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DIFFRACTIVE DISSOCIATION IN INCLUSIVE REACTIONS

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General features of diffractive dissociation in inclusive reaction are discussed especially in the case when it dominates the high energy total cross-section.

The subject of this School is diffractive dissociation, of which unfortunately I know very little. I am here not because I have anything to say but because I happen to have many Polish friends. I hope you will appreciate that it was not an easy task for me to scrape together sufficient material for these lectures.

While preparing for them, I was reminded of a story about a certain gentleman who was asked to write an article about Chinese metaphysics. Since he knew nothing about the subject, he looked up the Encyclopaedia Britannica and read the sections on China and on metaphysics respectively. Then he wrote his article on the combined material. I am afraid that in constructing these lectures, I have followed much the same tradition.

Since I am probably the first person to mention the word inclusive here, I shall start by explaining what this means. Inclusive experiments are those in which one detects and measures only some of the final particles in a collision without caring what else may also be produced. For example

$$a+b \to x \quad \sigma_T \tag{1}$$

$$a+b \to c+x \tag{2}$$

$$a+b \to c+d+x \text{ etc.} \tag{3}$$

These are to be distinguished from "exclusive" experiments where one detects and measures all the final particles, such as

$$a+b \to c+d+e. \tag{4}$$

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The inclusive approach to particle collisions is similar in spirit to what is usually done in a many-body system when the number of particles becomes large, as for example in a liquid. One is then not so interested in following in detail the motion of each individual particle. Rather, one focuses one's attention on collective quantities such as the density, the correlation functions, *etc*. Similarly, when the final multiplicity is high, it is sometimes more convenient to describe particle collisions in terms of inclusive cross-sections such as (1), (2), and (3) instead of exclusive cross-sections, such as (4).

In these lectures I shall be discussing only general properties of inclusive reactions. It is therefore sufficient for our purposes to consider only an idealized system in which all particles produced are of the same type. In fact this idealization is not too far from reality, since by far the greater number of particles produced in high energy collisions are just pions.

It is useful to introduce the quantities

$$\varrho_1 = \frac{1}{\sigma_T} E_c \frac{d\sigma}{dp_c^3} \tag{5}$$

$$\varrho_2 = \frac{1}{\sigma_T} E_c E_d \frac{d\sigma}{dp_c^3 dp_d^3} \tag{6}$$

where σ is the cross-section, $p_{c(d)}$ the momentum of the emitted particle $c(d)$ and $E_{c(d)}$ its energy. Since (5) and (6) are covariant, one needs not specify in what frame they are to be measured.

The quantity ϱ_1, may be interpreted as the probability density of finding a final particle c in the invariant phase space volume element dp_c^3/E_c. Thus, on integrating ϱ_1, over all phase space, one obtains the average multiplicity,

$$\int \varrho_1 dp_c^3/E_c = \langle n \rangle. \tag{7}$$

Similarly, ϱ_2 is the probability of finding one particle in the volume element dp_c^3/E_c and another in the element dp_d^3/E_d. The integral of ϱ_2 over all phase space then gives

$$\int \varrho_2 dp_c^3/E_c \cdot dp_d^3/E_d = \langle n(n-1) \rangle. \tag{8}$$

The quantities ϱ_i are functions of the longitudinal and transverse components of the final momenta $p_{\|c}, p_{\perp c}$ *etc*. Now it is a fundamental hypothesis in the theory of inclusive reactions that the distributions ϱ_i, when considered as functions of the variables $p_\perp$ and (* denotes CM system)

$$x = 2p_{\|}^{*}/\sqrt{s} \tag{9}$$

approach finite limits when the incoming energy $s \to \infty$ *e. g.*

$$\lim_{s\to\infty} \varrho_1(x_c, p_{\perp c}; s) = \varrho_1^{\infty}(x_c, p_{\perp c}). \tag{10}$$

In other words, the distributions in $p_\perp$ and the scaled CM momentum x become independent of the incoming energy when the latter is sufficiently high. Known as the scaling hypothesis of Feynman, this statement is supported by fairly strong theoretical arguments and also by the gathering experimental evidence from, for example, the ISR. For the sake of discussion therefore, I shall accept it as the truth in these lectures. The distributions ϱ_i then take on a more basic physical significance being properties intrinsic to the system, independent of the incoming energy.

