ON THE TWO-STEP PROCESS IN DIRECT (*d*, *t*) REACTION IN RARE-EARTH NUCLEI

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In the paper the two-step model of the (*d*, *t*) direct reaction in rare-earth nuclei is investigated. The inclusion of the strong coupling in the entrance and exit channels improves the agreement of theoretical angular distributions with experimental data, although a discrepancy in absolute strengths remains.

The currently used method of the analysis of the direct reaction is the single-step distorted wave Born approximation (DWBA). Such reaction have been studied in many regions of the periodic table and have yielded valuable nuclear structure information. However in nuclei in which there are strongly coupled excited states, such as in the region of deformed nuclei, two-step process *via* excited states can give important contributions to the direct reaction amplitude. In one particle transfer reaction when strong coupling is important, it is possible to approximate the effects of the coupling for allowed transitions while retaining the usual DWBA form [1]. For the case of deformed nuclei, when only the quadrupole deformation is taken into account, the only change is to change the potential radii R_0 to $R_0(1+K_{lj\Omega}\beta_2)$ where β_2 is the deformation parameter and

$$K_{lj\Omega} = \sqrt{2}\, C_{lj\Omega}^{-1} \sum_{l'j'} C_{l'j'\Omega}[5(2j'+1)/4\pi(2j+1)]^{1/2} \times$$
$$\times \langle 2j'o\Omega|j\Omega\rangle \langle 2j'o-\tfrac{1}{2}|j-\tfrac{1}{2}\rangle. \quad (1)$$

In the formula (1) the coefficients C_{lj} are defined by the relation

$$|N\Omega\rangle = \sum_{lj} C_{lj}(\Omega)|Nlj\Omega\rangle. \quad (2)$$

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The state $|N\Omega\rangle$ is the Nilsson one particle state in deformed potential and $|Nlj\Omega\rangle$ is the spherically-symmetric solution of the equation

$$-(\hbar\omega_0/2)(\nabla^2-\varrho^2)|Nlj\Omega\rangle =$$

$$= \hbar\omega_0(N+3/2)|Nlj\Omega\rangle, \qquad N = 0, 1, 2, \ldots \tag{3}$$

$$\varrho^2 = \sum_{k=1}^{3} \frac{m\omega_0}{\hbar} x_k^2.$$

The C_{lj} coefficients are related to $a_{l\Lambda}$ coefficients (defined by Nilsson [2]) by the relation

$$C_{lj}(\Omega) = \sum_{\Lambda} \langle l\Lambda \tfrac{1}{2} \Sigma \,|j\Omega\rangle a_{l\Lambda}. \tag{4}$$

In the paper by Kunz *et al.* [1] this method of the calculation of the two-step process in the framework of the usual DWBA-code is applied to the (p, d) reactions. It is interesting to check the above mentioned model for the (d, t) reactions on rare-earth deformed nuclei.

The two reactions (p, d) and (d, t) are one neutron transfer reactions quite well described by the usual DWBA model [3], [4], [5]. However for some transitions observed in (d, t) reactions on rare-earth nuclei a discrepancy between computed and measured angular distributions is observed [6], [7], [8]. In usual DWBA this discrepancy can be removed by means of arbitrary change of the optical model parameters in the entrance and exit channels. It can be expected that the part of this change can be explained by the inclusion of the strong coupling in entrance and exit channels. In our paper we assume that the angular distributions for the (d, t) reactions can be described as follows [9], [10].

$$\frac{d\sigma^{(-)}}{d\omega} = 2N^{(-)}C_{lj}^2\sigma_l^{(-)}(\theta)V^2. \tag{5}$$

TABLE I

Optical-model parameters for deuterons and tritons

	V (MeV)	W (MeV)	r_0 (fm)	a (fm)	r_0' (fm)	a' (fm)
Deuteron optical parameters	86	12	1.15	0.87	1.37	0.7
Triton optical parameters Ref. [6]	154	12	1.10	0.75	1.40	0.65
Ref. [14]	155	10	1.20	0.70	1.30	0.65
Ref. [15]	168	17	1.14	0.72	1.52	0.77
Ref. [16]	153	21	1.24	0.69	1.42	0.89
Ref. [17]	167	10	1.16	0.75	1.50	0.82

