CALCULATION OF NEUTRON EQUILIBRIUM PAIRING IN SPHERICAL SINGLE CLOSED SHELL NUCLEI

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Neutron pairing in Ni, Sn and Pb isotopes is calculated by minimalization of the total energy under the condition of constant volume. Satisfactory overall agreement with experimental pairing parameters is found in all three cases.

1. Introduction

Low energy collective modes of nuclear motion, which influence the structure of nuclear ground states, have, as a rule, a simple geometrical interpretation. The corresponding effective fields may be considered as symmetry distortions of the single particle potential. The strength of the fields may be connected with the parameters of the potential with the help of the self-consistency conditions of geometrical type, *e.g.* volume conservation condition.

Till now pairing has been considered as an exception from this rule. In view of the evident two particle, short range character of pairing correlations any simple connection of the effective pairing force $G\Sigma a_\nu^+ a_{-\nu}^+ a_{-\nu}\, a_\nu$ or the pairing field $\Delta\Sigma(a_\nu^+ a_{-\nu}^+ + a_{-\nu} a_\nu)$ with the parameters of the single particle potential seemed unprobable. However, it has been established [1, 2] that every field producing component of the residual interaction can be recovered from the self consistent single particle potential when the appropriate self-consistency condition is known. Thus, the question which should be answered in the case of pairing is rather that of the existence of a geometrical condition which the pairing field should obey. The simplest candidate for such a condition, which has been suggested in [3] and which should be carefully tried, is the volume conservation, assuring conservation of the average particle density in the nucleus. As it has been shown in [3], minimalization of the total energy under this condition gives reasonable values for static pairing parameters in the case of symmetric configurations and harmonic oscillator potential.

The physical significance of the volume conservation applied to the case of pairing correlations needs some discussion. First of all there is the question of how much pairing may be associated with the volume conserving components of the residual interaction. The pairing mode is a low energy mode and should not mix strongly with a rather high

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energy breathing mode, which covers the effect of fluctuations of average nuclear density and volume. On the other hand, pairing correlations are known to be due to the combined effect of many high multipolarity components of the residual interaction. Verry little is known till now about these components and their influence on nuclear radius. Their coherent low energy effect may very well be analogous in many respects to that of low energy, low multipolarity components. From this point of view it is instructive to see how large is the pairing components of an affective force calculated under the condition of constant volume. Applying the technique of [2, 3, 5] to the volume conserving symmetry distortions of the harmonic oscillator potential and taking harmonic radial dependence for distortions of all multipolarities, one gets:

$$F_{12} = \frac{\delta \mathcal{U}_1}{\delta \varrho_2} = \sum_{\lambda \neq 0} \sum_{\mu} \frac{4\pi}{3} \frac{m\omega^2}{AR^2} \hat{r}_1^2 \hat{r}_2^2 Y_{\lambda\mu}(\hat{\theta}_1) Y^*_{\lambda\mu}(\hat{\theta}_2) =$$

$$= \frac{4\pi}{3} \frac{m\omega^2}{AR^2} \hat{r}_1^2 \hat{r}_2^2 [\delta(\hat{\theta}_1 - \hat{\theta}_2) - Y_{00}(\hat{\theta}_1) Y^*_{00}(\theta_2)] \tag{1}$$

where m — the mass of a nucleon, ω — the oscillator frequency, A — atomic number of the nucleus considered, R — the nuclear radius, ϱ — density of particles. $\hat{r}$ and $\hat{\theta}$ are the radial and angular co-ordinates in the co-ordinate system in which single particle potential $\mathcal{U}$ is spherically symmetric. Their structure reflects the deformation dependence of the force (1). They become usual space co-ordinates for spherical nucleus.

Volume conservation eliminates from the expansion (1) the monopole term, which would otherwise cause a change in the nuclear radius, making the estimate of the force strength impossible. It also changes single particle potential $\mathcal{U}$ enlarging its strength ω by the amount proportional to the mean value of the square of the deformation change for each excitation mode. The single particle contribution to the nuclear radius gets thereby smaller by the amount by which the radius is enlarged due to the zero point vibrations in each mode.