For some purposes, it is more convenient to employ instead of x the so-called rapidity variable y defined as

$$y = \frac{1}{2}\log\frac{E^* + p^*_{||}}{E^* - p^*_{||}}. \tag{11}$$

This has the nice property that under a Lorentz transformation along the beam direction, it is changed only by the boost angle ω, thus

$$y \to y' = y + \omega. \tag{12}$$

Considering then ϱ_i as functions of y and $p_\perp$, one has from (7) and (8) that

$$\int \varrho_1(y_c)dy_c = \langle n \rangle \tag{13}$$

$$\int \varrho_2(y_c, y_d)dy_c dy_d = \langle n(n-1) \rangle \tag{14}$$

where the dependence of ϱ_i on $p_\perp$ and the integration over $p_\perp$ have both been suppressed.

It is often helpful to visualize the distributions of final particles in y and $p_\perp$ as being analogous to those of molecules in a liquid or a gas. Since the transverse momentum $p_\perp$ is known to be always restricted, ($\langle p_\perp \rangle \sim 300$ MeV), the molecules are confined to a narrow tube in space, resembling for most purposes a system in one dimension. The length of the tube is the kinematically allowed range of the rapidity variable y, which according to (11) is $Y \propto \log s$. The molecules near to the ends of the tube correspond to emitted particles which have small momenta in the rest frame of either the target or the projectile, or in other words the so-called "fragmentation products". Whereas, molecules in the middle portion of the tube are "central" particles whose momenta are large in the rest frames of both the target and the projectile.

As in the case of ordinary fluids, it is convenient here also to introduce the correlation functions such as

$$g_2(y_c, y_d) = \varrho_2(y_c, y_d) - \varrho_1(y_c)\varrho_1(y_d) \tag{15}$$

which may be interpreted as the change in probability of finding a particle at y_c due to the presence of another at y_d. The properties of g_2 depend on the interaction between the molecules. In most cases of actual liquids and gases, the correlation function vanishes when the separation between c and d becomes large. If the function vanishes exponentially with large separation, thus

$$g_2(y_c, y_d) \to \exp\{-\mu|y_c - y_d|\}$$

the correlation is said to be "short-ranged" with range $\lambda \sim \frac{1}{\mu}$. It may also vanish more slowly like a power in the separation $\sim |y_c-y_d|^{-r}$ as in the case of Coulomb interactions; or it may not vanish at all; the correlations are then termed "long-ranged". This concept of range can be generalized to higher order correlations involving several particles. For short range systems, the correlation function g_N for N — particles has cluster decomposition properties; namely, g_N vanishes whenever any subset of the particles become well--separated from the others. The characteristic distance over which the correlation function is effective is again given by the correlation length $\lambda = 1/\mu$.

Such systems with short range correlations are particularly easy to visualise. Consider a collision in which the energy s is so high that the length of the rapidity plot $Y \propto \log s$ is much larger than the correlation length λ. Particles in the central region of the plot being then many correlation lenghts away from the ends of the tube will not feel the effects of the boundary any more. One thus expects the density of the fluid there to be uniform namely that $\varrho_1(y_c)$ will be constant independent of y_c. The average multiplicity as given by (13) will then simply be proportional to the length of the tube, *i. e.* $\langle n\rangle \propto \log s$. Moreover, the two-body correlation function $g_2(y_c, y_d)$ will be translational invariant in the central region and depends only on the separation the two particles: y_c-y_d.

Now in the same way that the correlation functions between the molecules of a fluid yield information on the basic interactions between them, it may be expected that the correlation functions in hadron collisions may also teach us something about the mechanisms by which particles are produced. As we shall see, this is indeed the case.

