Vol. B 3 (1972) ACTA PHYSICA POLONICA No 2

LEE MODEL WITH V-PARTICLE HAVING CONTINUOUS SPECTRUM OF ASYMPTOTIC MASSES

J. LUKIERSKI AND M. OZIEWICZ

Institute of Theoretical Physics, University of Wrocław*

(Received May 20, 1971)

Problems related with the field-theoretic description of unstable particles are studied. The Lee model is introduced with the V-particle replaced by a general unstable elementary object with continuous mass spectrum. The model is completely solved in the lowest $N\theta$ sector. The analytic continuation on the second Riemann sheet of the scattering amplitudes and the analytically continued unitarity condition are considered.

The derivation of the reduction formula with complex mass shell is presented. Finally the equivalence is shown of this model in $N\theta$ sector with the conventional Lee model with four-leg direct interaction.

1. *Introduction*

In quantum field theory there are two different ways of introducing unstable particles as primary objects:

a) The method due to Peierls [1], based on the notion of complex mass shell, defined by the following restriction on the analytically continued values of the four-momentum p_μ:

$$p_\mu p^\mu = p_0^2 - \boldsymbol{p}^2 = M^2 = M_0^2 - i\Gamma, \Gamma > 0. \tag{1.1}$$

Such a formulation leads to the definition of a propagator of unstable particle by means of a complex pole on the second Riemann sheet. In particular, in group-theoretic language, unstable particles are described by irreducible, nonunitary representations of the Poincare group [2–3], corresponding to the complex eigenvalues of the four-momentum p_μ, satisfying the condition (1.1).

b) Another method, introducing more general unstable elementary objects, identifies the notion of the propagator for free unstable object with the generalized free field [4–5]. This way of introducing unstable particles has been advocated in the late fifties by Schwinger [6] and Matthews and Salam [7]. In particular, the notion of the mass shell for free unstable objects is defined as follows [8]:

$$p^2 = \kappa^2 \quad m_1^2 \leqslant \kappa^2 \leqslant m_2^2 \tag{1.2}$$

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Wrocław, Cybulskiego 36, Poland.

i. e. the mass-shell extends over the four-dimensional domain of the momentum space, contained between two hyperboloids $p^2 = m_1^2$ and $p^2 = m_2^2$. The interval $[m_1^2, m_2^2]$ describes the mass spectrum of the unstable elementary object. According to group theory the free, unstable objects can be defined as the multiplicity-free reducible unitary representations of the Poincare group [9].

The propagator of the free unstable elementary object $\Phi_0(x)$ is given by the formula[1]

$$\langle 0|T\left\{\Phi_0\left(\frac{x}{2}\right)\Phi_0\left(-\frac{x}{2}\right)\right\}|0\rangle = \frac{1}{i}\int\limits_{m_1^2}^{m_2^2} d\kappa^2 \varrho(\kappa^2)\Delta^c(x;\kappa^2) \tag{1.3}$$

where $\varrho(\kappa^2)$ is a continuous positive measure, which can be normalized as follows:

$$\int\limits_{m_1^2}^{m_2^2} \varrho(\kappa^2)d\kappa^2 = 1. \tag{1.4}$$

Such a way of description is much more general than the one, using the notion of a complex pole on the unphysical sheet. Indeed, in order to get exactly a single complex pole, one should introduce the following spectral function in (1.3):

$$\varrho_{M^2}(\kappa^2) = \frac{1}{\pi}\frac{\Gamma}{(\kappa^2 - M_0^2)+\Gamma^2} \tag{1.5}$$

and extend the integration over all positive and negative values of squared masses κ^2. This last requirement, violating the spectral condition, reflects one of the basic difficulties with Peierl's approach — the fact that the notion of free unstable particle introduced on the complex mass shell violates general principles of QFT[2]. We can only conclude, therefore, that the description by a complex pole is an approximate one, valid only for a limited range of masses around M_0^2 and under the assumption that $M_0^2 \gg \Gamma$. This deficiency is, however, not valid, if we introduce elementary unstable objects by means of the second approach. This method is so general, that any particular scattering channel with continuous spectrum of total energy can be represented as the generalized mass shell of the type (1.2) for a single unstable elementary object[3]. If one wishes to introduce in such formalism an unstable free particle, one should modify (1.5) as follows:

$$\varrho_{M^2}^{ph}(\kappa^2) = f(\kappa^2)\,\varrho_{M^2}(\kappa^2) \tag{1.6}$$

where $f(\kappa^2)$ has the following properties:

1) $f(\kappa^2) = 0$ for $\kappa^2 < m_{\text{thresh}}^2$
2) $f(\kappa^2) = 1$ for $\kappa^2 = M_0^2$

---

[1] We shall consider in this paper only spinless, unstable, elementary objects.

[2] Another point, raised recently against the idea of complex mass-shell approach, consists in presenting the examples of scattering amplitudes, satisfying all requirements of analytic S-matrix theory and producing sharp isolated resonances without accompanying Peierl's pole on the unphysical sheet. See, for example, G. Gallucci, L. Fonda, G. C. Ghirardi, *Phys. Rev.*, **166**, 1719 (1968).

[3] The discussion of the most general, free, unstable object has been given by J. Lukierski, *Acta Phys. Hungar.*, **26**, 217 (1969) (Proc. of Balaton Symposium on Hadron Spectroscopy, Kesthely, Hungary).

3) $f(\kappa^2)$ describes the correct threshold behaviour at $\kappa^2 = m^2_{\text{thresh}}$. Besides, one can normalize $f(\kappa^2)$ in such a way, that the relation (1.4) remains valid also for $\varrho^{ph}_{M^2}$. Before passing to some technical problems we would like to dwell shortly upon the physical meaning and utility of the notion of elementary unstable objects in relativistic QFT. The instability property and the occurence of decay products always implies, that for every theory with unstable particles there should exist an underlying asymptotically complete theory defined in the Fock space of stable asymptotic particles. Speaking in less rigorous, but more intuitive language, every unstable particle is indeed a composite object, formed from its decay products. It appears, however, that sometimes it is useful to forget about the underlying "basic" theory, containing only stable particles[4]. In particular, the notion of unstable elementary object makes sense if we wish to consider a perturbation expansion in the presence of resonances. It is easy to deduce that finite sum of perturbation theory diagrams is not able to explain the occurence of unstable objects. One can, therefore, try to separate the terms, which are not approximate by a finite number of perturbation theory terms. Such a procedure has been proposed in nonrelativistic scattering theory for the case of stable bound states by Weinberg[5], who introduced in such a way the so-called quasiparticles. If, however, the quasiparticle is unstable, for example a resonance, one should modify Weinberg's idea by introducing the free field operator, which is able to describe a continuous mass spectrum. In such a way one arrives at the notion of the field operator $\varphi(x; s)$ with an additional continuous parameter, which in the free case is characterized by the following equation of motion[6]

$$(\Box - s)\,\varphi_0(x; s) = 0 \tag{1.7}$$

and the following four-dimensional commutator

$$\left[\varphi_0\left(\frac{x}{2}; s\right),\ \varphi_0\left(-\frac{x}{2}; s'\right)\right] = i\Delta(x; s)\delta(s-s'). \tag{1.8}$$

