LETTERS TO THE EDITOR

COMMENTS ON HIGH ENERGY COHERENT DIFFRACTIVE PRODUCTION OF MULTI-BODY STATES ON NUCLEAR TARGETS

By W. Czyż

Institute of Nuclear Physics, Cracow*

And L. C. Maximon

National Bureau of Standards, Washington D. C.**

(*Received October 23, 1972*)

Arguments are given that the traditional, Glauber-like model for the multiple scattering of composite objects be modified to the extent of including relativistic deformation of the wave functions. It is argued that this modified formalism is a specific realization of Van Hove's model of coherent nuclear production of multi-body states and thus is sufficient to explain the astonishingly small nucleon total cross-sections that have been extracted from multi-boson production experiments. It is shown on the simple example of Lorentz-contracted oscillator wave functions that, to have Van Hove's effect present, the interaction between the components of the diffractively produced object must be of the order of magnitude of their masses.

The recent experiments of coherent diffractive production of three- and five-pion systems from nuclear targets are usually interpreted as showing very small absorption of such multi-pion systems (the total 3π-nucleon and 5π-nucleon total cross-sections extracted from such experiments are not larger than the pion-nucleon cross-section [1]).

There are a few theoretical papers suggesting various interpretations of this phenomenon (see *e. g.* Refs [2, 3, 4, 5]). In this note we discuss the more traditional (Glauber-like) models which construct the production amplitude from elastic multiple scattering of the components of the produced multibody states [6, 7, 8]. Some differences between such models and the Van Hove description of coherent production [3] were pointed out

* Address: Instytut Fizyki Jądrowej, Kraków 23, Radzikowskiego 152, Poland.

** Address: National Bureau of Standards, Washington D. C. 20234, USA.

recently by Gottfried [4]. In the present paper we shall rather emphasize similarities which go very far provided one introduces some relativistic deformations of the wave functions of the diffractively produced composite objects. Before going into the discussion of our model let us stress that if one assumes that the incident particle and the object produced are composed of the same number of components [2] the experimental results of [1] are obvious and there is no puzzle to explain. We shall, however, assume that the outgoing object has more components than the incident particle: *e. g.* the incident pion is to a good approximation one "bare" pion plus a small admixture of a three "bare" pion state, the outgoing system, however, is mostly a three "bare" pion state with a small admixture of a one "bare" pion state [8].

We analyse the problem using a Glauber-like formula [6] for the coherent production amplitude from a nucleus with A nucleons:

$$\mathfrak{M}_{fi}(\Delta) = \frac{ip}{2\pi}\int d^2b e^{i\Delta\cdot b}\langle f|\{1-\prod_{j=1}^{A}(1-\Gamma_j(b))\}|i\rangle \tag{1}$$

where Δ is the transverse momentum transfer (we shall work in the limit of very large incident momentum where the longitudinal momentum transfer can be neglected). $\Gamma_j(b)$ are the profile-operators of the target nucleons as functions of the impact parameter b; they act, in general, on the internal coordinates of the produced object which, during the process of scattering, gets excited and deexcited.

In order to clarify these concepts let us write explicit expression for Γ_j in the case where the produced object is composed of two elements [6]:

$$\begin{aligned}\Gamma_j(b, \zeta) = \int d^2s\varrho_i(s)\ [\gamma_{j1}(b-\tfrac{1}{2}\zeta+s)+\gamma_{j2}(b+\tfrac{1}{2}\zeta+s)-\\ -\gamma_{j1}(b-\tfrac{1}{2}\zeta+s)\gamma_{j2}(b+\tfrac{1}{2}\zeta+s)]\end{aligned} \tag{2}$$

where $\varrho_j(s) = \int_{-\infty}^{+\infty} dz\varrho_j(s, z)$ is the two dimensional single nucleon density distribution of j-th nucleon, γ_{j1} and γ_{j2} are the nucleon profiles of the two elements, and ζ is the transverse distance between these two elements. In this case, if we neglect spin and isospin *etc.* internal quantum numbers, the various states should be described by wave functions which depend only on the relative distance ζ.

We shall also be interested in the limit of (2) when the target nucleus becomes an infinitely long slab (or, alternatively, when the object undergoing scattering is much smaller than the target nucleus):

$$\begin{aligned}\Gamma_j(\zeta) = \bar{\varrho}[\int d^2s\gamma_{j1}(s)+\int d^2s\gamma_{j2}(s)-\\ -\int d^2s\gamma_{j1}(s-\tfrac{1}{2}\zeta)\gamma_{j2}(s+\tfrac{1}{2}\zeta)]\end{aligned} \tag{3}$$

where $\bar{\varrho}$ is the average single nucleon density.

