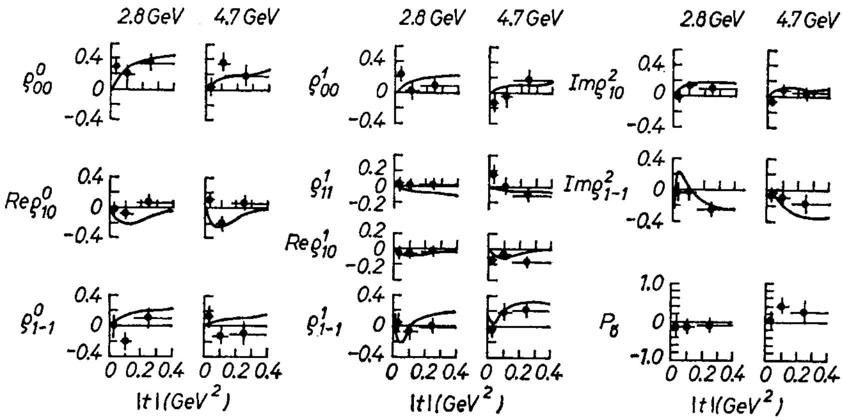


Fig. 5. The spin-density matrix elements of the ω and the parity asymmetry $P_\sigma = (\sigma_{\text{nat}} - \sigma_{\text{un}})/(\sigma_{\text{nat}} + \sigma_{\text{un}})$ for the reaction $\gamma p \to \omega p$ at 2.8 and 4.7 GeV as a function of t in the helicity system (taken from Ref. [18])

2.5.4. $\pi^+ p \to A_1{}^\pm p \to (\pi^\pm \pi^+ \pi^-) p$. Helicity conservation for this diffractive process has been studied by the ABBCCHLV-collaboration [19] for the 8 GeV/c π^+ and 16 GeV/c π^+ and π^- experiments. Assuming that spin-parity 1^+ is the dominant contribution in the A_1-mass region the predictions for the A_1-density matrix elements are

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{array}{l} \text{in Gottfried-Jackson system for TCHC} \\ \text{in helicity system for SCHC} \end{array}$$

The experimental results have been determined by calculating some moments and are almost compatible with TCHC. SCHC can be ruled out. Fig. 6 shows ϱ_{00} for A_1 (and also for Q^- which will be discussed in 2.5.5) for different values of the four-momentum transfer in the Gottfried-Jackson and helicity systems. The other density matrix elements are not plotted but they are not far away from the predictions of TCHC[2].

As we have seen in 2.4.2, a 0^- admixture does not drastically change our results, when we calculate the moments. The ϱ_{00} of spin 1^+ changes, but the remaining non-diagonal elements for 1^+ are only differently normalized and the corresponding moments have to vanish in the Gottfried-Jackson system for TCHC. A more refined analysis for the A_1^-

[2] In section 3.2.2 will be shown (from a more general analysis applied to these data) that there are indeed small systematical deviations from TCHC.

mass region was done in Illinois [20], [21]. I would like to keep the discussion of this analysis short because Kotański will treat it in detail in his lectures. For various experiments ($\pi^- p \to \pi^-\pi^+\pi^- p$ at 5 GeV/c, 7 GeV/c, 7.5 GeV/c, 11 GeV/c, 13 GeV/c, 20 GeV/c

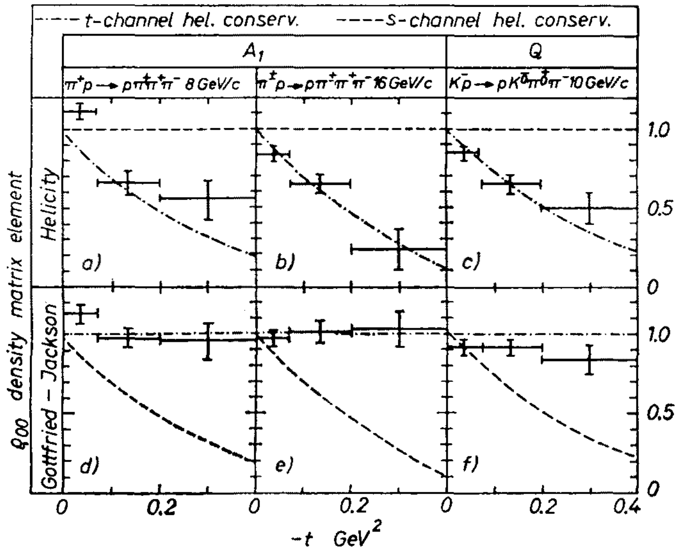
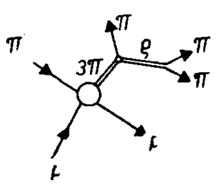


Fig. 6. The spin-density matrix element ϱ_{00} of the A_1 and Q in the helicity and Gottfried-Jackson systems for the reactions $\pi^+ p \to A_1^+ p$ at 8 and 16 GeV/c, $\pi^- p \to A_1^- p$ at 16 GeV/c and $K^- p \to Q^- p$ at 10 GeV/c as as a function of t. The dot-dash and dashed curves are predictions of TCHC and SCHC (taken from Ref. [19])

and 25 GeV/c) they made a partial wave analysis of the (3π)-system in the mass range 1.0 to 1.4 GeV. In this analysis the reaction is treated as a two step process: production of the (3π) system followed by a decay *via* $\varrho\pi$.



$$T = \sum_{\substack{\text{Spin-parity} \\ \text{of } (3\pi)}} T^{\text{Pr}}(\pi^- p \to (3\pi)p) \cdot T^{\text{Dec}}(3\pi \to \varrho\pi \to (\pi\pi)\pi)$$

The production amplitudes T^{Pr} define the spin states of the (3π) system and give the production density matrices.

$$\varrho_{S\eta\lambda, S'\eta'\lambda'}$$

The decay amplitude can easily be written down for the decay (3π) $\to$ ($\varrho\pi \to (\pi\pi)\pi$. By a maximum likelihood method the experimental data were fitted with the theoretical predictions to get the best fit for the density matrix elements.

The density matrix elements for the 1^+ state (A_1) agree very well with TCHC.

In the last mentioned method it is assumed that the production of the (3π) system in the A_1 region can be split into a production and a decay process, that means that the amplitude factorizes. This is an assumption which has to be tested.