The generalized free field $\Phi_0(x)$ is expressed by fields $\varphi_0(x; s)$ as follows:

$$\Phi_0(x) = \int_0^\infty \varrho^{1/2}(s)\varphi_0(x; s)ds. \tag{1.9}$$

The complete QFT containing unstable particles should be constructed in two steps:

a) One introduces a theory with an elementary unstable object, with arbitrarily chosen mass spectrum $\varrho(s)$.

b) One picks up from the underlying theory, describing the interactions of stable particles, the dynamical mechanism determining the spectral function.

---

[4] Sometimes one assumes, that the underlying "basic" theory is described by a nonobservable field operator and that only stable bound states have a physical interpretation. Heisenberg's theory of elementary particles is constructed in such a way. A similar scheme is valid in field-theoretic quark models.

[5] See for example S. Weinberg, *Brandeis Lectures*, 1964, Vol. 2, p. 289.

[6] The field operator $\varphi_0(x; s)$ has been introduced by Licht (A. Licht, *Ann. Phys.*, **34**, 161 (1965)). The notion of field operator with continuous spectrum of asymptotic masses has been introduced by Thirring (W. Thirring, *Phys. Rev.*, **126**, 1209 (1962)).

The general LSZ formulation of a field theory with basic field operator $\varphi(x; s)$, describing interacting elementary unstable object, has been given by one of the authors [5]. The notion of a free field in such QFT, describing elastic scattering, has been also investigated [10]. In both these papers only the first level of the theory was considered with the field operator $\varphi(x; s)$ as a primary one, and the spectral function $\varrho(s)$ not determined dynamically. In order to study both levels of the complete theory — the formulation with unstable elementary objects as well as the correspondence with the underlying theory of stable particles — we shall consider in this paper a generalization of the Lee model with the field operator $V(\boldsymbol{p})$, describing the V-particle, replaced by the operator $V(\boldsymbol{p}; E)$. We hope that using such a simple nonrelativistic model one can show the meaning and features of the unstable elementary object, described by the field operator

$$\mathscr{V}(\boldsymbol{p}) = \int_0^\infty \varrho^{1/2}(E) V(\boldsymbol{p}; E) dE. \tag{1.10}$$

In Sect. 2 we give the formulation of our model and in Sect. 3 we present an explicit solution of the $N\Theta$ sector.

We have the following three processes in $N\Theta$ sector:

$$\begin{aligned} N\Theta &\to N\Theta \\ N\Theta &\to \mathscr{V} \\ \mathscr{V} &\to \mathscr{V}. \end{aligned} \tag{1.11}$$

The first process describes the non-resonant part of $N\Theta$-scattering, the second process the transition between the non-resonant and resonant parts, and the third one describes the "free" resonance.

In Sect. 4 we discuss the relation between our description and the one representing the unstable V-particle as a complex pole on the unphysical sheet. We show also in Sect. 4 how to continue analytically on the second Riemann sheet the unitarity condition. The problems of comparison with the conventional Lee model with direct four-leg interaction are studied in Sect. 5.

2. *The formulation of the model*

We introduce the following Hamiltonian:

$$\mathrm{H} = H_0^V + H_0^N + H_0^\Theta + H_{\text{int}} \tag{2.1}$$

where [18]:

$$H_0^V = \int_{E_0}^\infty dE \cdot E \int d^3p V^+(\boldsymbol{p}, t; E) V(\boldsymbol{p}, t; E) \tag{2.1a}$$

$$H_0^N = m_N \int d^3p N^+(\boldsymbol{p}, t)\, N(\boldsymbol{p}, t) \tag{2.1b}$$

$$H_0^\Theta = \int d^3k \omega(k)\, \Theta^+(\boldsymbol{k}, t)\, \Theta(\boldsymbol{k}, t) \tag{2.1c}$$

and

$$H_{\rm int} = \frac{g_0}{(2\pi)^{3/2}} \int \frac{d^3k f(k)}{(2\omega(k))^{1/2}} \int d^3p \{\mathscr{V}^+(\boldsymbol{p}, t) N(\boldsymbol{p}-\boldsymbol{k}, t)\Theta(\boldsymbol{k}, t) + \text{h.c.}\} \tag{2.1d}$$[7]

$$\mathscr{V}(\boldsymbol{p}, t) = \int_{E_0}^{\infty} dE \varrho^{1/2}(E) V(\boldsymbol{p}, t; E) \tag{2.1e}$$

where $f(k)$ is real and $\int_{E_0}^{\infty} dE\, \varrho(E) = 1$.

The field operators satisfy the following commutation relations:

$$[V^+(\boldsymbol{p}, t; E), V(\boldsymbol{p}', t; E')] = \delta(\boldsymbol{p}-\boldsymbol{p}')\, \delta(E-E') \tag{2.2a}$$

$$[N^+(\boldsymbol{p}, t), N(\boldsymbol{p}', t)] = \delta(\boldsymbol{p}-\boldsymbol{p}') \tag{2.2b}$$

$$[\Theta^+(\boldsymbol{k}, t), \Theta(\boldsymbol{k}', t)] = \delta(\boldsymbol{k}-\boldsymbol{k}'). \tag{2.2c}$$

Other equal time commutator of the operators V, V^+, N, N^+, Θ, and Θ^+ do vanish.

In order to obtain the equations of motion, we should use the Heisenberg equations. One gets the following set of equations[8]

$$\left(\frac{1}{i}\frac{\partial}{\partial t} - E\right) V(\boldsymbol{p}, t; E) = \frac{g_0}{(2\pi)^{3/2}} \varrho^{1/2}(E) \int \frac{d^3k f(k)}{(2\omega(k))^{1/2}} N(\boldsymbol{p}-\boldsymbol{k}, t)\Theta(\boldsymbol{k}, t) \tag{2.3a}$$

---

[7] We denote $k = |\boldsymbol{k}|$ and $\omega(k) = (k^2+\mu^2)^{\frac{1}{2}}$. Besides, we shall assume further that $E_0 = m_N+\mu$.

