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A RELATIVISTIC QUARK MODEL FOR DIFFRACTIVE EXCITATION OF MESONS

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The Böhm, Joos, Krammer [7] relativistic quark model is used to obtain high energy meson scattering amplitudes from quark-quark elastic (absorptive) scattering amplitudes. A relativistic form of Glauber's multiple scattering expansion is assumed and some results reported.

1. Introduction

As Carlitz, Frautschi, and Zweig [1] pointed out, the non-relativistic $SU(6)$ quark model does not allow diffractive excitations of low mass mesons. However, the mesonic states known as the A_1, A_2 and Q are produced in the reactions $\pi p \to Ap$ and $Kp \to Qp$ with cross-sections $\sigma \simeq 0.1\, \sigma_{\text{elastic}}$; these cross-sections do not decrease much with increasing energy [2, 3].

In the non-relativistic quark model the mesons are represented by quark-antiquark states (see Table I) and baryons by qqq states. Let us assume that at high energies qq scattering is highly inelastic. In meson-baryon and baryon-baryon collisions, diffractive excitations occur owing to the absorptive effect of qq scattering [4]. If qq scattering is spin independent and we neglect spin-orbit coupling, $\pi \to A_1$ and $\pi \to A_2$ excitations do not occur because these are singlet-triplet transitions. If qq and $q\bar{q}$ scattering amplitudes are equal, $\pi \to B$ does not occur. Assuming this spin independence and equality of qq and $q\bar{q}$ amplitudes, one also has zero polarization in elastic πp scattering and equality of pp and $\bar{p}p$ cross-sections. However, at the energies at which productions are mainly observed, we observe spin dependence in elastic scattering [5] and considerable difference in pp and $\bar{p}p$ cross-sections. A quark model of diffractive excitations in which the qq amplitudes are spin dependent will give non-vanishing polarizations in meson-baryon and baryon-baryon scattering and also diffractive productions of A_1, A_2 and Q [6]. Similarly, if qq and $\bar{q}q$ amplitudes differ, one may obtain diffractive B meson productions.

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TABLE I

	J^{PC}	$q-\bar{q}$ state	Primary decay modes
π (140)	0^{-+}	$^1S_0$	
ϱ (750)	1^{--}	$^3S_1$	$\pi\pi$
δ (962)	0^{++}	$^3P_0$	$\pi\pi$
A_1 (1070)	1^{++}	$^3P_1$	$\varrho\pi$?
B (1220)	1^{+-}	$^1P_1$	$\omega\pi$
A_2 (1315)	2^{++}	$^3P_2$	$\varrho\pi$
A_3 (1640)	2^{-+}	$^1D_2$	$f^0\pi$
	J^P		
K (494)	0^-	$^1S_0$	
K^* (892)	1^-	$^3S_1$	$K\pi$
? Q (1280)	1^+	$^3P_1$	$K^*\pi$, $\varrho\pi$
? Q (1240)	1^+	$^1P_1$	3π

In the relativistic model I shall describe here, the meson-baryon and meson-meson amplitudes are approximated as a multiple scattering expansion in quark-quark elastic scattering amplitudes similar to the Glauber multiple scattering expansion; some years ago, Frautschi and I described how such an expansion might describe hadronic diffractive excitation [4]. At that time, we could not implement our ideas because we felt that relativistic effects would be important and we did not know a relativistic description of mesons and baryons as composite systems. Recently, Böhm, Joos, and Krammer [7] have given a fully relativistic model of the mesons as $q-\bar{q}$ bound states; their Bethe-Salpeter amplitudes are in direct correspondence with the non-relativistic $SU(6)$ model. Now, therefore, I would like to present a fully relativistic model of diffractive excitation and use the Bethe-Salpeter amplitudes of Böhm, Joos, and Krammer to obtain some results.