It means that we neglect the coupling phenomena (like Coriolis coupling). As it is well known the theoretical differential cross-sections are strongly affected by the optical model parameters. A critical examination of the described model of (d, t) reaction can be carried out only if the uncertainties in the parameters are removed as far as possible. Therefore as a zero-order starting point in our calculation we used the optical model parameters which best fit the measured angular distributions [6], [7], [8]. The review of optical model parameters used for the description of the reactions with tritons is given in the Table I. In our calculations of the cross-sections we used the modified Code G.A.P.2 [11] (without spin-orbit term) similar to that of the well known code JULIE. The coefficients C_{lj} for the appropriate Nilsson state were taken from the table given by Chi [12]. The deuteron optical parameters used are listed in Table I [6].

They have been successfully applied in analysis of (d, p) reactions on Yb, Er and Gd isotopes [3], [4], [5] and are essentially in agreement with standard deuteron parameters given by Perey and Perey [13].

In order to test the model proposed by Kunz *et al.* [1] we select three states 1/2 1/2⁻ [521], 7/2 3/2⁻ [521] and 3/2 3/2⁻ [521]. The calculated $K_{lj\Omega}$-values and ΔR_0 are listed in Table II.

TABLE II

The $K_{lj\Omega}$ coefficients for the investigated states

Nilsson assignment	$K_{l\beta_s\Omega}$	β	ΔR_0
1/2 1/2⁻[521]	0.4	0.3	0.12
3/2 3/2⁻[521]	0.45	0.3	0.13
7/2 3/2⁻[521]	0.08	0.3	0.02

The corresponding modified DWBA calculation of the angular distributions are presented in Figs 1–3. The agreement with the experimental results seems to be much better for the modified DWBA calculations (dashed lines) than for the normal DWBA calculations (solid lines).

The absolute values of the cross-sections for the (d, t) reactions which populate the states 1/2 1/2⁻ [521], 7/2 3/2⁻[521], 3/2 3/2⁻[521] can be also obtained from the formula (5). In order to compare the results given by the model proposed by Kunz *et al.* with the ordinary DWBA calculations the absolute values of the cross-sections were reduced to the same value of the $Q = -2$ MeV. The value of the normalization constant $N^{(-)}$ was chosen to be equal to 3.0 [18]. The parameters V^2 were the same as those used by Kanestrøm and Tjøm in DWBA calculation for (d, p) and (d, t) reactions [18], [19], [20]. In the application of the model proposed by Kunz *et al.* other effects, except the nonelastic scattering, which give the deviation from the one step direct reaction populating pure rotanional states must be excluded. The Coriolis coupling is the most important effect which disturbe the pure rotational structure of the investigated states. The influence of the Coriolis coupling on (d, p) and (d, t) cross-section has been studied for several nuclei in the mass region 155–171 in Ref. [20]. In our paper we assume that the states for which