There exists a third paper in which helicity conservation in A_1 production is studied. The Paris–Milano–Genova–Durham-collaboration [22] analyse the reactions

$$\left.\begin{array}{l}\pi^+ d \to (\pi^+\pi^-\pi^+)d \\ \pi^+ n \to (\pi^+\pi^+\pi^-)n \\ \pi^- p \to (\pi^-\pi^-\pi^+)p\end{array}\right\} \text{ at } 11 \text{ GeV}/c.$$

Assuming a pure 1^+ state for A_1, the density matrix elements are given in Fig. 7. They are calculated by using the normal to the (3π) decay plane as analyser. The non-diagonal elements are compatible with TCHC, but the diagonal matrix element ϱ_{00} lies away from 1 in the Gottfried-Jackson frame. The authors also use the unambiguous π as analyser

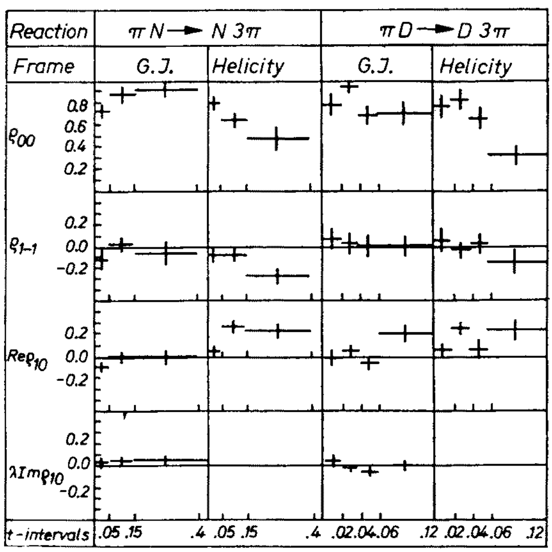


Fig. 7. The spin-density matrix elements of the A_1 in the helicity and Gottfried-Jackson systems for the reactions $\pi N \to A_1 N$ and $\pi^+ d \to A_1^+ d$ at 11 GeV/c as a function of t (taken from Ref. [22])

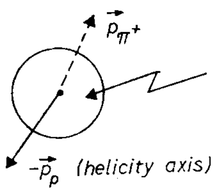
and find that 0^- waves contribute. They find an asymmetry in cos ϑ and a φ distribution which shows structure. Therefore non-TCHC-amplitudes must be present. In my opinion these deviations from TCHC are not too serious, especially because this data (at least in part) have been used in the analysis of the Illinois group who support TCHC.

2.5.5. $K^\pm p \to Q^\pm p \to (K^\pm\pi^+\pi^-)p$. When testing helicity conservation in the K^+ experiments, one has to realize that the Q^+ peak is obscured due to the presence of Δ^{++} from the reaction

$$K^+ p \to K^{*0}_{890}\Delta^{++} \to (K^+\pi^-)(\pi^+ p).$$

These Δ^{++}-events have to be excluded. If there is only 1 spin-parity state, then, due to parity conservation in the Q-decay, all distributions are identical when the momenta of the three-particles are reversed. One half of the distribution is in principle enough (see 2.4.2.) Therefore all events which satisfy the following cut in the Q rest-frame are excluded [23].

$$(\vec{p}_{\pi^+} \cdot \vec{p}_p) \geqslant 0$$



plane perpendicular to the momentum $\vec{p}_p$ (events behind this plane are excluded)

This cut removes all Δ^{++} events and the remaining distribution of the normal to the $(K\pi\pi)$ plane is not biased and can be used for a determination of the density matrix elements. It should be noted that this is true when only one spin-parity state contributes. The spin--density matrix elements are calculated assuming 1^+ for the Q-region and the results are seen in Fig. 8. The ϱ_{00} is plotted in the Gottfried-Jackson and helicity frames as a function of the four-momentum transfer for various values of the incident K^+ momentum. These values and also the other density matrix elements are compatible neither with

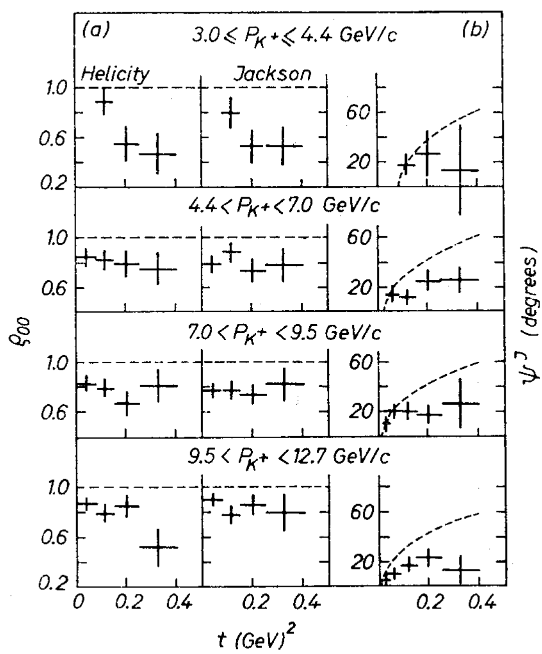


Fig. 8. The spin-density matrix element ϱ_{00} of the Q in the helicity and Gottfried-Jackson systems for the reaction $K^+p \to Q^+p$ and the position of the system with least azimuthal dependence for various incident K^+ momenta (1. 3.0–4.4 GeV/c, 2. 4.4–7.0 GeV/c, 3. 7.0–9.5 GeV/c, 4. 9.5–12.7 GeV/c) as a function of t (taken from Ref. [23])

TCHC nor with SCHC. The rotation angle (starting from the Gottfried-Jackson system) for the system for which the density matrix is most diagonal has also been determined. This system is always between the Gottfried-Jackson and helicity frames. A warning about this method: it is surely not true that there is only one spin state contributing despite the fact that probably one is dominating. The analysis is therefore doubtful.

The Q^- production reaction is more reliable for a test of helicity conservation. The problem with the Δ^{++} is not so important since in K^-p experiments much less Δ^{++} is produced than in K^+p experiments.

Two collaborations have published their data for the Q-region ($K^-p \to Q^-p$):
a) the ABBCCHLV-collaboration, at 10 GeV/c [19]
b) the ADLV-collaboration, at 8 GeV/c [24].

Assuming a pure 1^+ state the density matrix elements have been determined for the Q-region as a function of the four-momentum-transfer (or in Ref. [24] of the crossing