2. *Relativistic formulation*

We take the state $|a\rangle$ to be a $q\bar{q}$ bound state described by the Bethe-Salpeter amplitude [8]

$$\chi(q, P_a) = (2\pi)^{-5/2} \int d^4x e^{-iq\cdot x} \langle 0| T\{\psi(x/2)\psi(-x/2)\}|a\rangle \tag{1}$$

where $\psi(x)$ is the quark field and $P_a^2 = M_a^2$. The particles q and $\bar{q}$ in $|a\rangle$ have momenta p_1 and p_2 where

$$p_1 = \frac{P_a}{2} + q \quad \text{and} \quad p_2 = \frac{P_a}{2} - q; \tag{2}$$

χ is a matrix whose entries are labeled by the Dirac and $SU(3)$ components of ψ and $\bar{\psi}$. It satisfies an equation of the form

$$(\not{p}_1 - m)\chi(\not{p}_2 + m) = i \int d^4q \mathscr{K}(q, q', P)\chi(q', P). \tag{3}$$

Mandelstam [8] has shown how such amplitudes may be used to calculate the matrix elements of a relativistic field theory; *viz.*,

$$\langle b|\ T\{\psi(x_i)\ldots\bar{\psi}(y_i)\ldots\varphi(z_i)\ldots\}\ |a\rangle. \tag{4}$$

Using his prescription, one obtains for example in the ladder approximation the matrix element of the electric current $j_\mu = \bar{\psi}\gamma_\mu\lambda_Q\psi$ in state $|a\rangle$ as

$$(2\pi)^4\langle a|\ j_\mu(0)\ |a\rangle =$$

$$= \int d^4q\ \mathrm{Tr}\left\{-\bar{\chi}\left(q+\frac{k}{2},\ P_a+k\right)\gamma_\mu\lambda_Q\chi(q,P_a)\left(\frac{\not{P}_a}{2}-\not{q}+m\right)\right\}+$$

$$+\int d^4q\ \mathrm{Tr}\left\{\bar{\chi}\left(q-\frac{k}{2},\ P_a+k\right)\left(\frac{\not{P}_a}{2}+\not{q}-m\right)\chi(q,P_a)\gamma_\mu\lambda_Q\right\} \tag{5}$$

where $k = P_a' - P_a$ = momentum transfer. Each term of (5) is represented by a diagram

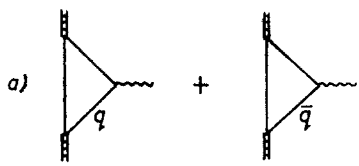
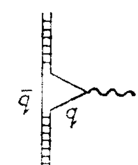


Fig. 1

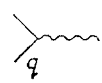
like Figure 1a. Note that the diagrams in Figure 1a are not Feynman diagrams. The rules for the contribution from



to $\langle a|j_\mu|a\rangle$ are:

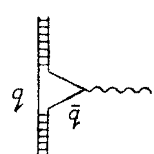
i) incoming ladder $= \chi_a$;

ii) non-interacting $\bar{q}$ line $= (-\not{p}_{\bar{q}}-m)$ to the right of χ;

iii)  $= \gamma_\mu\lambda_Q$ to the left of χ;

iv) outgoing ladder $= \bar{\chi}$.

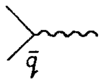
For the



contribution we have:

i') same as (i) above;

ii') non-interacting q line $= (\not{p}_q - m)$ to the left of χ;

iii')  $= \gamma_0 \gamma_\mu^+ \gamma_0 \lambda_Q^+ = \gamma_\mu \lambda_Q$ to the right of χ;

iv') same as (iv) above.

We shall use similar ladder approximations to calculate the other matrix elements of our model.

Let us now consider as an example $\pi^+\pi^-$ elastic scattering at high energy and small momentum transfer. Assume that the S-matrix may be approximated by the Glauber multiple scattering expansion [4]

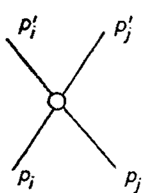
$$1 - S = T = \sum_{i,j} t_{ij} - \frac{1}{2!} \sum' t_{ij} t_{kl} + \frac{1}{3!} \sum' t_{ij} t_{kl} t_{mn} - \\ - \frac{1}{4!} \sum' t_{ij} t_{kl} t_{mn} t_{op} \qquad (6)$$

where t_{ij} is an elastic scattering T-matrix [9] for the scattering of quark i in π^+ by quark j in π^-. We assume that at high energies the t_{ij} matrix elements are real [9] and strongly favor small momentum transfer (the latter assumption allows us to neglect exchange effects). The contribution of a single scattering term t_{ij} to $\langle \pi^+\pi^- | T | \pi^+\pi^- \rangle \equiv T_{\pi^+\pi^-;\pi^+\pi^-}$ is shown

Fig. 1

in Figure 1b. Contributions to a double scattering term $t_{qq} t_{q\bar{q}}$ are shown in Figure 1c and Figure 2. The summation Σ' over the multiple scattering terms in (6) means sum over all indices omitting terms in which the same pair occurs in more than one factor.