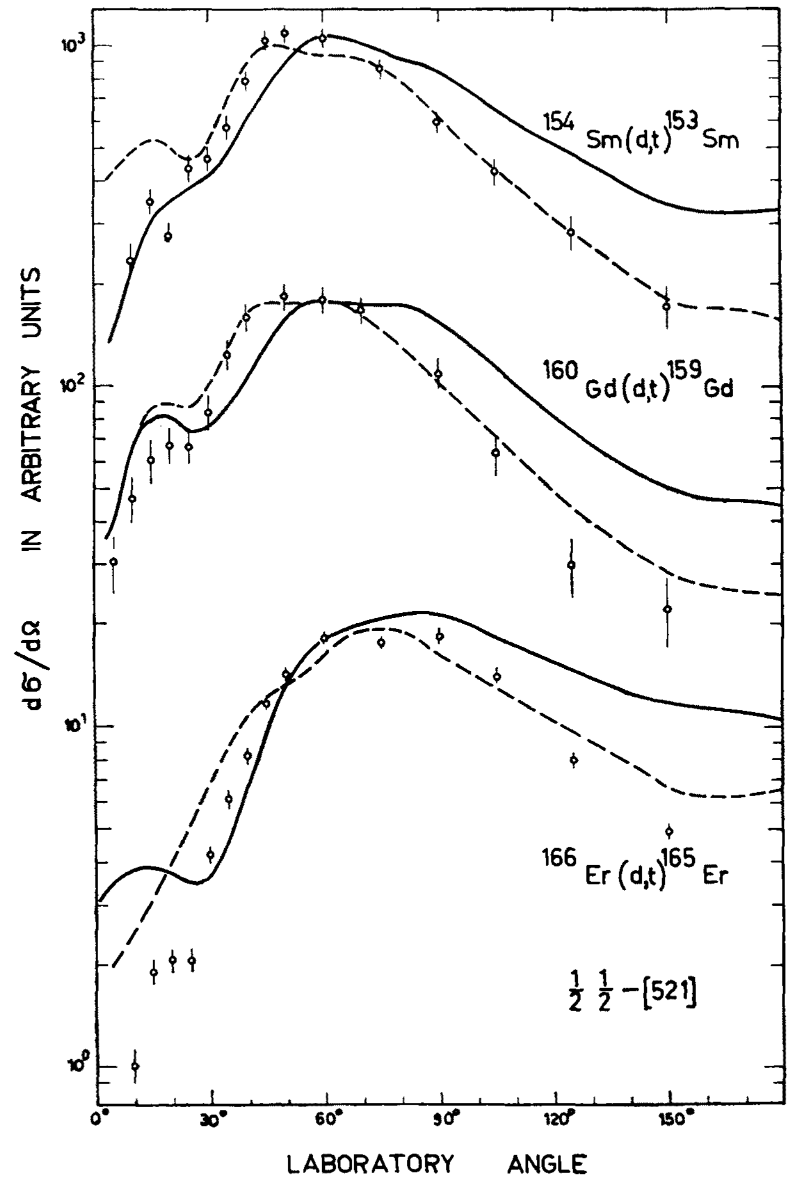


Fig. 1. Angular distributions for triton groups with $l = 1$ (Nilsson state 1/2 1/2⁻ [521]). The solid line shows the results of a DWBA calculation with the parameters listed in Table I [6]. The dashed line shows the modified DWBA calculation ($E_d = 12.1$ MeV)

the cross-sections computed with and without the inclusion of Coriolis coupling are nearly the same can be treated as free of the Coriolis coupling. In Table III we give the comparison of the theoretical, theoretical mixed, theoretical modified and experimental values of (d, t) cross-sections for appriopriate states. The theoretical values were computed