The delta function character of the interaction (1) suggests that it contains strong pairing component. The corresponding average pairing force strength may be estimated by taking $1/4\pi$ for the average value of the angular matrix element of (1) and replacing r^2 by its mean value at the Fermi surface:

$$\langle r^2 \rangle_F = \frac{\hbar}{m\omega} \left(N_F + \frac{3}{2} \right) \simeq \frac{\hbar}{m\omega} \cdot 1.1 A^{1/3}.$$

One gets:

$$G \simeq \frac{4\pi}{3} \frac{m\omega^2}{AR^2} \langle r^2 \rangle_F^2 \frac{1}{4\pi} \simeq \frac{11.5}{A} \text{MeV}. \tag{2}$$

This value should be compared with $G = 13$–14 MeV/A, which is needed to explain ~ 2 MeV energy gap in spherical nuclei, for the pairing force acting in the single particle space of 7–8 oscillator shells.

From the estimate of the volume conserving force (1) one learns first of all that it contains a pairing component strong enough to explain most, possibly all pairing observed

in nuclei. Furthermore, the pairing component is determined here by the same quantities which determine the single particle potential, namely by the size of the nucleus. It should, however, be kept in mind that the force (1) is the one appropriate for RPA calculation and it is an open question to what extent can it be used in the BCS-like approximations. Static pairing should rather be obtained in a way analogous to the one used in the case of static deformations, namely by minimalization of the total energy under the conditions which gave the force (1). As in the case of static deformation, the adjustments of the single particle field (introduced by the volume conservation requirement) should be important in the equilibrium pairing calculation. As a matter of fact, they determine the position of the energy minimum in the pairing case.

The mechanism by which pairing adjusts the single particle field is the rearrangement mechanism and for the generalized Hartree field it is given by the following expression for the self-consistent single-particle potential (see *e.g.* [6] and references given there):

$$U_{\nu'\nu''} = \sum_{\nu>0} \langle \nu'\nu | V_{12} | \nu\nu'' \rangle 2\mathscr{V}_\nu^2 \tag{3}$$

where $\mathscr{V}_\nu$ are pairing amplitudes for the single particle states ν. V_{12} is the nucleon-nucleon two-body interaction. In order to illustrate the effect of the rearrangement (3) let us assume that the harmonic oscillator potential is generated by a volume conserving monopole force $V_{12} = k\omega^2 r_1^2 r_2^2$ where k is a constant, and is chosen in such a way, that the value of $\langle r^2 \rangle$, calculated for pairing parameters corresponding to the pairing given by the real force V_{12}, is equal to its experimental value. The single particle field (3) can now be written as:

$$\mathscr{U}_1 = k\omega \frac{\hbar}{m} \left[\sum_\nu \left(N_\nu + \frac{3}{2} \right) 2\mathscr{V}_\nu^2 \right] r_1^2$$

and thus the self-consistent value of ω is given by

$$\omega(\Delta) = 2k \frac{\hbar}{m^2} \sum_{\nu>0} \left(N_\nu + \frac{3}{2} \right) 2\mathscr{V}_\nu^2 \tag{4}$$

where we stressed the dependence of ω on the pairing energy gap Δ which determines the amplitudes $\mathscr{V}_\nu$. When we force pairing to change, the expression (4) gives the corresponding rearrangement of the harmonic oscillator single particle potential. The oscillator frequency ω changes in such a way, that for any value of the pairing parameters the mean square radius $\langle r^2 \rangle = \dfrac{\hbar}{m\omega(\Delta)} \displaystyle\sum_{\nu>0} \left(N_\nu + \frac{3}{2} \right) 2\mathscr{V}_\nu^2$ remains constant. This is the condition which was used in [3] to get the equilibrium pairing.

In the present paper the calculation of the equilibrium pairing energy gap, performed under the condition (4) for a more realistic, Nilsson potential, are presented. Inclusion of the spin-orbit and l^2 terms in the single particle potential allows the calculation for the ground states of real nuclei. We present here the results for neutron pairing in spherical single closed shell nuclei with $Z = 28, 50$ and 82 (Ni, Sn and Pb isotopes). They show how the proposed method works in three different mass regions: $A \sim 60$, $A \sim 116$ and $A \sim 200$.