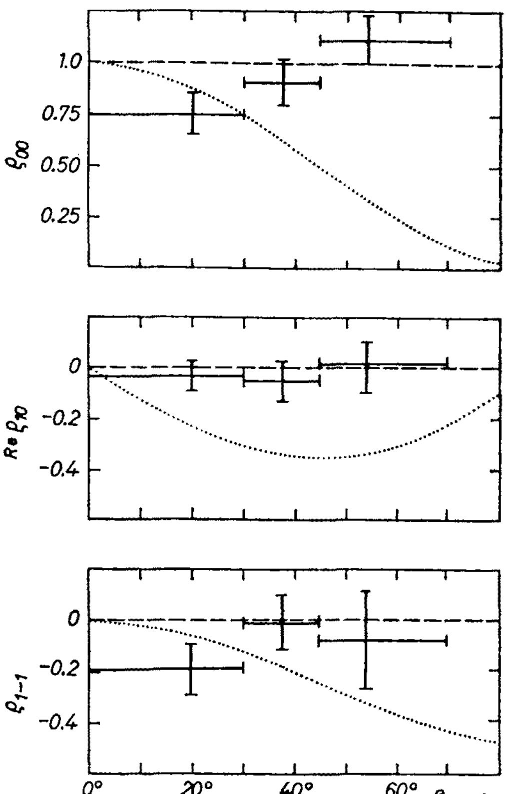


Fig. 9

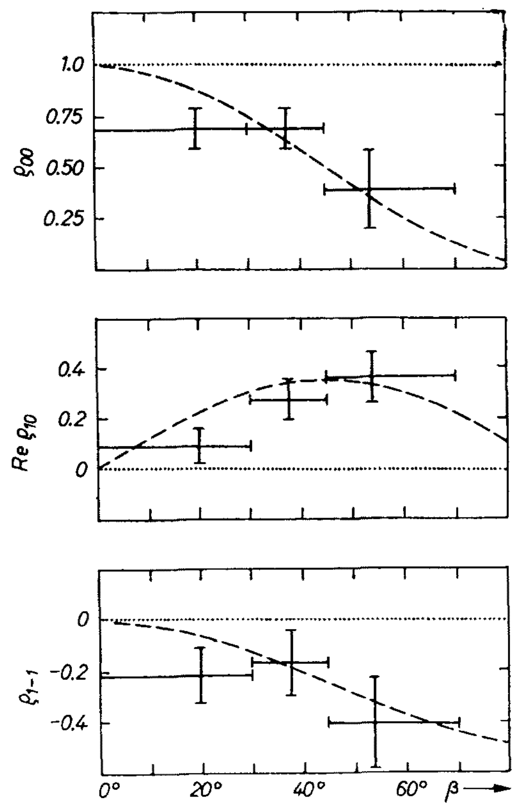


Fig. 10

Fig. 9. The spin-density matrix elements of the Q in the Gottfried-Jackson system for the reaction $K^-p \to Q^-p$ at 8.25 GeV/c as a function of the crossing angle. The predictions of SCHC (dotted line) and TCHC (broken line) are shown (taken from Ref. [24])

Fig. 10. The spin-density matrix elements of the Q in the helicity system for the reaction $K^-p \to Q^-p$ at 8.25 GeV/c as a function of the crossing angle. The predictions of SCHC (dotted line) and TCHC (broken line) are shown (taken from Ref. [24])

angle, which is a function of the four-momentum transfer for a fixed mass of the $(K\pi\pi)$ system, see Fig. 13).

The results are seen in Figs 6, 9 and 10 and agree in first approximation with TCHC. A 0^- admixture does not change drastically our results as explained in 2.4.2 and 2.5.4. Another type of bias, that just in the Q-region about 20% of the events are such that K^- and π^- cannot be distinguished (we have for such events two ambiguous hypotheses), has no effect on the density matrix elements ϱ_{00}, Re ϱ_{10} and ϱ_{1-1} as explained in 2.4.2.

2.5.6. $\pi^+p(d) \to A_3^+p(d) \to (\pi^+\pi^+\pi^-)p(d)$. Paler *et al.* [25] tried to determine the density matrix elements for the A_3 in the reactions

$$\left.\begin{array}{l}\pi^+d \to A_3^+d \\ \pi^+p \to A_3^+p\end{array}\right\} \text{ at 13 GeV}/c.$$

The background under the A_3 is not too large for the d-reaction and for the p-reaction only $(f^0\pi)$ events in the A_3-region were used. Assuming spin-parity 2^- the diagonal density matrix elements have been determined as a function of the four-momentum-transfer. The results for ϱ_{00} are seen in Fig. 11. They are compatible with TCHC.

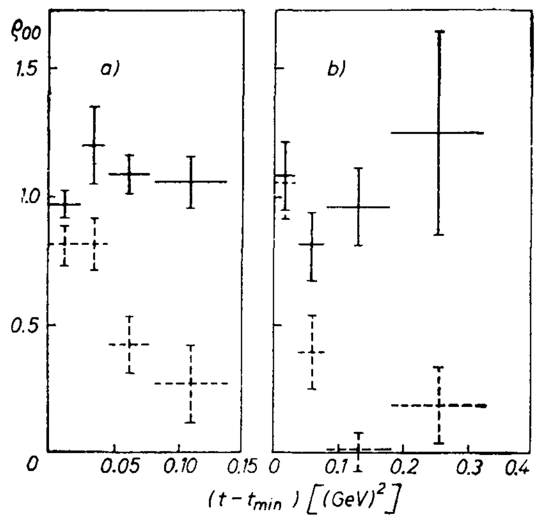


Fig. 11. The spin-density matrix element ϱ_{00} of the A_3 in the helicity (broken crosses) and Gottfried-Jackson systems (unbroken crosses) for the reactions $\pi^+d \to A_3^+d$ (a) and $\pi^+p \to A_3^+p$ (b) at 13 GeV/c as a function of $t' = t - t_{\min}$ (taken from Ref. [25])

2.5.7. $\pi^\pm p \to \pi^\pm N^*(1700) \to \pi^\pm(p\pi^+\pi^-)$. The Purdue-Notre Dame collaboration [26] have studied the diffractively produced $N^*(1700)$ in

$$\pi^+p \to \pi^+(\pi^+\pi^-p) \text{ at 13 and 18 GeV}/c$$

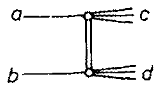
$$\pi^-p \to \pi^-(\pi^-\pi^+p) \text{ at 8 GeV}/c.$$

The $N^*(1700)$ is seen above a large background. The whole region is believed to be in a $5/2^+$ state for which density matrix elements have been determined. The results are compatible with TCHC.

3. *Helicity conservation for the general case*

3.1. Theoretical considerations

Up to now we have considered only two-body or quasi-two-body diffractive processes. Let us consider now the general case in which two particles, a and b produce diffractively two packets of particles c and d.



Such a process can always be described in the s-channel in terms of an infinite sum over amplitudes with all possible helicities of the packets c and d. SCHC means that there are only non-zero amplitudes when the helicity of c is that of a, the helicity of d is that of b in an s-channel picture. This give simple tests which can be checked, as pointed out by Cohen-Tannoudji *et al.* [27]. The distribution for each particle or particle combination (normal to two particles) of the packet c (or d) has to have a flat azimuthal distribution in the helicity frame of the packet c (or d). Similarly, TCHC can be tested since it predicts flat azimuthal distributions in the Gottfried-Jackson system of the packet c or d respectively. This last test is just the Treiman-Yang test [28].

I would like to derive the formulae needed for a special case which has been tested experimentally quite carefully.

$$ab \to (c_1c_2c_3)d\,.$$

For this case the packet c consists of 3 pseudoscalar particles and d is a nucleon (*e.g.* $K^{\pm}p \to (K^{\pm}\pi^{\mp}\pi^{-})p$ or $\pi^{\pm}p \to (\pi^{\pm}\pi^{+}\pi^{-})p$).