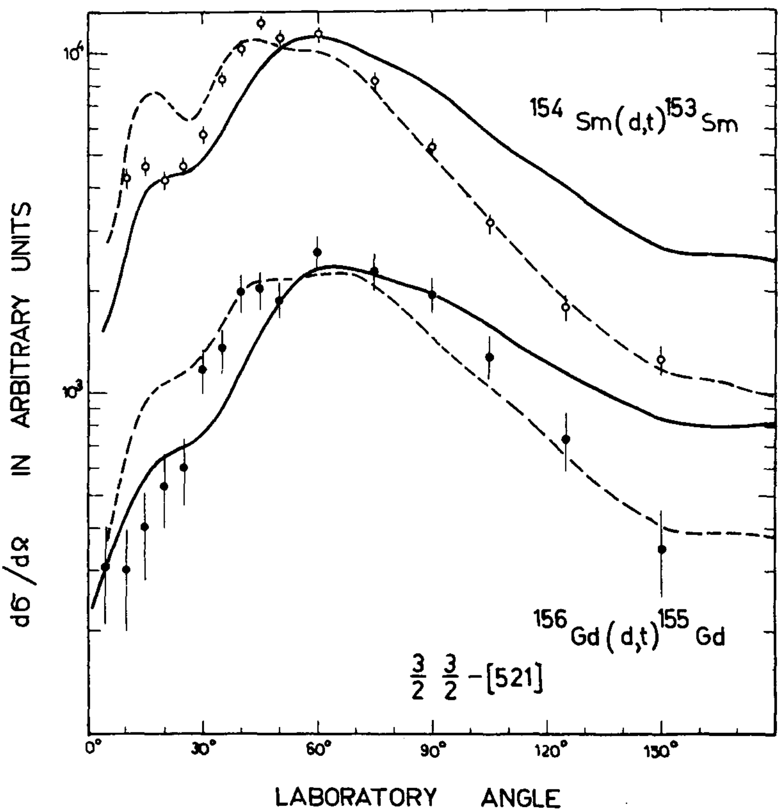
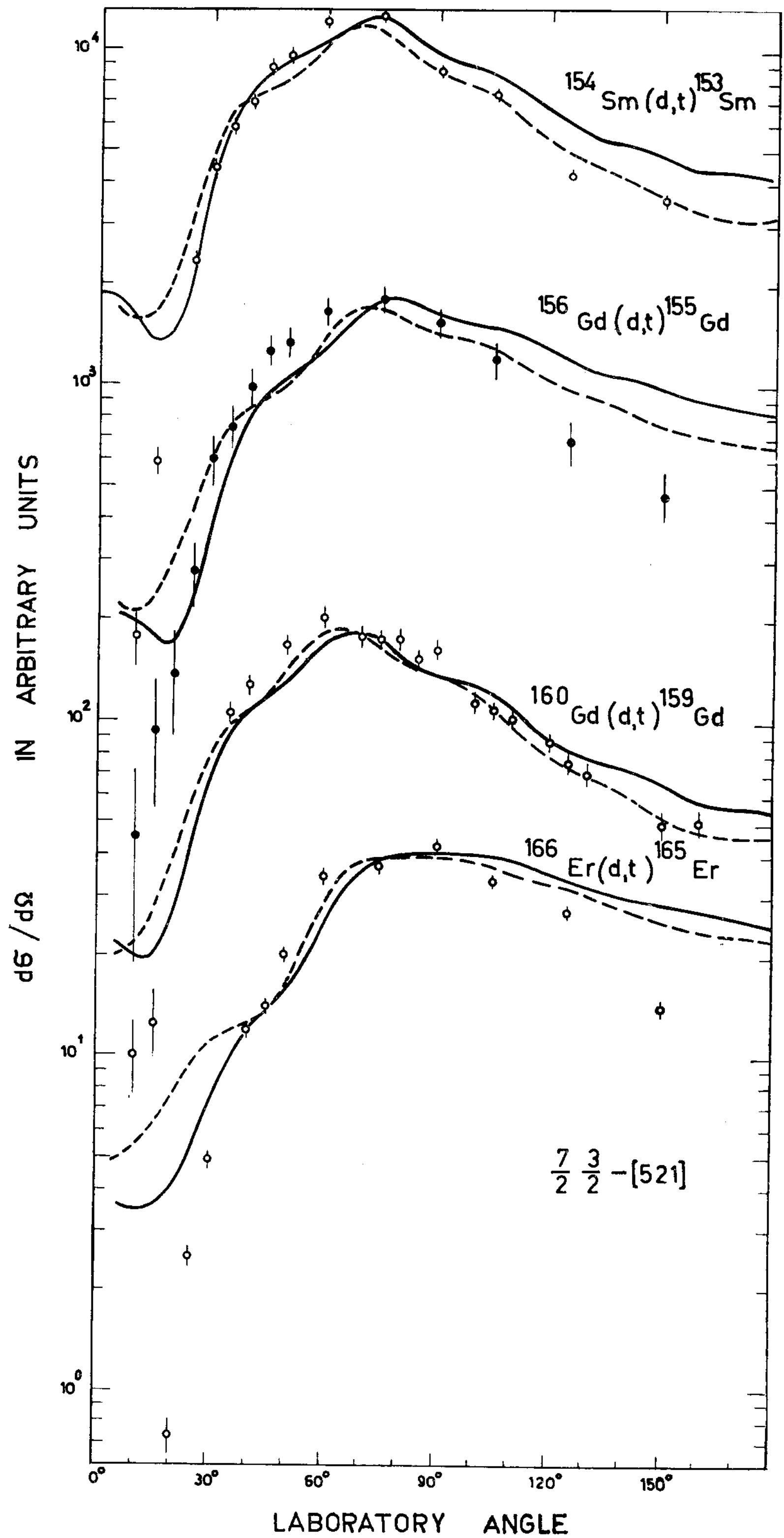