[8] One can also derive the equations (2.3) as Lagrange-Euler equations following the Lagrangean:

$$\mathscr{L} = \mathscr{L}_0^V + \mathscr{L}_0^N + \mathscr{L}_0^\Theta + \mathscr{L}_{\rm int}$$

where

$$\mathscr{L}_0^V = \frac{i}{2} \int_{E_0}^{\infty} \{V^+(\boldsymbol{x}, t; E) \overleftrightarrow{\partial_t} V(\boldsymbol{x}, t; E) + EV^+(\boldsymbol{x}, t; E)\, V(\boldsymbol{x}, t; E)\}\, dE$$

$$\mathscr{L}_0^N = \frac{\rm i}{2} \{N^+(\boldsymbol{x}, t) \overleftrightarrow{\partial_t} N(\boldsymbol{x}, t) + m_N N^+(\boldsymbol{x}, t)\, N(\boldsymbol{x}, t)\}$$

$$\mathscr{L}_0^\theta = \frac{i}{2} \{\Theta^+(\boldsymbol{x}, t) \overleftrightarrow{\partial_t} \Theta(\boldsymbol{x}, t) + \int d^3x' \Theta^+(\boldsymbol{x}, t)\omega^2(\boldsymbol{x}-\boldsymbol{x}')\, \Theta(\boldsymbol{x}' t,)\}$$

and

$$\mathscr{L}_{\rm int} = g_0 \int d^3x'\, F(\boldsymbol{x}-\boldsymbol{x}') \{\mathscr{V}^+(\boldsymbol{x}, t) N(\boldsymbol{x}', t)\, \Theta(\boldsymbol{x}', t) + \text{h. c.}\}$$

provided that

$$\omega_n(\boldsymbol{x}) = \frac{1}{(2\pi)^3} \int e^{i\boldsymbol{p}\boldsymbol{x}} (p^2+\mu^2)^n d^3p$$

$$F(\boldsymbol{x}) = \frac{1}{(2\pi)^3} \int f(k) e^{i\boldsymbol{k}\boldsymbol{x}}\, d^3k.$$

The field operators occurring in the Lagrangean satisfy the following E. T. commutation rela-

$$\left(\frac{1}{i}\frac{\partial}{\partial t}-m_N\right)N(\boldsymbol{p},t)=\frac{g_0}{(2\pi)^{3/2}}\int\frac{d^3kf(k)}{(2\omega(k))^{1/2}}\mathscr{V}(\boldsymbol{p}-\boldsymbol{k},t)\Theta^+(\boldsymbol{k},t) \tag{2.3b}$$

$$\left(\frac{1}{i}\frac{\partial}{\partial t}-\omega(k)\right)\Theta(\boldsymbol{k},t)=\frac{g_0}{(2\pi)^{3/2}}\int\frac{d^3pf(p)}{(2\omega(p))^{1/2}}\mathscr{V}(\boldsymbol{p},t)N^+(\boldsymbol{p}-\boldsymbol{k},t). \tag{2.3c}$$

Introducing the object $\mathscr{V}$ with continuous spectrum of asymptotic masses one preserves the basic properties of the conventional Lee model, *i. e.*:

a) The physical vacuum $|0\rangle$ is defined by means of the relation

$$N(\boldsymbol{p},t)|0\rangle=\Theta(\boldsymbol{p},t)|0\rangle=V(\boldsymbol{p},t;E)|0\rangle=0. \tag{2.4}$$

b) It is possible to define two conserved charges

$$Q_1=\int_{E_0}^{\infty}dE\int d^3pV^+(\boldsymbol{p},t;E)V(\boldsymbol{p},t;E)+\int d^3pN^+(\boldsymbol{p},t)N(\boldsymbol{p},t) \tag{2.5a}$$

$$Q_2=\int d^3pN^+(\boldsymbol{p},t)N(\boldsymbol{p},t)-\int d^3k\Theta^+(\boldsymbol{k},t)\Theta(\boldsymbol{k},t) \tag{2.5b}$$

satisfying the property:

$$[H,Q_1]=[H,Q_2]=0. \tag{2.6}$$

From the relations (2.6) it follows that in the model (2.1) one can introduce analogous sectors as in usual Lee model[9]

c) The local limit is obtained, if

$$f(k)\to 1. \tag{2.7}$$

In this paper we shall discuss in detail the $N\Theta$ sector of our model.

3. *The solution of $N\Theta$ sector*

It is known[6] that the Zachariasen [11] model can be described by a bilinear Lagrangean with the field operator describing S-wave pair with continuous mass parameter. The

---

tions:

$$[V^+(\boldsymbol{x},t;E),V(\boldsymbol{x}',t;E')]=\delta(\boldsymbol{x}-\boldsymbol{x}')\,\delta(E-E')$$
$$[N^+(\boldsymbol{x},t),N(\boldsymbol{x}',t)]=\delta(\boldsymbol{x}-\boldsymbol{x}')$$
$$[\Theta^+(\boldsymbol{x}\,t),\Theta(\boldsymbol{x}',t)]=\omega^{-1}(\boldsymbol{x}-\boldsymbol{x}').$$

The equal time commutators have been normalized in such a way that the local limit is described by the relations (2.1).

It should be stressed, that the nonlocality occuring in our Lagrangean is only with respect to the space directions and one can derive the equations (2.3) as Lagrange-Euler equations without any formal difficulties.

[9] See for example S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, Row, Peterson and Co, Evanston (USA), 1961.

Zachariasen model shows remarkable resemblance to the $N\Theta$ sector in the Lee model. It has been shown [12] that in Weinberg's reference frame, with infinite total three-momentum, the $N\Theta$ scattering amplitude in the Lee model is given exactly by the unrenormalized form of the scattering amplitude in the Zachariasen model. On the other hand it has been proved [10] that in relativistic theory the scattering amplitude in any elastic channel can be described as one-particle scattering of some field operator $\varphi_A(x; s)$, commuting to a c-number, where the parameter s describes the total mass spectrum. This general conclusion is also valid in nonrelativistic theory, provided that the covariant total mass square s is replaced by the total energy. In particular it has been demonstrated[10] that the $N\Theta$ sector (*i. e.* the formula for $N\Theta$ scattering and the formula for the propagator of V-particle) can be described in a completely equivalent way by a bilinear Hamiltonian of the Thirring type[6] the only difference being the nonrelativistic kinematics. Applying this method to our model, one can describe the dynamics of $N\Theta$ sector by the following Hamiltonian:

$$H = H_0^V + H_0^\varphi + H_{\text{int}}^{V\varphi} \tag{3.1}$$

where H_0^V is given by (2.1a),

$$H_0^\varphi = \int_{E_0}^{\infty} dE \cdot E \int d^3p \varphi^+(\boldsymbol{p}, t; E)\varphi(\boldsymbol{p}, t; E) \tag{3.1a}$$

$$H_{\text{int}}^{V\varphi} = g_0 \int d^3p \{\mathscr{V}^+(\boldsymbol{p}, t)\, \Phi(\boldsymbol{p}, t) + \text{h. c.}\} \tag{3.1b}$$

and

$$\Phi(\boldsymbol{p}, t) = \int_{E_0}^{\infty} dE \sigma^{1/2}(E)\varphi(\boldsymbol{p}, t; E). \tag{3.1c}$$