The transition amplitude for this process is as seen in 1.3 in the CM-system (z-axis along incident particle, a, y-axis perpendicular to the plane formed by the particles b and d):

$$\begin{aligned} f_{\lambda_d\lambda_b}(\vec{p}_a\vec{p}_b\vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_d) &= \langle \vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_a\lambda_d|T|\vec{p}_a\vec{p}_b\lambda_b\rangle = \\ &= f_{\lambda_d\lambda_b}(s,\, t,\, M_{123},\, \kappa_1,\, \kappa_2,\, \alpha\beta\gamma) = \\ &= \langle \vec{P} = 0,\, 0\vartheta M,\, \alpha\beta\gamma M_{123}\kappa_1\kappa_2,\, \lambda_d|T|\vec{P} = 0,\, 00M,\, \lambda_b\rangle = \\ &= \sum_{J_{123}\mu K} D^{J_{123}*}_{\mu K}(\alpha\beta\gamma)g^{J_{123},K}_{\mu\lambda_d\lambda_b}(s,\, t,\, M_{123},\, \kappa_1,\, \kappa_2) \end{aligned}$$

$\vec{p}_i$ are the momenta of the particles in the CM-system. J_{123} is the angular momentum of the system (c_1 c_2 c_3), μ is the helicity and K is the component of the angular momentum along the direction given by the two Euler-angles ($\alpha\beta$).

The Euler angles ($\alpha\beta\gamma$) are defined in the ($c_1c_2c_3$) rest frame with z-axis along the ($c_1c_2c_3$) helicity axis (therefore along $-\vec{p}_d$), y-axis perpendicular to the plane formed by the particles b and d, x-axis such that a right handed coordinate frame is obtained (this is the helicity system). They specify the orientation of the momentum triangle. ($\alpha\beta\gamma$) give either the directions of the normal plus particle or the directions of the particle plus normal to the ($c_1c_2c_3$) plane. Instead of the invariance of the T operator against parity operation it is more convenient to use the invariance against the reflection in the $x-z$-plane ($Y = P \cdot R(0, \pi, 0)$: $TY = YT$

$$f_{\lambda_d\lambda_b}(\vec{p}_a\vec{p}_b\vec{p}_1\vec{p}_2\vec{p}_3\vec{p}_d) =$$

$$= \frac{\eta_a\eta_b}{\eta_1\eta_2\eta_3\eta_d} \prod_{i=1} (-)^{S_i-\lambda_i} f_{-\lambda_d-\lambda_b}(\vec{p}_a\vec{p}_b\vec{p}_1^R\vec{p}_2^R\vec{p}_3^R\vec{p}_d) =$$

$$= (-)^{1-\lambda_b-\lambda_d} f_{-\lambda_d-\lambda_b}(\vec{p}_a\vec{p}_b\vec{p}_1^R\vec{p}_2^R\vec{p}_3^R\vec{p}_d)$$

p_i^R are the momenta of particles $c_1c_2c_3$ reflected in the $x-z$-plane. Note that p_a, p_b, p_d which define the $x-z$-plane, are unchanged.

The angular distribution of the particles c_1, c_2, c_3 for unpolarized initial state and no observation of the final polarization has therefore to be symmetric with respect to a reflection in the $x-z$-plane. This is also true for a Lorentz transformation within the $x-z$-plane, *e.g.* for the helicity or Gottfried-Jackson-systems.

In the helicity or Gottfried-Jackson system the distribution for the normal to the plane formed by the momenta of c_1, c_2 and c_3, has to be symmetric with respect to a rotation around the y-axis by π.

The angular distribution for a particle direction or the normal has the following symmetry relation (for a fixed value of s, t, M_{123}, κ_1, κ_2).

particle: $\overline{W}(\alpha, \beta) = \overline{W}(-\alpha, \beta)$

normal: $\overline{W}(\alpha, \beta) = \overline{W}(\pi-\alpha, \pi-\beta)$

$\alpha\beta$ are the first two Euler angles (or azimuthal and polar angle).

Applying SCHC to the example considered we have

$$g^{J_{123},K}_{\mu\lambda_d\lambda_b} = \bar{g}^{J_{123},K}_{\mu\lambda_d\lambda_b} \cdot \delta_{\mu 0} \cdot \delta_{\lambda_b\lambda_d}.$$

The amplitude $f_{\lambda_d\lambda_b}$ has therefore no α-dependence. SCHC requires a flat distribution of the three particle directions c_1, c_2, c_3 and of the normal to this plane around the helicity quantization axis.

The proof for a test of TCHC is exactly the same and has the same prediction for the t-channel quantization axis.

I would like to stress that these tests yield only necessary but not sufficient conditions for helicity conservation.

All experimental tests which have been published use in different forms the test of Cohen-Tannoudji *et al.*, for SCHC and the Treiman-Yang test for TCHC.
There are essentially three kinds for this test:

a) just check whether the α-dependence is flat

b) test it by a moments analysis.

Here the angular distribution is decomposed in spherical harmonics and the expansion coefficients are the moments:

$$W(\alpha, \beta) = \sum_{lm} \langle Y_{lm} \rangle Y^*_{lm}(\alpha\beta).$$

The following conditions have to be fulfilled for the moments:

	particle	normal
reality	$\langle Y_{j-m}\rangle = (-)^m \langle Y_{jm}\rangle^*$	$\langle Y_{j-m}\rangle = (-)^m \langle Y_{jm}\rangle^*$
parity conservation	$\langle Y_{j+m}\rangle^* = \langle Y_{jm}\rangle$	$\langle Y_{j+m}\rangle^* = (-)^j \langle Y_{jm}\rangle$
helicity conservation	$\langle Y_{jm}\rangle = 0$ for $m \neq 0$	$\langle Y_{jm}\rangle = 0$ for $m \neq 0$

The first condition reflects the fact that the angular distribution is a real function. The second condition follows from the symmetry due to parity conservation discussed before. The third condition is true for TCHC in Gottfried-Jackson and SCHC in the helicity system and follows from the fact that no α-dependence is expected. Furtheron the moments with $j > 2J$ are zero if J is the highest angular momentum present in the sample.

c) test in LPS

Distributions in LPS are often used to select diffractive parts of the reaction considered. Rotation of an event around the s-channel (or t-channel) helicity axis changes the position of the event in LPS. This rotation corresponds to a complicated curve in LPS. When helicity is conserved, the events are distributed isotropically around the helicity axis. An arbitrary rotation around the helicity axis does not therefore change the physical situation and the density of events on specific parts of the curve remains the same. SCHC (or TCHC) is therefore checked if the population in LPS stays the same in every region when an arbitrary rotation around the s-channel (t-channel) helicity axis is performed.

3.2. Experimental results

In this part we discuss the experimental results concerning helicity conservation for the general case. We make no specific assumptions on the spin and parity content of the packets c and d, nor on the production mechanism (it is not important whether the particles in c and d come from resonances *etc.*).