Fig. 2. Angular distribution for triton groups with $l = 1$ (Nilsson state 3/2 3/2⁻ [521]). The solid line shows the result of a DWBA calculation with the parameters listed in Table I [6]. The dashed line shows the modified DWBA calculation ($E_d = 12.1$ MeV)

TABLE III

Comparison of experimental with theoretical cross-sections in investigated nuclei

State	Isotope	Energy (keV)	$\frac{d\sigma}{d\omega}$ (d, t), 90°(μb/str), $Q = -2$ MeV			
			exp.	theory	mix.	mod.
1/2 1/2⁻[521]	^{153}Sm	698	22	25	27	17
	^{159}Gd	506	69	37	37	24
	^{165}Er	297	137	93	92	74
3/2 3/2⁻[521]	^{153}Sm	126	19	21	42	12
	^{155}Gd	0	71	41	68	25
7/2 3/2⁻[521]	^{153}Sm	262	20	40	48	38
	^{155}Gd	145	118	77	123	73
	^{159}Gd	120	165	189	153	175
	^{165}Er	372	384	234	366	214



dσ / dΩ IN ARBITRARY UNITS
LABORATORY ANGLE
154 Sm (d,t) 153 Sm
156 Gd (d,t) 155 Gd
160 Gd (d,t) 159 Gd
166 Er (d,t) 165 Er
7/2 3/2 −[521]
10⁴
10³
10²
10¹
10⁰
0°
30°
60°
90°
120°
150°

by means of the formula (5). The theoretical mixed cross-sections were computed in the paper [20] by means of the formula

$$\frac{d\sigma^{(-)}}{d\omega} = 2N^{(-)}\left(\sum_i C^i_{lj} a_i V_i (\sigma^i_l)^{1/2}\right)^2 \tag{6}$$

where the summation over index i is performed over bands which are coupled. The a_i are the mixing amplitudes due to Coriolis coupling. The angular functions σ^i_l are obtained from the DWBA calculations with the deuteron and triton optical model parameters given in [6]. The theoretical modified cross-sections were calculated from the formula (5) with the optical model parameters modified due to nonelastic scattering in exit and entrance channels. From Table III we see that states 1/2 1/2⁻ [521] in ^{153}Sm, ^{159}Gd and ^{165}Er nuclei can be treated as free of Coriolis coupling. The same can be stated for the state 7/2 3/2⁻ [521] in ^{153}Sm and ^{159}Gd. By the comparison of the values of the computed cross-sections we obtain that the inclusion of nonelastic channels leads to smaller values of the cross-sections than the cross-sections computed by means of the one-step formula (5) or (6). In the case of the states 7/2 3/2⁻ [521] in ^{153}Sm and ^{159}Gd the computed modified cross-sections are in good agreement with experimental data. Moreover the agreement of modified cross-sections is better than the theoretical and theoretical mixed cross-sections. From the inspection of the Table III we obtain that the states 3/2 3/2⁻ [521] in ^{153}Sm and ^{155}Gd states 7/2 3/2⁻ [521] in ^{155}Gd, ^{165}Er are strongly mixed due to Coriolis coupling. It means that these states are not well suited for the analysis of the influence of the nonelastic scattering as our formula (5) does not contain mixing of the bands.

In conclusion we can say that as far as the angular distributions are concerned the inclusion of the nonelastic scattering improves the agreement of the calculated values with experimental data. But the discrepancy in absolute strengths remains.

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Fig. 3. Angular distributions for triton groups with $l = 3$ (Nilsson state 7/2 3/2⁻ [521]). The solid line shows the result of a DWBA calculation with the parameters listed in Table I [6]. The dashed line shows the modified DWBA calculation ($E_d = 12.1$ MeV)

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