We turn now to diffractive dissociation. By this I shall mean at the moment merely these processes where the cross-section for a fixed multiplicity, σ_n say, becomes independent of the incoming energy as the latter becomes large. Our interpretation is of course in accord with the usual one in which the incoming particles become diffractively excited by the collision, and after that individually decay, emitting a number of secondaries. Since the excitation is "diffractive", the cross-section is by assumption energy independent, and since the excited fireballs are supposed to decay independently, each will yield a definite distribution in multiplicity, implying therefore constant σ_n. Our present definition of diffractive dissociation being so general, presumably few physicists would object to the existence of such processes. The question then is merely whether such processes are expected to dominate high energy collisions, or whether they form only an insignificant part of the total cross-sections. Our purpose here therefore is to help answer this question by looking for characteristic effects of diffractive dissociation in inclusive distributions.

The first thing to notice in this direction is the following fact: namely, diffractive dissociation necessarily implies long range correlations [1]. That this is so, can be seen as follows. Consider the mean square fluctuation in the final multiplicity defined as follows:

$$\langle (n-\langle n\rangle)^2\rangle = \sum_n (n-\langle n\rangle)^2 \frac{\sigma_n}{\sigma_T}. \tag{16}$$

By means of (13) and (14), this can be written as

$$\langle (n-\langle n\rangle)^2\rangle = \int \varrho_2(y_c, y_d)dy_c dy_d - \\ - \int \varrho_1(y_c)dy_c - \int \varrho_1(y_c)dy_c \int \varrho_1(y_d)dy_d. \tag{17}$$

Introducing the correlation function g_2 as defined in (15) one has then

$$\langle (n-\langle n\rangle)^2\rangle = \int g_2(y_c, y_d)dy_c dy_d - \int \varrho_1(y_c)dy_c. \tag{18}$$

If the correlation function g_2 is purely short range, previous arguments suggest that ϱ_i is constant and that g_2 is dependent only on $y_c - y_d$. One may therefore integrate y_c first over the length of the tube and write

$$\langle (n-\langle n\rangle)^2\rangle \propto \log s\left[\int g_2(y_c-y_d)d(y_c-y_d) - \text{const}\right]. \tag{19}$$

Since g_2 is short ranged, the remaining integral is also just a finite constant independent of the energy. Thus

$$\langle (n-\langle n\rangle)^2\rangle \propto \log s. \tag{20}$$

However, by (16), one has also for any fixed m

$$\langle (n-\langle n\rangle)^2\rangle \geqslant (m-\langle n\rangle)^2 \frac{\sigma_m}{\sigma_T} \propto (\log s)^2 \frac{\sigma_m}{\sigma_T}. \tag{21}$$

Combining (21) with (20), we then deduce that

$$\frac{\sigma_m}{\sigma_T} \leqslant \frac{\text{const}}{\log s} \tag{22}$$

which implies that σ_m cannot have a diffractive component. It is easily seen that assuming a slow logarithmic increase in s of σ_T or a similar decrease of σ_m will not help. Indeed, by considering higher moments of the multiplicity fluctuations, very similar arguments will show that short range correlations necessarily imply

$$\frac{\sigma_m}{\sigma_T} \leqslant \text{const} \times (\log s)^{-r} \tag{23}$$

for any m and r, which is certainly in contradiction with any idea of diffractive dissociation.

Therefore one way of recognizing diffractive effects in inclusive distributions is to look for long range correlations. Unfortunately, however, attempts in this direction so far have yielded indefinite results. Firstly as remarked above, the single particle distribution $\varrho_1(y_c)$ is expected to be the independent of y_c if the correlations are purely short range. Whereas, if long range correlations exist, no such "plateau" is predicted. Now recent experimental results from the ISR at CERN have yielded rapidity plots which are remarkably flat in the central region, as illustrated in Fig. 1, quite consistent with only short range correlations [2]. Secondly, if correlations are purely short ranged, the particles at one end of the tube will not feel the presence of the other end. In other words, the distribution of emitted particles with small momenta in the target frame will not depend on the nature of the projectile. One example of such so-called tests of factorization is shown in Fig. 2

where the data are seen to be in good agreement with the preceeding predictions and again show no evidence of long range correlations [3].