The one particle state

$$|\boldsymbol{p}, t; E\rangle = \varphi(\boldsymbol{p}, t; E)|0\rangle \tag{3.2}$$

describes an S-wave $N\Theta$ pair with the total energy E. In the subspace representing the $N\Theta$ sector in the Lee model one can introduce the following substitution:

$$|\boldsymbol{p}, t; E\rangle = \frac{\sigma_0^{-1/2}(E)}{(2\pi)^{3/2}} \int \frac{d^3k}{(2\omega(k))^{1/2}}\, \delta(E-\omega(k)-m_N)N(\boldsymbol{p}-\boldsymbol{k}, t)\Theta(\boldsymbol{k}, t)|0\rangle. \tag{3.3}$$

Because

$$[\varphi^+(\boldsymbol{p}, t; E), \varphi(\boldsymbol{p}', t; E')] = \delta(\boldsymbol{p}-\boldsymbol{p}')\, \delta(E-E') \tag{3.4}$$

we have

$$\langle \boldsymbol{p}, t; E|\boldsymbol{p}', t; E'\rangle = \delta(\boldsymbol{p}-\boldsymbol{p}')\, \delta(E-E') \tag{3.5}$$

[10] One can also introduce the Lagrangean leading to the equations (3.9) as the Lagrange-Euler equations. See footnote 9 and J. Lukierski, M. Oziewicz, *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astron. Phys.*, **18**, 695 (1970).

and from (3.3), (3.5) and (2.2) one gets

$$\sigma_0(E) = \frac{1}{4\pi m_N}\sqrt{(E-m_N)^2-\mu^2}\,. \tag{3.6}$$

Using the relation

$$\frac{1}{(2\pi)^{3/2}}\int \frac{d^3k\cdot f(k)}{(2\omega(k))^{1/2}}\, N(\boldsymbol{p}-\boldsymbol{k},t)\Theta(\boldsymbol{k},t)|0\rangle =$$

$$= \frac{1}{(2\pi)^{3/2}}\int_{E_0}^{\infty} dE \int \frac{d^3k\cdot f(k)}{(2\omega(k))^{1/2}}\,\delta(E-\omega(k)-m_N)N(\boldsymbol{p}-\boldsymbol{k},t)\Theta(\boldsymbol{k},t)|0\rangle =$$

$$= \int_{E_0}^{\infty} dE\sigma_0^{1/2}(E)F(E)|\boldsymbol{p},t;E\rangle \tag{3.7}$$

and

$$F(E) \equiv f([(E-m_N)^2-\mu^2]^{1/2}) \tag{3.7a}$$

we see from (2.1d), that one should put in (3.1c) the following value of $\sigma(E)$:

$$\sigma(E) = \sigma_0(E)\cdot F^2(E). \tag{3.8}$$

Using the canonical commutation relations, one obtains from (3.1) the following Heisenberg equations of motion:

$$\left(\frac{1}{i}\frac{\partial}{\partial t}-E\right)\varphi(\boldsymbol{p},t;E) = g_0\sigma^{1/2}(E)\int_{E_0}^{\infty} dE\varrho^{1/2}(E)V(\boldsymbol{p},t;E) \tag{3.9a}$$

$$\left(\frac{1}{i}\frac{\partial}{\partial t}-E\right)V(\boldsymbol{p},t;E) = g_0\varrho^{1/2}(E)\int_{E_0}^{\infty} dE\sigma^{1/2}(E)\varphi(\boldsymbol{p},t;E). \tag{3.9b}$$

One can introduce the following four Green functions

$$\begin{aligned}
\tau_{\bar{V},V}(x;E,E') &= -i\,\langle 0|T\{V^+(x;E)\,V(x;E')\}|0\rangle \\
\tau_{\overline{N\Theta},N\Theta}(x;E,E') &= -i\,\langle 0|T\{\varphi^+(x;E)\,\varphi(0;E')\}|0\rangle \\
\tau_{\bar{V},N\Theta}(x;E,E'). &= -i\,\langle 0|T\{V^+(x;E)\,\varphi(0;E')\}|0\rangle \\
\tau_{\overline{N\Theta},V}(x;E,E') &= -i\,\langle 0|T\{\varphi^+(x;E)\,V(0;E')\}|0\rangle.
\end{aligned} \tag{3.10}$$

Using the equations (3.9) and the canonical commutation relations:

$$\begin{aligned}
[V^+(x;E),\,V(0,E')]_{t=0} &= \delta(\boldsymbol{x})\,\delta(E-E') \\
[\varphi^+(x;E),\,\varphi)0,E')]_{t=0} &= \delta(\boldsymbol{x})\,\delta(E-E')
\end{aligned} \tag{3.11}$$

one obtains for the Fourier transforms of (3.10) the following set of equations:

$$(p_0-E)\tau_{\bar{V}V}(p;E,E') = \delta(E-E')+g_0\varrho^{1/2}(E)\int_{E_0}^{\infty} dE''\sigma^{1/2}(E'')\tau_{\overline{N\Theta},V}(p;E'',E') \quad (3.12a)$$

$$(p_0-E)\tau_{\overline{N\Theta},V}(p;E,E') = g_0\sigma^{1/2}(E)\int_{E_0}^{\infty} dE''\varrho^{1/2}(E'')\tau_{\bar{V}V}(p;E'',E') \quad (3.12b)$$

$$(p_0-E)\tau_{\overline{N\Theta},N\Theta}(p;E,E') =$$

$$= \delta(E-E')+g_0\sigma^{1/2}(E)\int_{E_0}^{\infty} dE''\varrho^{1/2}(E'')\tau_{\bar{V},N\Theta}(p;E'',E') \quad (3.12c)$$

$$(p_0-E)\tau_{\bar{V},N\Theta}(p;E,E') = g_0\varrho^{1/2}(E)\int_{E_0}^{\infty} dE''\sigma^{1/2}(E'')\tau_{\overline{N\Theta},N\Theta}(p;E'',E') \quad (3.12d)$$

the solution of the system of equations is as follows:

$$\tau_{\bar{V}V}(p;E,E') = \frac{\delta(E-E')}{p_0-E+i\varepsilon}+$$

$$+g_0^2\frac{\varrho^{1/2}(E)\varrho^{1/2}(E')}{(p_0-E+i\varepsilon)(p_0-E'+i\varepsilon)}\cdot\frac{\tau_{\mathscr{V}}(p)}{1-g_0^2\cdot\tau_{\Phi}(p)\cdot\tau_{\mathscr{V}}(p)} \quad (3.13a)$$

$$\tau_{\overline{N\Theta},V}(p;E,E') = g_0\frac{\sigma^{1/2}(E)\cdot\varrho^{1/2}(E')}{(p_0-E+i\varepsilon)(p_0-E'+i\varepsilon)}\cdot\frac{1}{1-g_0^2\cdot\tau_{\Phi}(p)\cdot\tau_{\mathscr{V}}(p)} \quad (3.13b)$$

$$\tau_{\bar{V},N\Theta}(p;E,E') = g_0\frac{\varrho^{1/2}(E)\cdot\sigma^{1/2}(E')}{(p_0-E+i\varepsilon)(p_0-E'+i\varepsilon)}\cdot\frac{1}{1-g_0^2\cdot\tau_{\Phi}(p)\cdot\tau_{\mathscr{V}}(p)} \quad (3.13c)$$

$$\tau_{\overline{N\Theta},N\Theta}(p;E,E') = \frac{\delta(E-E')}{p_0-E+i\varepsilon}+$$

$$+g_0^2\frac{\sigma^{1/2}(E)\cdot\sigma^{1/2}(E')}{(p_0-E+i\varepsilon)(p_0-E'+i\varepsilon)}\cdot\frac{\tau_{\Phi}(p)}{1-g_0^2\cdot\tau_{\Phi}(p)\cdot\tau_{\mathscr{V}}(p)} \quad (3.13d)$$

where

$$\tau_{\mathscr{V}}(p) = -i\int d^4x e^{-ipx}\langle 0|T\{\mathscr{V}(x)\mathscr{V}(0)\}|0\rangle = \int_{E_0}^{\infty}\frac{\varrho(E)dE}{p_0-E+i\varepsilon} \quad (3.14a)$$

$$\tau_{\Phi}(p) = -i\int d^4x e^{-ipx}\langle 0|T\{\Phi(x)\Phi(0)\}|0\rangle = \int_{E_0}^{\infty}\frac{\sigma(E)dE}{p_0-E+i\varepsilon}. \quad (3.14b)$$

In order to obtain the scattering amplitudes for the processes (1.11) one should use the following reduction formula[11]

$$\frac{1}{\pi} T(\boldsymbol{p}, p_0) = \lim_{\substack{E \to p_0 \\ E' \to p_0}} \{(p_0 - E)(p_0 - E')\tau(\boldsymbol{p}, p_0; E, E')\}. \tag{3.15}$$

It is easy to see that the scattering amplitudes do not depend on $\boldsymbol{p}$, *i. e.* $T(\boldsymbol{p}, p_0) \equiv T(p_0)$ and similarly $\tau_{\mathscr{V}}(p) \equiv \tau_{\mathscr{V}}(p_0)$, $\tau_{\Phi}(p) \equiv \tau_{\Phi}(p_0)$.

We have, using (3.13)–(3.15):

$$T_{\overline{V},V}(p_0) = \pi \frac{g_0^2 \varrho(p_0)\tau_{\Phi}(p_0)}{1 - g_0^2 \cdot \tau_{\Phi}(p_0) \cdot \tau_{\mathscr{V}}(p_0)} \tag{3.16a}$$

$$T_{\overline{N\Theta},V}(p_0) = T_{\overline{V},N\Theta}(p_0) = \pi \frac{g_0 \sigma^{1/2}(p_0)\varrho^{1/2}(p_0)}{1 - g_0^2 \cdot \tau_{\Phi}(p_0) \cdot \tau_{\mathscr{V}}(p_0)} \tag{3.16b}$$

$$T_{\overline{N\Theta},N\Theta}(p_0) = \pi \frac{g_0^2 \sigma(p_0)\tau_{\mathscr{V}}(p_0)}{1 - g_0^2 \cdot \tau_{\Phi}(p_0) \cdot \tau_{\mathscr{V}}(p_0)}. \tag{3.16c}$$

It is easy to notice from (3.16b) that if

$$\operatorname{supp} \varrho \, U \operatorname{supp} \sigma \neq 0 \tag{3.17}$$

there occurs a transition $N + \Theta \to V$.

In order to arrive at a physical interpretation of the scattering amplitudes (3.16) we should have the following two unitarity conditions satisfied:

$$\operatorname{Im} T_{\overline{V}V}(p_0) = |T_{\overline{V},N\Theta}(p_0)|^2 + |T_{\overline{V}V}(p_0)|^2 \tag{3.18a}$$

$$\operatorname{Im} T_{\overline{N\Theta},N\Theta}(p_0) = |T_{N\Theta,V}(p_0)|^2 + |T_{\overline{N\Theta},N\Theta}(p_0)|^2. \tag{3.18b}$$

One can easily check, using the formulae

$$\operatorname{Im} \tau_{\mathscr{V}}(p_0) = \pi\varrho(p_0)\, \Theta(E_0 - p_0) \tag{3.19a}$$

$$\operatorname{Im} \tau_{\Phi}(p_0) = \pi\sigma(p_0)\Theta\, (E_0 - p_0) \tag{3.19b}$$

that the relations (3.18) are valid.

We would like to observe, that it is possible to write the unitarity relations (3.18) also in the following way:

$$\operatorname{Im} \tilde{T}_{\overline{V}V}(p_0) = \sigma(p_0)|\tilde{T}_{\overline{V},N\Theta}(p_0)|^2 + \varrho(p_0)|\tilde{T}_{\overline{V}V}(p_0)|^2 \tag{3.20a}$$

$$\operatorname{Im} \tilde{T}_{\overline{N\Theta},N\Theta}(p_0) = \varrho(p_0)|\tilde{T}_{\overline{N\Theta},V}(p_0)|^2 + \sigma(p_0)|\tilde{T}_{\overline{N\Theta},N\Theta}(p_0)|^2 \tag{3.20b}$$

---

[11] See for example Ref. [5]. See also
a) M. S. Maxon, R. B. Curtis, *Phys. Rev.*, **137B**, 996 (1965),
b) J. Lukierski, L. Turko, *Phys. Acad. Sci. Polon.*, **16**, 905 (1968).