The following experiments investigated the question of helicity conservation (data are only available for single diffraction dissociation, that means one of the packets c or d consists only of one particle):

a) the system c consists of two particles

$$\left.\begin{aligned} \pi^+p &\to (n\pi^+)\pi^+ \\ &\to (p\pi^0)\pi^+ \\ \pi^-p &\to (p\pi^0)\pi^- \end{aligned}\right\} \text{ at 11 GeV}/c \text{ (Durham–Genova–DESY–Milano–Saclay–collaboration)}$$

b) the system c consists of three particles

$$\left.\begin{aligned} K^-p &\to (K^-\pi^+\pi^-)p \\ &\quad\ (p\pi^+\pi^-)K^- \end{aligned}\right\} \text{ at 10 GeV}/c \text{ (ABBCCHLVW-collaboration)}$$

$\pi^{\pm}p \rightarrow (\pi^{\pm}\pi^{+}\pi^{-})p$ ⎫ at 8 and 16 GeV/c (ABBCCHLVW and ABBCH-collaboration)
$\qquad (p\pi^{+}\pi^{-})\pi^{\pm}$ ⎭ at 11 GeV/c (DGDMS-collaboration)

$pp \rightarrow (p\pi^{+}\pi^{-})p$ at 16 GeV/c (Cambridge–IC London collaboration)
at 19 GeV/c (Scand. collab.)

3.2.1. $\pi^{\pm}p \rightarrow \pi^{\pm}(N\pi)^{+}$. The Durham–Genova–Milano–DESY–Saclay collaboration tested helicity conservation in the 3 processes just mentioned [29]. They believe that the reaction

$$\pi^{+}p \rightarrow \pi^{+}\pi^{+}n$$

is the best candidate for this analysis among those considered. This is due essentially to the very weak production of resonances between the particles. To select mainly diffractive events, the following cut, which does not introduce any biases in the distributions, was used

$$M(n\pi^{+}_{\text{slow}}) < 1.8 \text{ GeV}$$
$$t\pi^{+}\pi^{+}_{\text{fast}} < 0.7 \text{ GeV}^2.$$

Calculating the azimuthal distribution of the π^{+}_{slow} in the Gottfried-Jackson and helicity frames they find the distributions in Fig. 12. The distributions in both frames are for all three reactions. The dashed histogram shows the azimuthal distribution in the strictly forward direction ($t_{\pi\pi} < 0.08$ GeV^2). As you see the distributions (especially in the Gottfried-Jackson frame) are quite flat except just at $\alpha = 0^0$ (the n and π^{+}_{S} are then in the production plane). The authors believe that the spike in the first bin is a true physical effect and conclude that both SCHC and TCHC are violated.

A warning: all these reactions are 1-constraint fits and the data are less reliable. It seems to me peculiar that there is a spike just in one bin. Could it not be possible that these events are not real $n\pi^{+}\pi^{+}$ events? Forgetting the first bin, I think, the data are compatible with TCHC.

3.2.2. $\pi^{\pm}p \rightarrow (\pi^{\pm}\pi^{+}\pi^{-})p$ and $K^{-}p \rightarrow (K^{-}\pi^{+}\pi^{-})p$. The ABBCCHLVW-collaboration studied helicity conservation with $\pi^{-}p$, $\pi^{+}p$ interactions at 16 GeV/c and with $K^{-}p$ interactions at 10 GeV/c [30], [31]. The low mass $(\pi^{\pm}\pi^{+}\pi^{-})$ and $(K^{-}\pi^{+}\pi^{-})$ diffractive events were used for such an investigation. Starting from the Gottfried-Jackson-system for the $(\pi\pi\pi)$ or $(K\pi\pi)$ respectively, a series of coordinate frames are defined by a rotation around the y-axis (the helicity system is reached after a rotation which is equal to the crossing angle). In these rotated coordinate systems, moments analyses for the three particle directions and their normal have been carried out. Among the various rotated coordinate systems that coordinate frame is determined which gives the most isotropic distribution, by calculating the minimum of the following χ^2 as a function of the rotation angle ψ:

$$\chi^2(\psi) = \sum_{l,m\neq 0}^{l=6} \left(\frac{\langle Y_{lm}\rangle}{\Delta\langle Y_{lm}\rangle}\right)^2 .$$

The moments of this expansion have to be zero (within their errors) if no azimuthal dependence is seen. The angle $\psi = 0$ corresponds to the Gottfried-Jackson system and ψ is

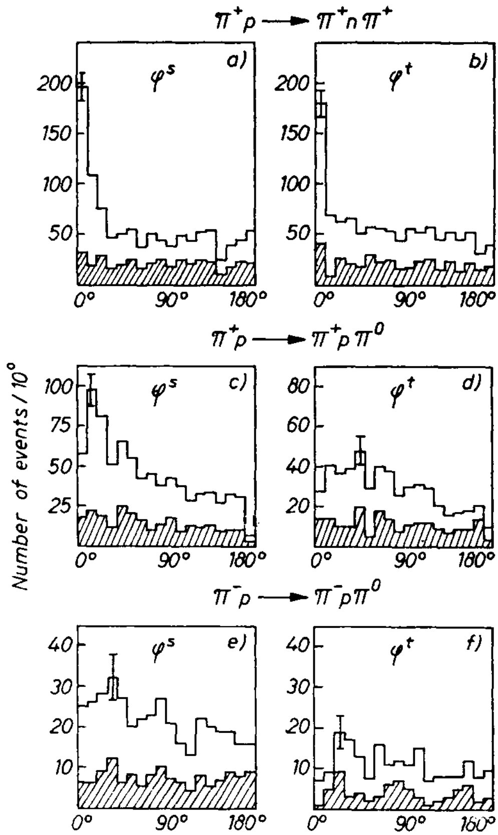


Fig. 12. s-channel (φ^s) and t-channel (φ^t) azimuthal distributions for reactions $\pi^+p \to \pi^+n\pi^+$ (1), $\pi^+p \to$ $\to \pi^+p\pi^0$ (2) and $\pi^-p \to \pi^-p\pi^0$ (3) at 11 GeV/c. The following cuts have been applied: (1) mass ($n\pi^+_{\text{slow}}$) < < 1.8 GeV and $t_{\pi^+\pi_{\text{fast}}}$ <0.7 GeV^2 or < 0.08 GeV^2 (shaded histogram) (2) for φ^s: mass ($p\pi^0$) <2.0 GeV, mass ($\pi^+\pi^0$)>1.0 GeV and $t_{\pi^+\pi^+}$ <0.7 GeV^2 or 0.06 GeV^2 (shaded histogram); for φ^t: mass ($p\pi^0$) <1.5 GeV, $t_{\pi^+\pi^+}$ <0.7 GeV^2 or 0.06 GeV^2 (shaded histogram); (3) for φ^s: mass ($p\pi^0$) <2.0 GeV, mass ($\pi^-\pi^0$) >1.0 GeV and $t_{\pi^-\pi^-}$< 0.7 GeV^2 or 0.06 GeV^2 (shaded histogram); for φ^t: mass ($p\pi^0$) <1.4 GeV and $t_{\pi^-\pi^-}$ <0.7 GeV^2 or 0.06 GeV^2 (shaded histogram) (taken from Ref. [29])

the crossing angle for the helicity frame.[3] The maximum value of l in the above sum has been limited to six, since the moments analysis shows that higher values of l do not contribute in the mass regions considered. The χ^2 is calculated for various ($\pi\pi\pi$) and ($K\pi\pi$) masses and various four-momentum transfers (we actually calculate the χ^2 as a function of the crossing angle which is related to the four-momentum transfer as seen in Fig. 13). The distributions for the K^--experiment are seen in Fig. 14. From these results we conclude that when the K^-, π^- momenta and the normal to these directions are taken as analysers, the minimum system (the frame with the least azimuthal dependence) lies close to the Gottfried-Jackson frame, whereas with the π^+ momentum as analyser one has

[3] It is to be noted that this method is sensitive to all α dependences in (cos $\beta - \alpha$) distribution, even including those which vanish when one projects out the α-distribution (method a) in section 3.1).