The examples quoted above, though doubtless worthy of consideration, are not decisive. Beside the lack of accuracy in the experiments, the apparent absence of correlations is observed only between the emitted particles and the ends of the tube, and not

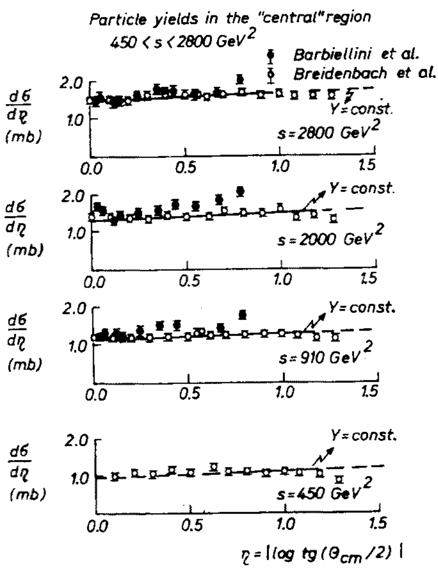

Fig. 1

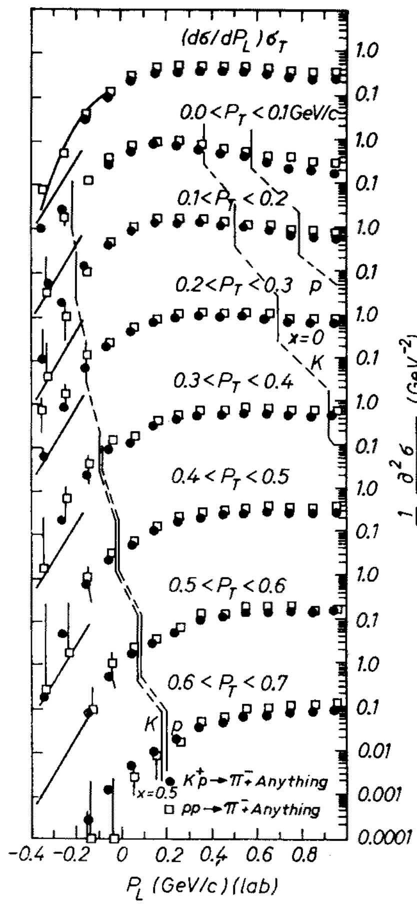

Fig. 2

Fig. 1. Distribution of charged particles in rapidity in the central region. Data from the ISR

Fig. 2. Normalized distributions ϱ_1 of π^- with small momenta in the target frame from pp and K^+p collisions. Similarity of distributions supports short range correlations

between the particles themselves. Whereas, in deriving (23), it was the second type of correlations which really matters. To resolve this, one will have to wait for the next generation of ISR experiments. Analysis performed at up to Serpukhov energy have indeed indicated strong correlation between emitted particles. However, because of the limited energy, it is not known whether these correlations will still persist at large separations.

In order to proceed further, one has to be more specific in defining what one means by diffractive dissociation. Let us first take the extreme view that the total cross-section is ultimately all dominated by diffractive processes so that

$$\sigma_T = \sum_n \sigma_n \tag{24}$$

where each σ_n is a constant with respect to the incoming energy s. The sum over n in (24) is presumably to be carried to the kinematic limit, namely to $N(s) = \sqrt{s}/m$ where m is the mass of the produced (identical) particles. One requirement on diffractive dominance models is thus that the sum (24) be convergent. Next, the average multiplicity $\langle n \rangle$ can be written as

$$\langle n \rangle = \sum_n n\sigma_n/\sigma_T. \tag{25}$$

This is known to increase slowly with the incoming energy, with most theoreticians biased towards a logarithmic dependence: $\langle n \rangle \propto \log s$. One natural assumption then is to take for large n [4].

$$\sigma_n \propto \frac{1}{n^2} \tag{26}$$

which on summing yields a finite σ_T whereas the average multiplicity (25) diverges logarithmically with the upper limit $N(s) \propto \sqrt{s}$, as required. The conclusion (26) is not strictly necessary. However, in order to satisfy both the imposed criteria on the total cross-section and the average multiplicity, σ_n is not likely to have very different behaviour.