The choice of the constant factor $\frac{1}{\pi}$ is related with the simplest form (4.2) of the unitarity condition.

where

$$T_{\bar{V}V}(p_0) = \varrho(p_0)\tilde{T}_{\bar{V}V}(p_0)$$

$$T_{\overline{N\Theta},V}(p_0) = \varrho^{1/2}(p_0)\cdot\sigma^{1/2}(p_0)\tilde{T}_{\overline{N\Theta},V}(p_0)$$

$$T_{\overline{N\Theta},N\Theta}(p_0) = \sigma(p_0)\tilde{T}_{\overline{N\Theta},N\Theta}(p_0). \tag{3.21}$$

The coefficients $\sigma(p_0)$ and $\varrho(p_0)$ have the meaning of the phase spaces of the intermediate Φ($N\Theta$-pair!) and $\mathscr{V}$ objects inserted between $\hat{T}^+$ and $\hat{T}$ in the unitarity relation.

4. Analytic continuation of the energy plane and the unstable V-particle as a complex pole on the unphysical sheet

Let us introduce the field operator describing the unstable particle, by means of the formula (2.1e). We define the following Green functions:

$$\tau_{\bar{\mathscr{V}},\mathscr{V}}(x) = -i\langle 0|T\{\mathscr{V}^+(x)\mathscr{V}(0)\}|0\rangle = \int dE\int dE'\varrho^{1/2}(E)\varrho^{1/2}(E')\tau_{\bar{V},V}(x;E,E')$$

$$\tau_{\bar{\mathscr{V}},N\Theta}(x;E) = -i\langle 0|T\{\mathscr{V}^+(x)\varphi(0;E)\}|0\rangle = \int dE'\varrho^{1/2}(E')\tau_{\bar{V},N\Theta}(x;E,E')$$

$$\tau_{\overline{N\Theta},\mathscr{V}}(x;E) = -i\langle 0|T\{\varphi^+(x;E)\mathscr{V}(0)\}|0\rangle = \int dE'\varrho^{1/2}(E')\tau_{\overline{N\Theta},V}(x;E,E'). \tag{4.1}$$

Using the formulae (3.13) for the Green functions, one obtains the following results for the Fourier transforms of the Green functions (4.1):

$$\tau_{\bar{\mathscr{V}}\mathscr{V}}(p_0) = \tau_{\mathscr{V}}(p_0)+\tau_{\mathscr{V}}(p_0)\frac{1}{\pi}\tilde{T}_{\bar{V}V}(p_0) \tag{4.2a}$$

$$\tau_{\bar{\mathscr{V}},N\Theta}(p_0;E) = \tau_{\overline{N\Theta},\mathscr{V}}(p_0;E) = \frac{\sigma^{1/2}(E)}{p_0-E+i\varepsilon}\tau_{\mathscr{V}}(p_0)\frac{1}{\pi}\tilde{T}_{\bar{V},N\Theta}(p_0). \tag{4.2b}$$

The Green functions (4.2a) and (4.2b) are described by the analytic functions in the complex energy plane with a cut extending from E_0 to ∞, and they can be analytically continued into the second sheet. We have the following formulae for the propagators (3.14) on the second sheet ($\mathrm{Im}\ z_0 < 0$):

$$\tau^{\mathrm{II}}_{\mathscr{V}}(z) = \tau^{\mathrm{I}}_{\mathscr{V}}(z)-2\pi i\varrho(z) \tag{4.3a}$$

$$\tau^{\mathrm{II}}_{\Phi}(z) = \tau^{\mathrm{I}}_{\Phi}(z)-2\pi i\sigma(z) \tag{4.3b}$$

where $\tau^{\mathrm{I}}_{\mathscr{V}}(z)$ and $\tau^{\mathrm{I}}_{\Phi}(z)$ are obtained from the formulae (3.14) by the replacement $p_0+i\varepsilon\rightarrow z$, and analogously:

$$\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z) = \pi\frac{g_0^2\tau^{\mathrm{II}}_{\Phi}(z)}{1-g_0^2\cdot\tau^{\mathrm{II}}_{\Phi}(z)\cdot\tau^{\mathrm{II}}_{\mathscr{V}}(z)} \tag{4.4a}$$

$$\tilde{T}^{\mathrm{II}}_{\overline{N\Theta},V}(z) = T^{\mathrm{II}}_{\bar{V},N\Theta}(z) = \pi\frac{g_0}{1-g_0^2\cdot\tau^{\mathrm{II}}_{\Phi}(z)\cdot\tau^{\mathrm{II}}_{\mathscr{V}}(z)}. \tag{4.4b}$$

The analytic continuations of the Green function (4.2) is as follows:

$$\tau^{\mathrm{II}}_{\bar{\mathscr{V}}\mathscr{V}}(z) = \tau^{\mathrm{II}}_{\mathscr{V}}(z)+\tau^{\mathrm{II}}_{\mathscr{V}}(z)\cdot\tau^{\mathrm{II}}_{\mathscr{V}}(z)\frac{1}{\pi}\,\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z) \tag{4.5a}$$

$$\tau^{\mathrm{II}}_{\bar{\mathscr{V}},N\Theta}(z;E) = \tau^{\mathrm{II}}_{\overline{N\Theta},\mathscr{V}}(z;E) = \frac{\sigma^{1/2}(E)}{z-E}\,\tau^{\mathrm{II}}_{\mathscr{V}}(z)\frac{1}{\pi}\,\tilde{T}^{\mathrm{II}}_{\bar{V},N\Theta}(z). \tag{4.5b}$$

Let us assume that the propagator $\tau^{\mathrm{II}}_{\mathscr{V}}(z)$ has a pole on the second Riemann sheet at the point $z = M = M_0-i\Gamma$.

Assuming that its residuum is normalized to unity, we have

$$\lim_{z\to M}(z-M)\tau^{\mathrm{II}}_{\mathscr{V}}(z) = 1. \tag{4.6}$$

Using the relation (4.6), one can introduce the following reduction formula for the Green functions (4.5), defined on the second sheet

$$\lim_{z\to M}\pi(z-M)^2\tau^{\mathrm{II}}_{\bar{\mathscr{V}}\mathscr{V}}(z) = \tilde{T}^{\mathrm{II}}_{\bar{V}V}(M) \tag{4.7a}$$

$$\lim_{z\to M}\pi(z-M)\,(z-E)\tau^{\mathrm{II}}_{\bar{\mathscr{V}},N\Theta}(z) = \sigma^{1/2}(E)\tilde{T}^{\mathrm{II}}_{\bar{V},N\Theta}(M). \tag{4.7b}$$

The reduction formulae (4.7) give the value of the scattering amplitudes (4.4a)-(4.4b) at the complex point $z = M_0-i\Gamma$. Such a point determines the complex energy shell on the second sheet, introducing Peierl's notion of unstable particle.