2. Energy expression

The most important problem in the calculation, in which energy is minimized with respect to the pairing parameters, is of course, the expression for the total energy of the nucleus, which should give a sufficiently accurate energy change due to the change of pairing. As we include rearrangement (4) explicitly, the harmonic oscillator contribution to the single particle energy term must be written with the factor 3/4, in order that the potential energy change is counted once. In the Nilsson potential the $l\cdot s$ and l^2 terms are present in addition to the oscillator term. They do not contribute to the single particle radius, and so the rearrangement (3) in them could be included in a standard way, by taking their contribution to the energy change with the factor 1 instead of 1/2. However, the self-consistency conditions under which the force (1) was obtained include the assumption, that the shape of the potential follows the shape of the density distribution. In our case this would mean that a suitably defined square radius of the potential should also be conserved. One can define a quantity which measures the effective size of the Nilsson potential as

$$\frac{1}{m\omega^2}\sum_{\nu<0}\varepsilon_\nu 2\mathscr{V}_\nu^2$$

where ε_ν are the single particle energies. In te limit of a pure harmonic oscillator it is equal to the mean square radius of the density distribution. When the $l\cdot s$ and l^2 terms are included, it measures the average square radius of the surface of equal potential at energies corresponding to the occupied single particle states. In the BCS approximation with one value of Δ for all the states it is impossible to keep constant both the density radius and the above defined radius of the potential. However, the main change which is introduced in the energy expression by the conservation of the potential radius in comparison with the density radius conservation is that it defines also the rearrangement in the $l\cdot s$ and l^2 terms, replacing the factor 1/2 by 5/4. As the rearrangement in these terms should really be connected with the change of parameters specific to them and not with the change of ω, one can keep the density radius constant and try to include the effect of the more complete self-consistency condition by using the factor 5/4 instead of the standard factor 1 in the $l\cdot s$ and l^2 contribution to the energy change. The single particle contribution to the energy takes, therefore, the form:

$$E_{\text{s.p.}} = \hbar\omega(\Delta)\left\{\tfrac{3}{4}\sum_{\nu>0}(N_\nu+\tfrac{3}{2})2\mathscr{V}_\nu^2+\tfrac{5}{4}\kappa\sum_{\nu>0}\left[-2l\cdot s-\mu(l^2-\langle l^2\rangle)\right]_{\nu\nu}2\mathscr{V}_\nu^2\right\} \qquad (5)$$

where N_ν is the harmonic oscillator quantum number of the state ν, κ and μ are the Nilsson model parameters.

The pairing energy contribution is taken in the form corrected for the zero-point pairing vibrations in the BCS+RPA approximation [7]

$$E_{\text{pairing}} = -\Delta\sum_{\nu>0}U_\nu\mathscr{V}_\nu+\hbar\sum_\sigma\omega_\sigma-\sum_{\nu>0}E_\nu \qquad (6)$$

where U, $\mathcal{V}$ and E are the BCS pairing amplitudes and quasiparticles energies, ω_σ are the roots of the BCS+RPA equation for the frequencies of the pairing vibration mode:

$$[4\Delta^2-(\hbar\omega_\sigma)^2]\sum_{\nu>0}\frac{1}{2E_\nu[4E_\nu^2-(\hbar\omega_\sigma)^2]}=0. \tag{7}$$

The last two terms in [6] represent the difference between BCS+RPA and BCS ground state energies and include in particular the $G\mathcal{V}^4$ term of BCS theory.

The complete expression which should give a right energy change with the change of the volume conserving pairing is:

$$\tilde{E}(\Delta)=\frac{\omega(\Delta)}{\omega(\Delta=0)}[E_{\text{s.p.}}+E_{\text{pairing}}] \tag{8}$$

where $E_{\text{s.p}}$ and E_{pairing} are measured in $\hbar\omega(\Delta)$ units, while $\tilde{E}(\Delta)$ is measured in $\hbar\omega(\Delta=0)$.

The energy (8) was minimized with respect to the gap parameter Δ. The equilibrium value of this parameter should describe the physical pairing in the nucleus.

It should be stressed here, that the energy expression (8) should be used with care when energy differences between different nuclear states or different nuclei are considered (*e.g.* odd-even mass differences). These are not restricted by the volume conservation condition and thus very accurate knowledge of the Nilsson model parameters change would be required to use the expression (8). It is more practical to calculate those energy changes in a standard way, namely by comparing the sums of the single particle energies and pairing contributions at a fixed value of the Nilsson model parameters.

3. Details of the calculation

The parameters of the Nilsson potential are the only parameters in our calculation. They are known quite well for the regions of large deformation. This fixes their values for neutrons in $N=5$ and $N=6$ oscillator shells — the Fermi surface region for Pb isotopes. For neutrons in $N=3$ and $N=4$ shells, the Fermi surface regions for Ni and Sn, the Nilsson model parameters were obtained by extrapolation from the deformed regions.