To illustrate how one might use (6) to calculate for example $T_{\pi^+\pi^-;\pi^+\pi^-}$ (or $T_{A^+\pi^-;\pi^+\pi^-}$), let us assume that the invariant amplitude t_{ij} is given by (for definiteness we assume both i and j are quarks)

$$t_{ij} =$$ 

$$= s_{ij} f(t) \bar{u}(p_i') u(p_i) \bar{u}(p_j') u(p_j) \qquad (7)$$

where $s_{ij} = (p_i+p_j)^2$ and $t = (p_i-p_i')^2$. Then the contribution from t_{ij} to $T_{\pi^+\pi^-;\pi^+\pi^-}$ is

$$f(t)\ \frac{s}{4}\left[\int \frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left\{-\bar{\chi}\left(q+\frac{k}{2},\ P_+ +k\right)\chi(q,P_+)\left(\frac{\not{P}_+}{2}-\not{q}+m\right)\right\}\right]\times$$

$$\times\left[\int \frac{d^4q'}{(2\pi)^4}\,\mathrm{Tr}\left\{-\bar{\chi}\left(q'-\frac{k}{2},\ P_- -k\right)\chi(q',P_-)\left(\frac{\not{P}_-}{2}-\not{q}'+m\right)\right\}\right]. \tag{8}$$

Here $t = k^2$ and $s = (P_++P_-)^2$. Note that the quark-quark energy variable $s_{ij} \neq s$; *viz.*,

$$s_{ij} = \frac{s}{4} + (P_+ + P_-)\cdot(q+q') + (q+q')^2. \tag{9}$$

In (8), we have approximated s_{ij} by $\frac{s}{4}$. The second term in (9) is a kinematic correction which for example gives the π^+ system information concerning the direction of motion of the incident π^-. This gives a so-called p_z (or P_c) correction to the non-relativistic Glauber theory (we noted that such corrections might arise in Ref. [4]). A double scattering contribution, for example, $t_{qq}t_{q\bar{q}}$ (Figure 1c) may be approximated by (see below)

$$\left(\frac{s}{4}\right)^2\int\frac{d^4k_1}{(2\pi)^4}f(k_1^2)f((k-k_1)^2)\left[\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left\{-\bar{\chi}\left(q+\frac{k}{2},\ P_+ +k\right)\times\right.\right.$$

$$\left.\left.\times\left(\frac{\not{P}_+}{2}+\not{q}+\not{k}_1-m\right)^{-1}\chi(q,P_+)\left(\frac{\not{P}_+}{2}-\not{q}+m\right)\right\}\right]\times$$

$$\times\left[\int\frac{d^4q'}{(2\pi)^4}\,\mathrm{Tr}\left\{\bar{\chi}\left(q'-\frac{k}{2},\ P_- -k\right)\chi(q',P_-)\right\}\right]. \tag{10}$$

(We have for simplicity assumed $f(k^2)$ is the same function for qq, $\bar{q}q$ and $\bar{q}\bar{q}$.) In (8) and (10), we use the rules (*i*) — (*iv*) and (*i'*) — (*iv'*) for q and $\bar{q}$, respectively, replacing $\gamma_\mu\lambda_Q$ by unity and in (10) insert a free particle propagator for the q propagating between its successive scatterings. Vertex corrections and $q\bar{q}$ interactions for the $q\bar{q}$ pair comprising a given meson between successive scatterings have been neglected. This may not be a good approximation. It is presented here only as an example of a relativistic double scattering correction. A more adequate approximation would take $q\bar{q}$ interactions between successive scatterings into account; *e. g.*, a sum over intermediate states α and β of diagrams like that shown in Figure 2. If we assume the quark-quark scattering strongly favors small momentum transfer, the interactions taken into account in the Böhm, Joos, Krammer amplitudes might be a sufficiently good approximation for $q\bar{q}$ interactions between successive scatterings. In this case, such amplitudes might be used for the mass shell contribution to the intermediate state sum (over α and β) of Figure 2 diagrams, while (7) is used to approximate the high energy qq scattering. However, (7) represents highly absorptive qq scattering. It seems resonable to suppose, therefore, that intermediate states other than two-body might be important in double and higher multiple scattering terms. We do not