It is interesting to see that the values of $\tilde{T}^{\mathrm{II}}_{\bar{V}V}(M)$ and $\tilde{T}^{\mathrm{II}}_{\bar{V},N\Theta}(M)$ are restricted by the unitarity condition, analytically continued onto the second sheet. Let us introduce the phase space $\Sigma_{\mathscr{V}}(z,z^*)$ and $\Sigma_{\Phi}(z,z^*)$ for the objects $\mathscr{V}$ and Φ, having complex energy $z = p_0+iq_0$ and represented on the second sheet by the formulae (4.5). We have the following formulae:

$$\Sigma_{\mathscr{V}}(z,z^*) = \Sigma^*_{\mathscr{V}}(z^*,z) \tag{4.8a}$$

$$\Sigma_{\Phi}(z,z^*) = \Sigma^*_{\Phi}(z^*,z) \tag{4.8b}$$

and, if $z\to p_0$, one should get (see (3.19)):

$$\Sigma_{\mathscr{V}}(z,z^*) \underset{z\to p_0}{\to} \varrho(p_0)\Theta(E-p_0) \tag{4.9a}$$

$$\Sigma_{\Phi}(z,z^*) \underset{z\to p_0}{\to} \sigma(p_0)\Theta(E-p_0). \tag{4.9b}$$

The unitarity condition, analytically continued on the second sheet, can be written as follows:

$$\mathrm{Im}\,\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z) = \Sigma_{\Phi}(z,z^*)|\tilde{T}^{\mathrm{II}}_{\bar{V},N\Theta}(z)|^2+\Sigma_{\mathscr{V}}(z,z^*)|\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z)|^2 \tag{4.10a}$$

$$\mathrm{Im}\,\tilde{T}^{\mathrm{II}}_{\overline{N\Theta},N\Theta}(z) = \Sigma_{\mathscr{V}}(z,z^*)|\tilde{T}^{\mathrm{II}}_{\overline{N\Theta},V}(z)|^2+\Sigma_{\Phi}(z,z^*)|\tilde{T}^{\mathrm{II}}_{\overline{N\Theta},N\Theta}(z)|^2 \tag{4.10b}$$

where the phase spaces are given by the formulae

$$\Sigma_{\mathscr{V}}(z, z^*) = \frac{1}{\pi} \operatorname{Im} \tau^{\mathrm{II}}_{\mathscr{V}}(z) \tag{4.11a}$$

$$\Sigma_{\Phi}(z, z^*) = \frac{1}{\pi} \operatorname{Im} \tau^{\mathrm{II}}_{\Phi}(z). \tag{4.11b}$$

If we assume the behaviour (4.6) on the second sheet, the phase space $\Sigma_{\mathscr{V}}$ is singular at $z = M$, and for the neighbourhood of $z = M$ one can write

$$\operatorname{Im} \tilde{T}^{\mathrm{II}}_{\bar{V}V}(z) \underset{z=M}{\approx} \frac{1}{\pi} \operatorname{Im} \tau^{\mathrm{II}}_{\mathscr{V}}(z) |\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z)|^2 \tag{4.12}$$

or

$$\operatorname{Im} [\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z)]^{-1} \underset{z=M}{\approx} -\frac{1}{\pi} \operatorname{Im} \tau^{\mathrm{II}}_{\mathscr{V}}(z). \tag{4.13}$$

One gets, therefore, near the singularity described by (4.6), that

$$\tilde{T}^{\mathrm{II}}_{\bar{V}V}(z) \underset{z=M}{\approx} -\pi(z-M). \tag{4.14}$$

The result (4.14) can be derived also directly from (4.4a). We can conclude, therefore

$$T^{\mathrm{II}}_{\bar{V}V}(M) = 0 \tag{4.15a}$$

$$\frac{d}{dz} \tilde{T}^{\mathrm{II}}_{\bar{V}V}(z)|_{z=M} = -\pi. \tag{4.15b}$$

It is interesting to note that one obtains the formula (4.6), if one assumes

$$\varrho(p_0) = \frac{1}{\pi} \frac{\Gamma}{(p_0 - M_0)^2 + \Gamma^2}. \tag{4.16}$$

In such a case one gets besides the pole (4.6) also a second pole, at $t = M^*$. Because one can write

$$\varrho(z) = \frac{\Gamma}{\pi} \frac{1}{(z-M)(z-M^*)} = -\frac{1}{2\pi i}\left\{\frac{1}{z-M} - \frac{1}{z-M^*}\right\} \tag{4.17}$$

we see, using (4.3a), that

$$\tau^{\mathrm{II}}_{\mathscr{V}}(z) = \begin{cases} \dfrac{1}{z-M} & \text{for } z \approx M \\ -\dfrac{1}{z-M^*} & \text{for } z \approx M^*. \end{cases} \tag{4.18}$$

The pole at $z = M^*$ is very far away from the physical region and does not influence the value of the scattering amplitude on the real axis.

One can introduce also the generalizations of the notion of a complex mass shell, defined by the formulae (4.7). It is possible to generalize (4.7) for any singular point of $\tau^{\mathrm{II}}_{\mathscr{V}}(z)$. Assuming for example, that near the threshold

$$\varrho^{1/2}(E) \underset{E=E_0}{\approx} a\Theta(E-E_0) \tag{4.19}$$

one gets the following behaviour of $\tau^{\mathrm{II}}_{\mathscr{V}}(z)$

$$\tau^{\mathrm{II}}_{\mathscr{V}}(z) \underset{E=E_0}{\approx} \frac{a}{2\pi i} \ln (E_0-z). \tag{4.20}$$

The reduction formula, describing the value of the scattering amplitude $\tilde{T}^{\mathrm{II}}_{VV}$ at the threshold (the scattering length) takes the form

$$\lim_{z\to E_0} \frac{4\pi^3}{a} \ln^{-2} (E_0-z)\tau^{\mathrm{II}}_{\overline{\mathscr{V}}\mathscr{V}}(z) = \tilde{T}^{\mathrm{II}}_{VV}(E_0). \tag{4.21}$$

The reduction formula, defining the scattering amplitude (4.4a) for all complex values of z, can be described as follows:

$$T^{\mathrm{II}}_{VV}(z) = \pi[\tau^{\mathrm{II}}_{\mathscr{V}}(z)[^{-2} \{\tau_{\overline{\mathscr{V}}\mathscr{V}}(z)-\boldsymbol{\tau}^{\mathrm{II}}_{\mathscr{V}}(z)\} \tag{4.22}$$

and can be used for any choice of $\tau^{\mathrm{II}}_{\mathscr{V}}(z)$.