Let us go back to the general formula (1) and write it in an approximation which contains as special case both; the standard Kölbig-Margolis formula [9] and Van Hove's formula [3] for coherent nuclear production. We assume that the transition from the incident particle to the excited composite object is weak and we include it in the lowest order. The transitions between various states of the outgoing object, however, are included to all orders. Fig. 1 illustrates such process. The amplitude for the process shown in Fig. 1 one can write inserting various intermediate states $|m\rangle$ in between the operators Γ_j:

$$\mathfrak{M}_{fi}(\Delta) = \frac{ip}{2\pi}\int d^2b e^{i\Delta\cdot b} \underset{\substack{\text{sum over all}\\ \text{"histories"}}}{S} \sum_{c=1}^{A} (1-\langle f|\Gamma|m\rangle)(1-\langle m|\Gamma|m'\rangle)\ldots$$

$$\ldots\langle m_c|\Gamma_c|i\rangle\,(1-\langle i|\Gamma|i\rangle)\ldots(1-\langle i|\Gamma|i\rangle), \tag{4}$$

where by a "history" we understand a possible sequence of intermediate states: $|m\rangle$, $|m'\rangle\ \ldots\ |m_c\rangle$ after the c step took place (see Fig. 1). We obtain the Kölbig and Margolis

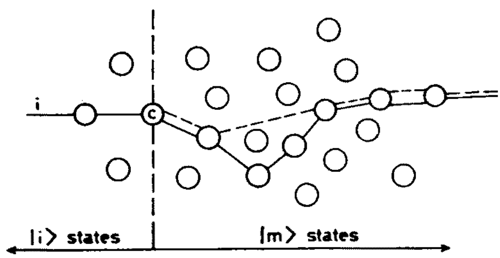

Fig. 1

formula when all intermediate states $|m\rangle$ are identical with the final state $|f\rangle$, and Van Hove's formula if we perform the sum over all possible "histories" of the produced object. This sum one can perform [3] diagonalizing the operator $\langle m'|\Gamma|m\rangle$:

$$\Gamma|\alpha\rangle = \lambda_\alpha|\alpha\rangle.$$

Then

$$\underset{\substack{\text{sum over all}\\ \text{"histories"}}}{S}\ \ldots = \sum_\alpha \sum_{c=1}^{A} \langle f|\alpha\rangle\,(1-\lambda_\alpha)^{c-1}\,\langle\alpha|\Gamma_c|i\rangle\,(1-\langle i|\Gamma|i\rangle)^{A-c} =$$

$$= \sum_\alpha \frac{(1-\langle i|\Gamma|i\rangle)^A-(1-\lambda_\alpha)^A}{\langle i|\Gamma|i\rangle-\lambda_\alpha}\,\langle f|\alpha\rangle\,\langle\alpha|\Gamma|i\rangle.$$

The form used in Ref. [3] is obtained by going to the optical limit: $(1-x)^A\to e^{-Ax}$ and we finally get

$$\mathfrak{M}_{fi}(\Delta) = \frac{ip}{2\pi}\int d^2b e^{i\Delta\cdot b}\sum_\alpha \langle f|\alpha\rangle\langle\alpha|\Gamma|i\rangle\,\frac{e^{-A\langle i|\Gamma|i\rangle}-e^{-A\lambda_\alpha}}{\langle i|\Gamma|i\rangle-\lambda_\alpha}\,. \tag{5}$$

As was discussed in [3] and [4], the spectrum of eigenvalues determines attenuation properties of the outgoing composite system. From (5) one sees that the λ_α's should be small to have, as observed in experiment, low absorption of the outgoing system. But in order to have that, $\langle m'|\Gamma|m\rangle$ should be „smooth" and in particular it must not contain a delta component or any other "spikes" [3, 4].

Let us first point out that in the limit of a very large nucleus (Eq. (3)) and if one uses orthogonal wave functions of the two states $|m\rangle$ and $|m'\rangle$ the matrix element $\langle m'|\Gamma|m\rangle$ does have $\delta_{m',m}$ which comes from the first term of (3) (which does not depend on ζ). A similar argument was given in [4] to stress the difference between the Van Hove and Glauber-like models. One should, however, make, at this stage, a point which stems from Refs [10] and [11]: In diffractive production processes in the limit of very high energy the energies of the produced system before (mass M) and after (mass M') collision with one nucleon of the target are:

$$E = \sqrt{M^2+p^2} \qquad (m \text{ state}), \quad E' = \sqrt{M'^2+p^2} \qquad (m' \text{ state}).$$

(There is no longitudinal momentum transfer in this limit and small transverse momentum transfer can be neglected.) Hence the Lorentz contraction factors are different for these two states, the ratio being:

$$\frac{\gamma'}{\gamma} = \frac{E'/M'}{E/M} = \frac{M}{M'}.$$

Thus, in general, the relativistic deformations of the states with different invariant masses are different and hence these states are not orthogonal to each other, with the consequence that $\langle m'|\Gamma|m\rangle$ does not contain the unwanted $\delta_{m',m}$ term. In other words, we obtain a specific realization of the Van Hove model, provided we introduce relativistic deformations in the wave functions of the diffractively produced object.