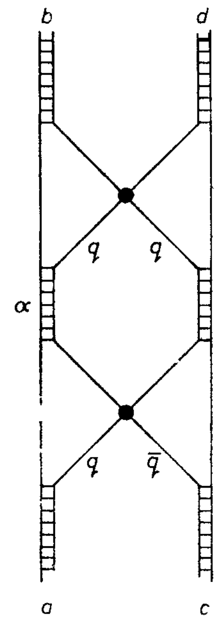


Fig. 2

know how to estimate such effects. Our model therefore is inadequate as a real model for high energy interactions. In the following section, we report mainly results of the model in the single scattering approximation $T \cong \Sigma\, t_{ij}$.

3. Some results of this model

With the above formulation of a relativistic multiple scattering model and the Bethe--Salpeter amplitudes of Böhm, Joos, and Krammer, we can calculate some effects.

First let us see the form of some amplitudes and cross-sections using the simple ansatz (7). In their rest frame $P = (M_J, 0, 0, 0)$, the Bethe-Salpeter amplitudes for the singlet mesons $\pi(0^{-+})$, $B(1^{+-})$, $A_3(2^{-+})$ ..., (J^{PC}), ... are [7]

$$\chi_{J,J_3}(q, P) = \left(\frac{4\pi 2^n}{(2n+1)!!}\right)^{1/2} \left(\frac{q^2}{\sqrt{\beta}}\right)^{n/2} Y_{JJ_3}(\hat{q})\chi_{00} \tag{11}$$

where the masses satisfy $M_J^2 = M_0^2 + \frac{3}{8} J \sqrt{\beta}$ and [10]

$$\chi_{00} = \left(1 + \frac{\not{P}}{m}\right) \gamma_5 (2\pi)^2 \sqrt{1/6\pi\beta}\; e^{-q_E^2/2\sqrt{\beta}}. \tag{12}$$

The single scattering approximation for $\pi\pi$ elastic scattering yields [11]

$$T^{(1)}_{\pi\pi;\pi\pi} \cong s f(t) \left(\frac{4m}{3}\right)^2 e^{t/8\sqrt{\beta}}. \tag{13}$$

The electromagnetic form factor of the pion is from (5)

$$F_\pi(t) = e^{t/16\sqrt{\beta}}. \tag{14}$$

In this approximation, the differential cross-section for $\pi\pi$ elastic scattering is

$$\left(\frac{d\sigma}{dt}\right)_{\pi\pi} \cong f(t)^2 F_\pi(t)^4 \frac{m^4}{\pi}\frac{16}{81}. \tag{15}$$

The corresponding qq elastic scattering cross-section is

$$\left(\frac{d\sigma}{dt}\right)_{qq} = f(t)^2 \frac{m^4}{\pi}. \tag{16}$$

(Note: $f(t)$ has dimension m^{-4}.)

The production reaction $\pi\pi \to B\pi$ vanishes in the above simple case; however, if $f_{qq} \neq f_{\bar{q}\bar{q}}$, we obtain

$$\left(\frac{d\sigma}{dt}\right)_{\pi^+\pi^-\to B^+\pi^-} = \frac{|\boldsymbol{k}|^2}{8\sqrt{\beta}}\left(\frac{d\sigma}{dt}\right)_{\pi\pi\to\pi\pi}\left|\frac{f_{qq}-f_{\bar{q}\bar{q}}}{f_{qq}+2f_{\bar{q}q}+f_{\bar{q}\bar{q}}}\right|^2, \tag{17}$$

(the TCP theorem requires equality of qq and $\bar{q}\bar{q}$ amplitudes. We exhibit equation (17) for illustrative purposes only because TCP theorem allows B-production in pion-proton collisions (see Eq. (19))), where $\boldsymbol{k}$ = three momentum of π^+ in rest frame of B^+; *viz.*,

$$|\boldsymbol{k}|^2 = [(M_B+M_\pi)^2-t]\,[(M_B-M_\pi)^2-t]/4M_B^2. \tag{18}$$

According to the mass formula: $\frac{8}{3}\sqrt{\beta} = M_B^2 - M_\pi^2$. For $\pi^+p \to B^+p$, the amplitude is proportional to $f_{qq}-f_{\bar{q}q}$ and the differential cross-section has the form

$$\left(\frac{d\sigma}{dt}\right)_{\pi^+p\to B^+p} \sim |f_{qq}-f_{\bar{q}q}|^2 \frac{|\boldsymbol{k}|^2}{8\sqrt{\beta}} F_\pi(t)^2 G_P(t)^2 \tag{19}$$

where $G_P(t)$ describes the momentum transfer to the proton. In both these reactions, t-channel helicity is conserved; *i. e.*, the B is produced with spin along the direction of the incoming π^+ in the B rest frame.