The results turn out to be practically independent of the subshell structure far away from the Fermi surface. Therefore, we can work with one single-particle level scheme for all the nuclei, provided the appropriate Nilsson model parameters are taken for each oscillator shell. The parameters for the relevant shells, also the values used for proton states, are listed below. The choice of the proton parameters is of minor importance for calculation of the neutron energy gaps. The values used in the calculation were taken from the deformed regions.

Neutrons

$$\kappa=0.0637,\ \mu=\begin{cases}0.48 & \text{for } N=3\\ 0.46 & \text{for } N=4\\ 0.42 & \text{for } N=5\\ 0.325 & \text{for } N=6\end{cases}$$

Protons

$\kappa=0.0637,\ \mu=0.60$ for $N=3,4,5$

$\kappa=0.0577,\ \mu=0.65$ for $N=6$

As the aim of the calculation was to check the proposed method of determining the pairing, no attempt has been made at this stage to choose the best parameters for each region considered. In order to get some feeling about the sensitivity of the results to the relative positions of the single-particle states the calculation was performed also with $\mu = 0.38$ for $N = 3$ (Ni isotopes) and with $\mu = 0.48$ and 0.42 for $N = 4$ (Sn isotopes). The results are compared in Fig. 1.

The pairing problem for $\Delta = 0.01, 0.02, 0.03, \ldots 0.30\,\hbar\omega$ was solved in the BCS approximation with the equation for the chemical potential:

$$\sum_{\nu>0} 2v_\nu^2 = \text{number of neutrons.} \tag{9}$$

Equation (9) was solved in the single particle space of 7 oscillator shells in the case of Ni and Sn isotopes and 8 oscillator shells in the case of Pb. The pairing amplitudes and quasiparticle energies obtained in this way were used in Eq. (7) to obtain the frequencies ω_σ.

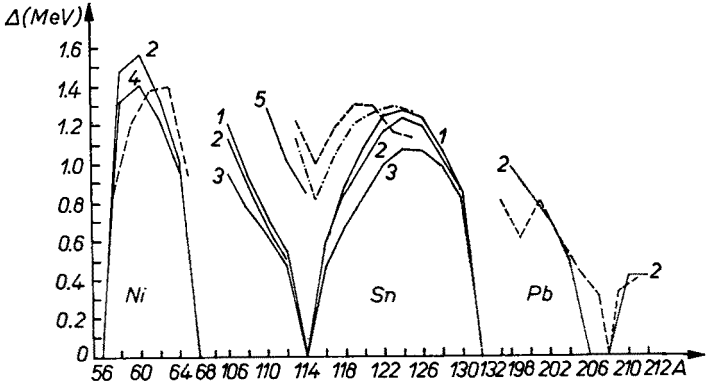

Fig. 1. Equilibrium energy gaps calculated in this paper are compared with values obtained by the inverse gap equation method in [8]. Dashed lines show the values of Δ obtained in [8] for the level nearest to the Fermi surface. The dotted-dashed line shows the gaps of [8] averaged over $d_{5/2}, g_{7/2}, s_{1/2}, d_{3/2}$ and $h_{11/2}$ levels for each isotope of Sn. Full lines show our results obtained with fixed Nilsson model parameters, as listed in the text (line marked *2*), $\mu = 0.48$ and 0.42 for $N = 4$ (*1* and *3* respectively), $\mu = 0.38$ for $N = 3$ (*4*) and with $s_{1/2}, d_{3/2}$ levels shifted down, as described in the text (line *5*)

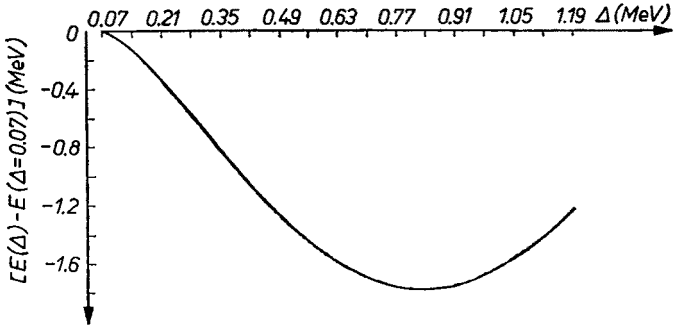

Fig. 2. The curve $\tilde{E}(\Delta)$ for ^{200}Pb

and in expressions (4), (8) to obtain the ratio $\omega(\Delta)/\omega(\Delta = 0)$ and energy $\tilde{E}(\Delta)$ for each value Δ. The equilibrium value Δ_{eq} was then obtained by finding the minimum of the function $\tilde{E}(\Delta)$. An example of the $\tilde{E}(\Delta)$ curve is given in Fig. 2.