We do not know of any consistent way of introducing the effects of relativistic deformations in the wave functions of our model. Merely as an illustration which may give some idea of the importance of such effects let us mention the simplest possible effect (introduced in [10] and [11]) of contracting the longitudinal variable z which, in a standard Glauber-like formula, gets integrated over and does not play any role: in the case of different Lorentz factors in the initial and final states one scales differently the z variables ($z \to \gamma z$ in $|m\rangle$ and $z \to \gamma' z$ in $|m'\rangle$), and hence the states are no longer orthogonal. The results of Refs [10, 11] show that indeed such a simple operation produces an excellent agreement between the quark model calculations of diffractive N^* productions (on nucleons) and experiment.

A general comment is in order here. By introducing a set of relativistically deformed, non-orthogonal states $|m\rangle$, we depart quite far from the standard Glauber-like picture. The most important deformation is introduced into the longitudinal degrees of freedom on which the conventional profiles Γ do not act at all (they act on transverse degrees of freedom only). So, by constructing $\langle m'|\Gamma|m\rangle$ with very strongly overlapping states $|m\rangle$, $|m'\rangle$ (see below), we make its diagonalization non-trivial (Γ becomes, in general, non-diagonal in the position space representation), and weakly dependent on the specific form of Γ (strong overlap also results in weak dependence on m, m').

In order to illustrate our points we also computed the transition matrix elements $\langle m'|\Gamma|m\rangle$ of the operator (3) in the case of a Lorentz contracted harmonic oscillator. That is to say, we scaled the z coordinates of the relative distance between the two sub-units of our diffractively produced composite objects using the prescription given above. The quantum numbers of the two states $|m\rangle$ and $|m'\rangle$ were identified with the oscillator quantum numbers n_x, n_y, n_z and n'_x, n'_y, n'_z. Hence the masses of the excited states are given by

$$M = M_0+n\omega, \qquad n = n_x+n_y+n_z, \tag{6}$$

where M_0 (the mass of the ground state) and ω (difference in mass of the neighbouring excited states) are accepted as free parameters. From (3) with gaussian elementary profiles and the oscillator potential wave functions we obtain the result that $\langle m'|\Gamma|m\rangle$ is a function of the vectors $\boldsymbol{n}(n_x, n_y, n_z)$, $\boldsymbol{n}(n'_x, n'_y, n'_z)$ and $\mu = \omega/M_0$. The matrix element $\langle \boldsymbol{n}'|\Gamma|\boldsymbol{n}\rangle$ of (3) is (due to the different Lorentz contractions of the two states) a broad function of M and M' if $\mu \gtrsim 1$: The M' dependence for a given M is weak and varies slowly with M. When $\mu \ll 1$, $\langle \boldsymbol{n}'|\Gamma|\boldsymbol{n}\rangle$ becomes very sharp (δ-like) function of M, M' with the peak at $M' = M$. If the parameters of the two elementary gaussian profiles are chosen to reproduce approximately the pion-nucleon elastic cross-section (compare *e.g.* [6]), the single scattering term of (3) dominates and double scattering contributes only about 10%.

We interpret these results as follows: The larger is μ the stronger is the interaction between the components of the diffractively produced object. Hence, in order to have Van Hove's effect [3], the interaction between these components must be strong ($\mu \gtrsim 1$). When the interaction is weak ($\mu \ll 1$) there is a sharp peak in $\langle \boldsymbol{n}'|\Gamma|\boldsymbol{n}\rangle$ and the conditions imposed on $\langle \boldsymbol{n}'|\Gamma|\boldsymbol{n}\rangle$ by Van Hove [3] are not satisfied. It would seem therefore that the effect of anomalously small absorption will not show up in the case of diffractive dissociation of small nuclei on large nuclei (it is presumably a $\mu \ll 1$ case): one has to have interaction energies comparable to the rest masses of the components. In a more realistic model of the spectrum of a diffractively produced system, one should rather have a ladder of quasistationary states which overlap very strongly with each other. Although the computations described above were done for a not very realistic case, our qualitative conclusions are presumably quite general.

Our last remark is that relativistic deformation of the wave functions of diffractively produced objects is not the only effect which makes $\langle m'|\Gamma|m\rangle$ smooth and produce low absorption of these objects. The finite sizes of such objects (relative to the target nucleus size) also tend to reduce absorption as was shown in [8]. However, the relative importance of these effects is still not well known.

One of the authors (W. C.) is grateful to R. H. Dalitz and K. Gottfried because the ideas presented in this note were triggered by his listening to their lectures given at the XII$^{\text{th}}$ Cracow School of Theoretical Physics, Zakopane, June 1972. He is also grateful to A. Białas for his very helpful remarks and criticism and to many colleagues from Cracow for stimulating discussions. Discussions of these problems with, and the constructive criticism by L. Van Hove and K. Gottfried are gratefully acknowledged.

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