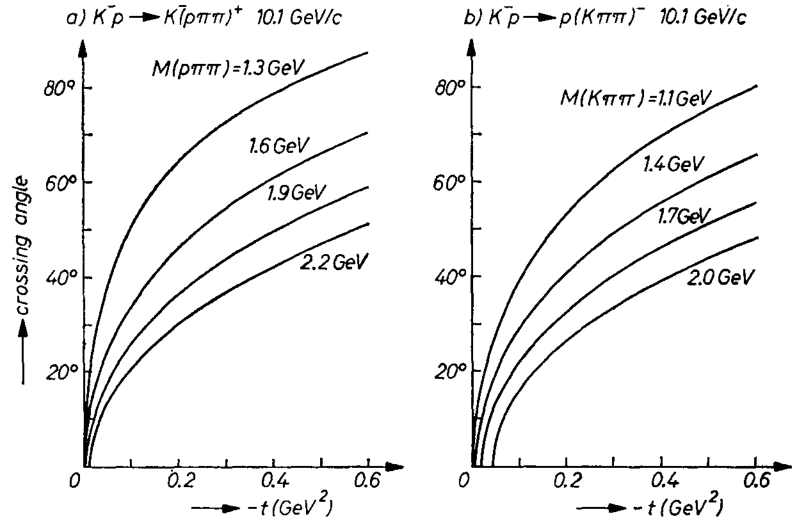


Fig. 13. The relation between crossing angle and the four momentum transfer t for various values of the masses ($p\pi^+\pi^-$) and ($K^-\pi^+\pi^-$) for the reaction $K^-p \to K^-\pi^+\pi^-p$ at 10 GeV/c (taken from Ref. [31])

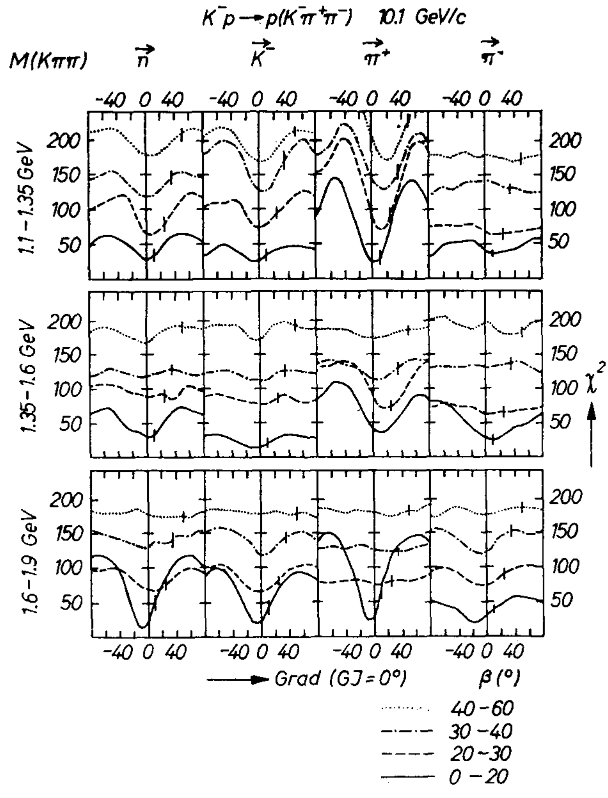


Fig. 14. $\chi^2(\psi)$ (as explained in the text) for the analysers K^-, π^+, π^- and the normal n to the ($K\pi\pi$) plane for various values of the masses ($K\pi\pi$) and crossing angles for the reaction $K^-p \to (K^-\pi^+\pi^-)p$ at 10 GeV/c (taken from Ref. [31])

a minimum system which systematically lies between the helicity and Gottfried-Jackson frames. One analyser which does not fulfil the predictions of helicity conservation is enough to exclude this hypothesis. We therefore conclude from this analysis that neither SCHC nor TCHC holds. The results are similar for the $(\pi\pi\pi)$ system. Except for the unlike pion analyser, the minimum system tends to lie close to the Gottfried-Jackson frame. The unlike pion fails TCHC.

The Paris–Milano–Genova–Durham-collaboration [22] studied the reactions

$$\pi^+ n \to (\pi^+\pi^+\pi^-)n$$
$$\pi^- p \to (\pi^+\pi^-\pi^-)p$$

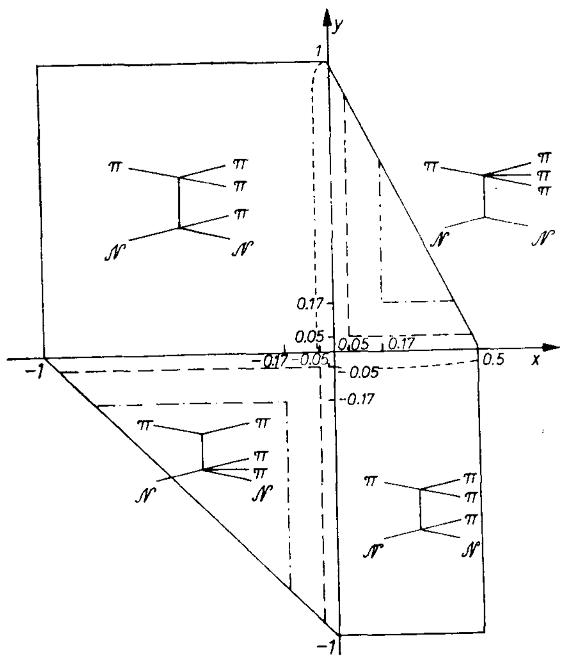


Fig. 15. Longitudinal phase space (LPS) plot. X and Y are the reduced longitudinal momenta of the slow charge — ambiguous pion and of the unambiguous pion. The dashed and dot-dashed lines correspond to the LPS region used to draw the azimuthal distributions (taken from Ref. [22])

at 11 GeV/c and investigated the events in the longitudinal momentum plot (see Fig. 15), where diffraction dissociation is dominant. They checked for this sample of events whether the azimuthal distributions are flat. The phase space part selected is itself a bias. If one chooses a particular event belonging to this region and rotates it (that means, rotates the momenta of the particles around the relevant helicity axis), the event may eventually leave the selected region. The selection of these events produces a cut on the azimuthal distribution and the distribution is not flat even if helicity is conserved. To avoid this bias, only events are used which do not leave the selected region when the rotation is performed. The results are seen in Fig. 16 where the azimuthal distributions of the normal