On the other hand, the single scattering approximation to our model with $f_{qq} = f_{\bar{q}q} = f_{\bar{q}\bar{q}}$ yields large cross-sections ($\sim\sigma_{\text{elastic}}$) for $\pi \to A_3$ and for the series $0^{-+} \to 2^{-+}, 4^{-+}, \ldots$. In fact, as $J \to \infty$ the cross-section rises like e^J! Of course, high J are very massive states and Böhm, Joos, and Krammer do not expect their approximate interaction to be valid for these states [7].

The triplet series $1^{++}, 2^{--}, 3^{++}, \ldots$, is described by amplitudes analogous to (11). The ground state (A_1) is given by [7]

$$\chi_{1J_3} = (2\pi)^2\left(1+\frac{\not{p}}{m}\right)(\vec{\gamma}\times\vec{q})_{J_3}[2/\sqrt{\beta^v}]^{1/2}\sqrt{1/6\pi\beta}\; e^{-q_E^2/2\sqrt{\beta^v}} \tag{20}$$

and the masses satisfy $M_J^2 = M_0^2 + \frac{8}{3} J \sqrt{\beta^v}$. The single scattering matrix elements connecting A_1 with π are of order $\sqrt{\beta^v}/m^2$ compared with $\pi \to A_3$ or $\pi \to B$ and should be neglected [11]. This reflects the fact that spin-orbit effects are negligible in the Böhm, Joos, Krammer model. If we had included some spin dependence in qq scattering instead of using (7), matrix elements for $\pi \to A_1$ could be appreciable. There are many different types of spin dependence. We have not yet investigated these possibilities.

As I am last lecturer in this XII Cracow School, I should like to close the school with an expression of appreciation on behalf of all of us who have come from abroad for the warm hospitality we have received. In particular, our thanks are offered to Dr E. Obryk and his excellent staff for all they have done to make our stay here so pleasant and worthwhile. The spirit and excellence of physics here will not soon be forgotten. We hope physics in Poland will not again suffer the tragic loss sustained last year from the untimely deaths of the experimentalists O. Czyżewski and L. Michejda. Their loss was a loss to us all and this School has fittingly honoured them by including the excellent experimental talks we have heard.

REFERENCES

[1] R. Carlitz, S. Frautschi, G. Zweig, *Phys. Rev. Letters*, **23**, 1134 (1969).

[2] J. Ballam, G. B. Chadwick *et al.*, SLAC-PUB-900 (1971); to be published in *Phys. Rev.*

[3] G. W. Brandenburg, A. D. Brody *et al.*, SLAC-PUB-1038 (1972).

[4] N. Byers, S. Frautschi, *Quanta*; *Essays in Theoretical Physics Dedicated to Gregor Wentzel*, Univ. of Chicago Press, 1970.

[5] L. B. Dick, *Phenomenology in Particle Physics*, Calif. Inst. of Technology 1971.

[6] A non-relativistic model (with some relativistic correction) using this effect has been presented by A. Le Yaouanc, L. Oliver, O. Pene, J. C. Raynal, Orsay *preprint* LPTHE 72/7.

[7] M. Böhm, H. Joos, M. Krammer, DESY *preprint* 72/11.

[8] S. Mandelstam, *Proc. Roy. Soc.*, **A223**, 248 (1955).

[9] Note our definition of T. Usual definition of T is $T = i(S-1)$. Thus, when elastic scattering amplitudes are pure imaginary (high inelasticity), elastic matrix elements of our T are pure real.

[10] All Bethe-Salpeter amplitudes obtained in Ref. [7] were obtained in the Wick rotated q-space where $q^2 = q_E^2 = q_4^2 + \vec{q}^2$. The integrations in (5), (8), and (10) may all be performed in Wick rotated q-space; thus there is no need here to worry about transforming back into the Lorentz space.

[11] We report here only leading terms; *i.e.*, terms of order $1/m$ and $1/m^2$ are neglected to conform with approximations made in Ref. [7].