Now, if σ_n is energy independent, the condition (21) already implies that $\langle (n - \langle n \rangle)^2 \rangle$ should diverge faster than $(\log s)^2$. If we accept further the condition (26), one has the more definite predictions

$$\langle (n - \langle n \rangle)^2 \rangle \sim \sum_n n^2 \sigma_n \propto \sqrt{s}. \tag{27}$$

This fluctuation is much bigger than that expected for short range systems, which according to (20) is proportional only to $\log s$. That this is so, can easily be seen as follows.

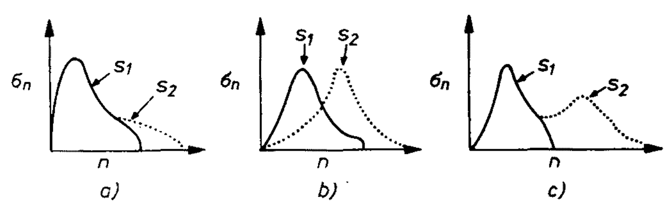

Fig. 3. σ_n as a function of n for a) purely diffractive, b) purely short-range, c) mixed mechanisms

Since σ_n is constant with respect to energy, the distribution of σ_n with n can change with s only at the upper limit as illustrated in Fig. 3a. In order to obtain an increase in average multiplicity as required it is necessary for σ_n to have a long tail in n. Now in calculating the fluctuation, the long tail is even more emphasized since σ_n is multiplied by n^2 in averaging, leading this to a strong s dependence as in (27). Similarly, one sees that higher moment fluctuations have even stronger energy dependence, *e. g.* $\langle (n-\langle n\rangle)^3\rangle \propto s$. This is very different from the case of short range systems where the increase in average multiplicity comes mainly from a shift in the peak of σ_n, as illustrated in Fig. 3b.

The fluctuations can then remain small as the energy increases. This difference between short-range systems and systems dominated by diffractive dissociation is so dramatic, that probably one will not have to wait very long before experiments from the ISR will tell us which alternative is favoured by nature.

Next, we inject into our discussion the original picture of diffractive dissociation, as first an excitation of the incoming particles into fireballs followed later by their decays. If the fireballs excited are always of finite mass the momenta ot their decay products will always be finite in their respective rest frames. As the incoming energy increases, the fireballs are separated by higher energies also. On the rapidity plot, therefore they move further and further apart, with their decay product sticking, as it were to the ends of the tube. In the central region, there will thus be a hole in the distribution. Experimentally as we have seen, this is very far from the truth.

In order to describe the experimental situation adequately, therefore, one must allow the excitation of high mass fireballs, which being produced with a small momentum in the CM system, will produce slow secondaries to fill up the central region. One does not expect these heavy fireballs to be excited frequently. None the less, their excitation probability $P(M)$, say, for fireball mass M, must have a sufficiently long tail in M to produce the desired effect. A favourite choice for workers in this field is that for large M

$$P(M) \sim 1/M^2. \tag{28}$$

Now once this choice is made, the decay pattern of the fireball is no longer arbitrary, since one has also to satisfy (26). Indeed, if $\bar{N}(M)$ were the average multiplicity in the decay of a fireball with mass M, it is seen that one must have

$$\bar{N}(M) \propto M. \tag{29}$$

This would be the case indeed if the fireball decays isotropically with the final particles sharing equally the available energy. However, one may well begin to doubt the propriety of this assumption, if the fireballs were to decay for example preferentially by cascade. Let us none the less accept it at present as a working hypothesis.