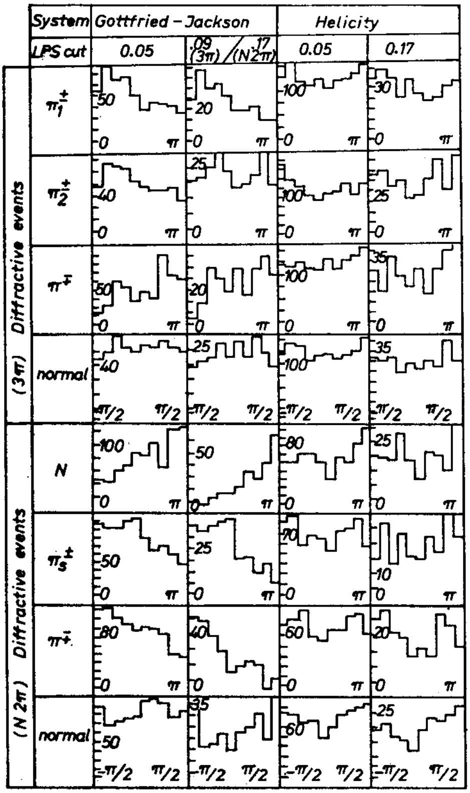


Fig. 16. Azimuthal distributions of the particles and the normal to the three-particle system for the LPS-selected events (see Fig. 15) around the s- and t-channel helicity axis for the reactions $\pi^+d \to \pi^+\pi^+\pi^-d$, $\pi^+n \to \pi^+\pi^+\pi^-n$ and $\pi^-p \to \pi^-\pi^+\pi^-p$ at 11 GeV/c (taken from Ref. [22])

and the three particle directions are plotted in the two frames. The various columns correspond to the selected region as seen in Fig. 15.

The data show a much better compatibility with SCHC than with TCHC which is very surprising, since the density matrix elements for the A_1 for the same data violate TCHC only weakly.

The reason for this result is the selection used. With the cut used about 20% of the events leave the appropriate LPS region for rotation around the s-channel helicity axis and 75% for rotation around the t-channel helicity axis. But even the 20% are of a certain type (they remain in the same LPS region in case of a rotation for $t_{pp\text{-minimum}}$) as pointed out by Meunnier and Plaut [32]. These two authors show that for a model for which SCHC is strongly violated (reggeised Deck effect [33]) the cuts applied give compatibility with SCHC. The results of Ref. [22] are therefore doubtful.

A similar analysis in the LPS has been done by the ABBCH collaboration [34] for the reactions

$$\pi^-p \to (\pi^-\pi^-\pi^+)p \quad \text{at 16 GeV}/c$$
$$\pi^+p \to (\pi^+\pi^-\pi^+)p \quad \text{at 8 and 16 GeV}/c.$$

The events have been studied in LPS. As explained in section 3.1 helicity conservation requires that the rotation around the helicity axis does not change the population in LPS. The effect can especially well be seen for rotation around the t-channel helicity axis because the events change strongly their positions.

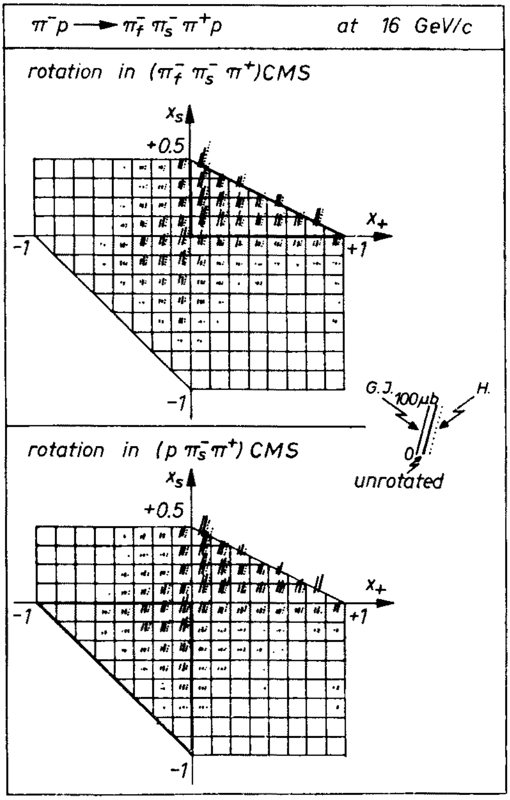


Fig. 17. LPS distribution of the events $\pi^- p \to \pi^-\pi^-\pi^+ p$ at 16 GeV/c before and after their simultaneous rotation by 180° in the Gottfried-Jackson frame (thin bars) and in the helicity frame (dashed bars) defined in the (3π) rest frame and ($p\pi^+\pi^-$) rest frame. The sector of particular interest is framed by a dark line (taken from Ref. [34])

The results are seen in Fig. 17. Strong violation of SCHC is confirmed. A small, but systematic violation fo TCHC is found for low (3π) masses.

3.2.3. $pK^-[\pi^\pm, p] \to (p\pi^+\pi^-)K^-[\pi^\pm, p]$. The same analysis as for the diffractive (3π)-system was used by the Paris–Milano–Genova–Durham-collaboration (Ref. [22]) to study the diffractive ($N\pi\pi$) system in the reactions

$$\left.\begin{array}{l}\pi^+ n \to \pi^+(\pi^+_{\text{slow}}\pi^- n)\\ \pi^- p \to \pi^-(\pi^-_{\text{slow}}\pi^+ p)\end{array}\right\} \text{at 11 GeV}/c$$

in the LPS. The results are seen in Fig. 15 and they are more compatible with SCHC than with TCHC. But the method used gives unreliable results as explained in 3.2.2.

The ABBCCHLVW-collaboration [30], [31] used the same analysis as for the diffractively produced three-meson-system for the diffractive $(p\pi\pi)$-system in K^-(10 GeV/c), π^+ and π^- (16 GeV/c) experiments (see section 3.2.2). The $\chi^2(\psi)$ distribution for different masses and four-momentum transfers (actually plotted in crossing angles) are seen in Fig. 18 for the four analysers (the directions of the particles p, π^+, π^- and the normal $\vec{n}$

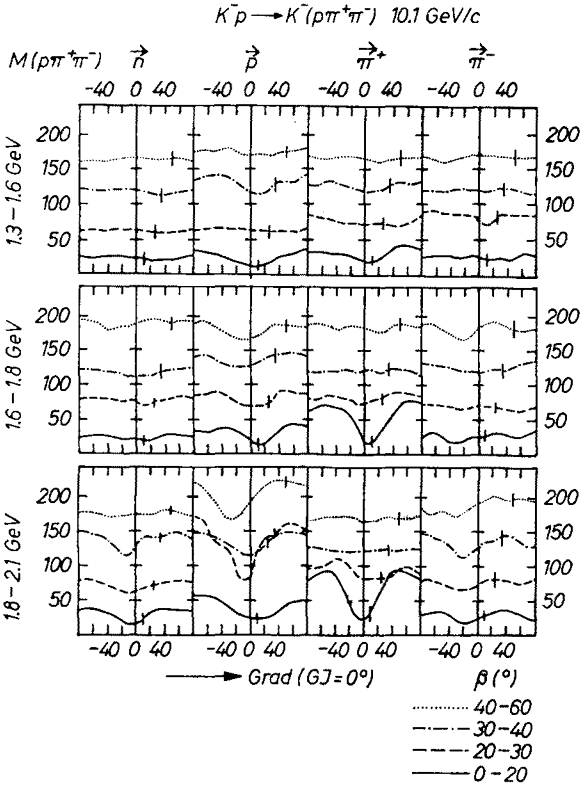


Fig. 18. $\chi^2(\psi)$ (as explained in the text) for the analysers p, π^+, π^- and the normal n to the $(p\pi\pi)$ plane for various values of the masses $(p\pi\pi)$ and crossing angles for the reaction $K^-p \to K^-(p\pi^+\pi^-)$ at 10 GeV/c (taken from Ref. [31])

to two of these particles). The results for all three experiments lead us to the conclusion that the data are compatible with TCHC.