4. Results and discussion

The calculated values of Δ_{eq} are listed in column 2 of the Table I. Column 3 of the Table gives the effective strength G_{eff} of the pairing force which would lead to $\Delta = \Delta_{eq}$ according to the BCS gap equation

$$\frac{2}{G_{eff}} = \sum_{\nu>0} \frac{1}{\sqrt{(\varepsilon_\nu - \lambda)^2 + \Delta_{eq}^2}}. \tag{10}$$

In the column 4 the values of G_{eff}, averaged over all isotopes, are given for each element. They show that the $1/A$ overall dependence of G is confirmed in our calculation. Apart

TABLE I

	1	2	3	4
	A	Δ_{eq} (MeV)	G_{eff} ($\hbar\omega$)	$\bar{G}_{eff} \times A$ (MeV)
Ni	58	1.48	0.0204	
	60	1.57	0.0194	11.8
	62	1.35	0.0179	
	64	1.02	0.0167	
Sn	106	1.13	0.0140	
	108	0.90	0.0124	
	110	0.68	0.0112	
	112	0.51	0.0106	
	114	0.00		
	116	0.59	0.0107	12.3
	118	0.84	0.0115	
	120	1.00	0.0120	
	122	1.16	0.0128	
	124	1.23	0.0133	
	126	1.19	0.0132	
	128	1.02	0.0134	
	130	0.85	0.0135	
Pb	198	0.98	0.0100	
	200	0.84	0.0097	
	202	0.70	0.0095	
	204	0.49	0.0090	12.9
	206	0.00		
	210	0.41	0.0086	
	212	0.41	0.0078	

from this dependence there is an additional dependence on the subshell structure near the Fermi surface. It forces the effective interaction to vary from one isotope to another and it has its counterpart in the variation of the effective force when it is obtained by solving the inverse gap equation of Ref. [8]. If the distance to the next subshell is larger then the average distance between the subshells in the shell considered G_{eff} gets smaller at the subshell closure. In some cases, *e.g.* at $g_{7/2}$ subshell closure in ^{114}Sn, one gets $\Delta_{\text{eq}} = 0$ as in the case of a closed shell nucleus. The approximations which are used here get worse for small Δ, but it seems that the only way to eliminate $\Delta_{\text{eq}} = 0$ point in the Sn isotopes would be to shift $s_{1/2}$ or $d_{3/2}$ level, or both down, close to the $g_{7/2}$ for $A < 118$ and let them go much higher in the $A \geqslant 118$ isotopes. Such changes in the relative position of the single particle levels would be in agreement with the analysis of Ref. [8]. However, $\Delta_{\text{eq}} = 0$ for one of the Sn isotopes is also possible. It would be clearly manifested only in such experiments as *e.g.* two nucleon transfer reactions, and ^{114}Sn behaves very much like a closed shell nucleus in these experiments.

In Fig. 1 the calculated energy gaps are compared with the results of [8], where the inverse gap equation method was used to obtain pairing parameters. The input data in this method are the experimental odd-even mass differences and experimental energies of the single quasiparticle levels. The energy gaps obtained in this way should, therefore, represent experimental pairing.

In view of the fact that the Nilsson single particle potential with fixed parameters is too simple a model for spherical nuclei, the agreement of our results with the experimental pairing may be considered as satisfactory in all the three, completely different mass regions. As it is seen from the Fig. 1, not only average pairing but also main tendencies are reproduced correctly. Further improvement of the results can be achieved by improving the single particle potential.

In the present form the method should be usefull in the case of deformed nuclei where the details of the subshell structure are averaged out by the deformation. One may hope to be able to parametrize all the low energy structure of these nuclei exclusively by the parameters of the spherical part of the single particle potential and obtain such effects as *e.g.* deformation dependence of the effective pairing force strength.

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