With the conditions (28) and (29), one is indeed able to reproduce the known qualitative features of the experimental single particle distributions. As pointed out before, however, the flat distribution in the central region does not come out naturally in this picture, but more as a detail feature of certain particular models. Indeed, in the context of the present framework, the rapidity variable y loses its significance. Its main virtue was its simple transformation property (12) under Lorentz boosts along the beam directions and it is useful when there is translational invariance in the central region as in the case of short range systems. In the present case, it is much more convenient to use instead the Feynman x-variables, defined in (9) which is the CM longitudinal momentum scaled by the maximum momentum allowed kinematically.

The variable x ranges from -1 to $+1$. The region $x < 0$ (> 0) corresponds asymptotically to the fragments of the target (projectile). One can see therefore that the correlation between particles emitted in the same hemisphere must be very different from that between one particle from the left ($x < 0$) and one from the right ($x > 0$). Indeed a particle with $x < 0$, being a decay product of the target fireball, need not be associated at all with a decay product of the projectile fireball with $x > 0$. Whereas two secondaries in the same hemisphere are expected to be strongly correlated. Thus, for example one expects that as $s \to \infty$ [5]

$$\frac{\langle n^R n^L \rangle}{\langle n^R \rangle \langle n^L \rangle} \to 1 \tag{30}$$

but as in (27)

$$\frac{\langle (n^R)^2 \rangle}{\langle n^R \rangle^2} \sim \sqrt{s}. \tag{31}$$

The predictions (30) and (31) already indicate that the point $x = 0$ is a singular point in the present picture. This is further emphasized by considering the two particle distribution ϱ_2 as a function of the variables x_c and x_d. As explained above, particles emitted near $x = 0$ come from the decay of heavy fireballs which prefers a high final multiplicity with the final particles having relatively low momenta to one another. Thus given a particle at x_c, it is highly probable that one finds another close to it at x_d coming from the same event. In other words, one expects $\varrho_2(x_c, x_d)$ to become very large when both x_c and x_d are close to zero in the same hemisphere. Indeed, based on more specific arguments some authors have suggested that one has a singularity in ϱ_2 of the form:

$$\lim \varrho_2(x_c, x_d) \sim \frac{1}{x_c + x_d} \tag{32}$$

as either $x_c, x_d \to 0^+$ or $x_c, x_d \to 0^-$. Notice, however, that this argument applies only to particles in the same hemisphere coming from the decay of the same fireballs. In contrast, for example, the limit of $\varrho_2(x_c, x_d)$ as $x_c \to 0^-$ and $x_d \to 0^+$ is expected to be finite.

One sees therefore that the dominance of diffractive processes at high energy will lead to strong predictions drastically different from those expected for short ranged systems. In the end only further experiment will tell us which picture is correct. Personally, however, I must say I doubt that diffractive processes will dominate entirely. On the other hand, I am also quite confident that some diffractive processes will persist. It is quite possible that even at very high energy, one has still partly diffractive dissociation, and partly short-ranged mechanisms such as multiperipheralism. Diffractive dissociation may contribute significantly only to low multiplicity events, and only to the production of particles with small momenta in the target or projectile rest frames. In other words, the excitation probability function $P(M)$ introduced in (28) has a much shorter tail than that indicated. Whereas, in the central region, the dominant processes may be short-ranged, giving us naturally the observed plateau in y. The many predictions of factorization could then still be approximately valid. I am certainly neither the only nor the first person to hold such views. The pioneer in this way of thinking, as indeed in much of what I have said, is K. Wilson who has made a penetrating study of the problems involved and even suggested an experimental program for investigation [6]. I shall only mention one of the features he proposed, namely the shape of σ_n at high energy if such a picture were true. This is illustrated in Fig. 3c. I understand also that Białas, Zalewski and other members of the Kraków group are making a careful study of similar questions and have already made several concrete proposals. There is no better way of concluding these lectures than by inviting them to share their wisdom with us during the discussion.

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