The ABBCH-collaboration finds the same conclusion for the diffractive $(p\pi\pi)$ system in π^+p (at 8 and 16 GeV/c) and π^-p (at 16 GeV/c) experiments [34]. SCHC is strongly violated but TCHC holds.

There are two reports on helicity conservation in pp-experiments. One comes from the Scandinavian collaboration [35] and they investigate the process

$$pp \to p\pi^-\Delta^{++}$$

at 19 GeV/c incident p momentum. They checked the azimuthal distribution of the Δ^{++} (which is the same as that of the π^-) and found it to be not very isotropic in both Gottfried-

Jackson and helicity frames. Fig. 19 shows the ratio R as a function of the $(p\pi\pi)$ mass, where R is defined as

$$R = \frac{B-F}{B+F}$$

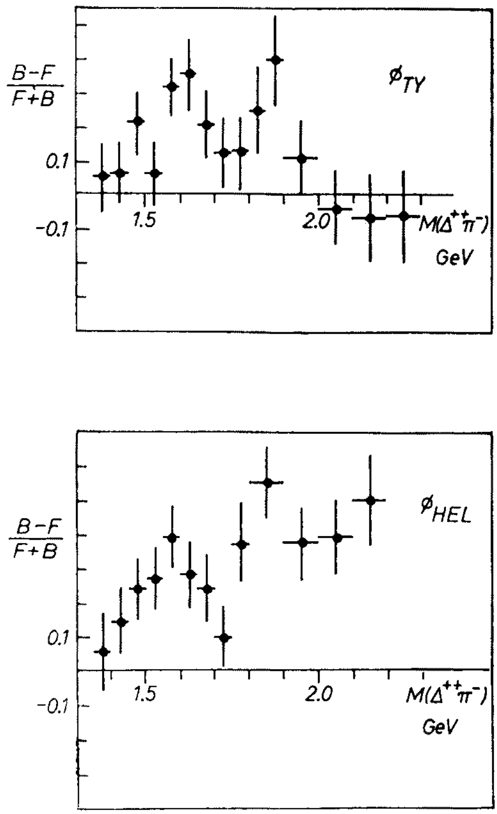


Fig. 19. Ratio $R = (B-F)/(B+F)$ (as explained in the text) as a function of the mass $(\Delta^{++}\pi^-)$ in the Gottfried-Jackson and helicity frames for the reaction $pp \to p\Delta^{++}\pi^-$ at 19 GeV/c (taken from Ref. [35])

with B: events with $\pi/2 \leqslant \alpha < \pi$

F: events with $0 \leqslant \alpha < \pi/2$

(the α-distribution is folded since parity conservation in the production process requires $W(\alpha) = W(-\alpha)$).

TCHC or SCHC means $R = 0$ in the Gottfried-Jackson or helicity frame respectively and both seem not to be in agreement with the data.

Another pp experiment at 16 GeV/c [36] investigates the reaction

$$pp \to pp\pi^+\pi^-.$$

They claim that their data are compatible with TCHC. As an example they show for three mass regions of the $(p\pi\pi)$ system (always taking the lower mass of the two possible masses: up to 1.55 GeV, 1.55 – 1.80 GeV, 1.80 – 2.05 GeV) the azimuthal distribution of the proton in the Gottfried-Jackson and helicity frames. This is seen in Fig. 20. The shaded histograms correspond to events with $t \geqslant 0.05$ GeV^2 (for $t = t_{min}$, the Gottfried-Jackson and the helicity frame coincide). The distribution of the normal to the ($p\pi\pi$) system looks

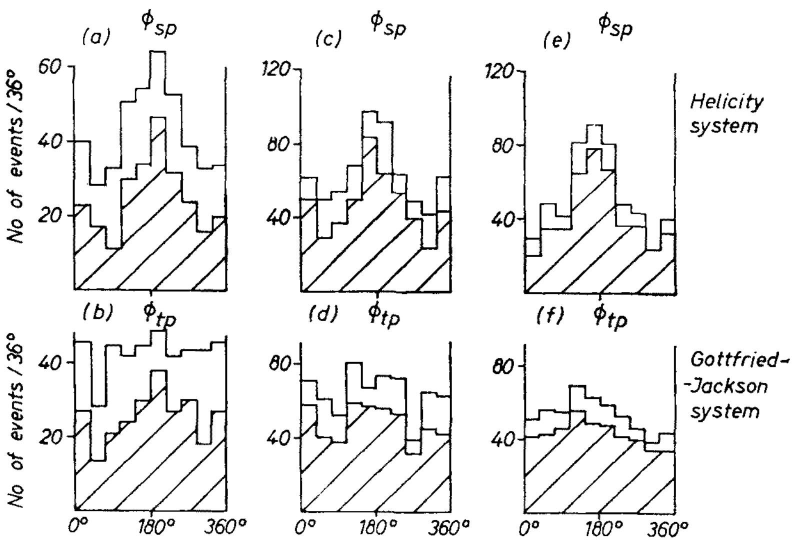


Fig. 20. Azimuthal distributions around the s-channel (φ_s) and t-channel helicity axis (φ_t) for the reaction $pp \rightarrow pp\pi^+\pi^-$ at 16 GeV/c as a function of the ($p\pi^+\pi^-$) mass (< 1.55 GeV, 1.55–1.80 GeV, 1.80–2.05 GeV). The shaded histograms correspond to events with $t > 0.05$ GeV^2 (taken from Ref. [36])

isotropic in both Gottfried-Jackson and helicity frames, and the distributions for the two pions are not given.

The distributions are repeated with a Δ^{++} selection and the analyser is now the Δ^{++}. For low ($p\pi\pi$) masses the azimuthal distribution looks anisotropic in both Gottfried--Jackson and helicity frames, but for higher masses the Gottfried-Jackson system is again the preferred one.

As a conclusion I would like to say that SCHC holds for elastic diffraction scattering (I include here the photoproduction of the ϱ-meson), but nearly all inelastic diffractive processes are incompatible with this hypothesis. On the other hand TCHC is either fulfilled for these reactions or only small deviations from this hypothesis are observed. A conclusive answer is expected when we have more high energy data.

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