Finally we shall consider the eigenvalue equation for the physical complex mass $\tilde{M} = \tilde{M}_0 - i\tilde{\Gamma}$ of the unstable $\mathscr{V}$-particle. The propagator $\tilde{\tau}^{\mathrm{II}}_{\mathscr{V}}(z)$ of the interacting $\mathscr{V}$-particle is given by the formula

$$[\tilde{\tau}^{\mathrm{II}}_{\mathscr{V}}(z)]^{-1} = [\tau^{\mathrm{II}}_{\mathscr{V}}(z)]^{-1} - g_0^2\tau^{\mathrm{II}}_{\Phi}(z) \tag{4.23}$$

and we have

$$[\tau^{\mathrm{II}}_{\mathscr{V}}(\tilde{M})]^{-1} = g_0^2\tau^{\mathrm{II}}_{\Phi}(\tilde{M}). \tag{4.24}$$

The equation (4.24) is the generalization of the usual formula for the mass renormalization in the ordinary Lee model

$$\delta m = \tilde{M}_0 - M_0 = g_0^2\tau^{\mathrm{II}}_{\Phi}(\tilde{M}_0) \tag{4.25}$$

and describes the case, when already unperturbed M_0 is a complex number with negative imaginary part.

5. *The equivalence with conventional field-theoretic model*

From the considerations of Section 3 it follows that in the $N\Theta$ sector one can describe the two-particle state

$$\frac{1}{(2\pi)^{3/2}} \int \frac{d^3k f(k)}{(2\omega(k))^{1/2}} N(\boldsymbol{p}-\boldsymbol{k}, t)\Theta(\boldsymbol{k}, t)|0\rangle \tag{5.1}$$

as the following one-particle state

$$\int_{E_0}^{\infty} dE\sigma^{1/2}(E)\varphi(\boldsymbol{p}, t; E)|0\rangle \tag{5.2}$$

where $\sigma(E)$ is given by the formula (3.8).

In this section we are interested in the inverse problem, *i. e.* how to find two-particle state corresponding to the following state

$$\mathscr{V}(\boldsymbol{p}, t)|0\rangle = \int_{E_0}^{\infty} dE\varrho^{1/2}(E)V(\boldsymbol{p}, t; E)|0\rangle \tag{5.3}$$

where $\varrho(E)$ is given as a primary quantity. It is easy to see that one can represent the state (5.3) as follows:

$$\frac{1}{(2\pi)^{3/2}}\int \frac{d^3kg(k)}{(2\omega(k))^{1/2}}\tilde{N}(\boldsymbol{p}-\boldsymbol{k}, t)\tilde{\Theta}(\boldsymbol{k}, t)|0\rangle \tag{5.4}$$

where

$$g^2(k) = \frac{\varrho(\omega(k)+m_N)}{\sigma(\omega(k)+m_N)} \tag{5.5}$$

and the operators $\tilde{N}$, $\tilde{N}^+$ and $\tilde{\Theta}$, $\tilde{\Theta}^+$ do commute with N, N^+ and Θ, Θ^+. The Hamiltonian, leading in the lowest sector to three scattering amplitudes

$$N\Theta \to N\Theta$$

$$N\Theta \to N'\Theta'$$

$$N'\Theta' \to N'\Theta' \tag{5.6}$$

identical with corresponding three scattering amplitudes (1.11) looks as follows

$$H = H_0 + H_{\text{int}} \tag{5.7}$$

where

$$H_0 = H_0^N + H_0^\Theta + N_0^{N'} + H_0^{\Theta'} \tag{5.7a}$$

and

$$H_{\text{int}} = \frac{g_0}{(2\pi)^{3/2}}\int \frac{d^3kf(k)}{(2\omega(k))^{1/2}}\int \frac{d^3k'g(k')}{(2\omega(k'))^{1/2}} \times$$

$$\times\{\tilde{N}^+(\boldsymbol{p}-\boldsymbol{k}', t)\,\tilde{\Theta}^+(\boldsymbol{k}', t)\,N(\boldsymbol{p}-\boldsymbol{k}, t)\,\Theta(\boldsymbol{k}, t)+\text{h. c.}\}. \tag{5.7b}$$

In particular, it is easy to see from the formula (5.5). that

$$\text{supp}\,\varrho(E) \subset [m_{\tilde{N}}+\tilde{\mu}, \infty] \tag{5.8}$$

i. e. no corresponding field-theoretic model exists, which allows to introduce an elementary unstable particle represented by a single pole on the unphysical Riemann sheet.

The equivalence between the lowest sectors of the Hamiltonians (2.1) and (5.7) cannot be extended, however, to arbitrarily high sectors. This problem is related with the general question, how to relate the generalized free fields with the free field operator, describing free stable scalar particles[12]. It has been shown recently by Turko [13] that only the lowest, one-particle sector, generated by the generalized, free field from the vacuum, can be represented by usual multiparticle states in the Fock space. If we consider the general state created by the products of generalized free field operators, an equivalent description in the Fock space of stable particles requires the introduction of parastatistics. If we consider, therefore, the $\mathscr{V}\Theta$-$N\Theta\Theta$ sector in our model (2.1), the corresponding generalized Lee model is further given by (5.7). The lack of equivalence appears in the channel where at least two $\mathscr{V}$-objects can appear. An example of such a sector with lowest number of particles is the one containing $NN\Theta\Theta$ states.

REFERENCES

[1] R. E. Peierls, *Proc. of the 1954 Glasgow Conference*, London 1955, p. 296.
[2] D. Zwanzinger, *Phys. Rev.*, **131**, 888 (1963).
[3] L. S. Schulman, *Ann. Phys.*, **59**, 201 (1970).
[4] J. Lukierski, *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astron. Phys.*, **16**, 343 (1968).
[5] J. Lukierski, *Nuovo Cimento*, **60A**, 353 (1969).
[6] J. Schwinger, *Ann. Phys.*, **9**, 169 (1960).
[7] P. T. Matthews, A. Salam, *Phys. Rev.*, **112**, 283 (1958).
[8] See Ref. [4], J. Lukierski, *Bull. Acad. Polon. Sci., Ser. Sci. Math. Astron. Phys.*, **16**, 431 (1968).
[9] F. Lurcat, *Phys. Rev.*, **173**, B 1461 (1968).
[10] W. Karwowski, J. Lukierski, N. Sznajder, *Nuovo Cimento*, **63A**, 509 (1969).
[11] F. Zachariasen, *Phys. Rev.*, **121**, 1851 (1961).
[12] H. Osborn, S. Zieman, *Nuclear Phys.*, **B1**, 180 (1967).
[13] L. Turko, *Ph. D. Thesis*, Wrocław University (to be published).

[12] See for example L. Streit, *Helv. Phys. Acta*, **39**, 65 (1965), R. A. Brandt, O. W. Greenberg. *J. Math. Phys.*, **10**, 1168 (1969) or A. L. Licht, Max Planck Institute *preprint*, 1970.