A SEPARABLE MODEL FOR HYPERON NUCLEON INTERACTIONS

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A separable two-channel potential model is used to describe hyperon-nucleon reactions. The parameters of the potential are set to reproduce the low energy sigma (lambda) nucleon scattering, hypernuclear data and the virtual bound state of the sigma and nucleon.

1. *Introduction*

There are two main sources of information on the hyperon nucleon forces, elementary scattering and hypernuclear physics. Considerable progress in understanding low energy interactions has recently been achieved by measurements of sigma and lambda reactions on protons [1, 7, 8]. The few body hyperonic interactions, however, involve sligthly more physics due to off-shell scattering. This requires a definite model, and the aim of this paper is to obtain a separable, two channel, potential model which would reproduce all the experimental information. Having in mind a further application in the few body physics we choose a form of the potential to be the simplest possible, with a minimum number or parameters.

The paper consists of three sections. In Section 2 we recollect the properties of the separable potential model and its relation to the reaction matrix approach. In Section 3 the description of the hyperon (Σ, Λ) and nucleon (N) reaction is provided. The region of interest begins at the ΛN threshold and reaches the low energy ΣN scattering. We are interested in the S wave only, as the information on the higher partial waves is practically null. A number of constraints is put on the potential. The most important are: reproductions of the ΛN and ΣN scattering lengths and effective ranges, reproduction of the ΣN virtual bound state in the triplet ΛN scattering and the effect of suppresion of the lambda nucleon forces in the odd A light hypernuclei, [2, 6]. A set of parameters of the potential is given and properties of the solution are discussed.

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2. *Basic properties of the separable model*

We review in this section the essential features of the many channel separable potential model and its relation to the reaction matrix model.

Let us consider the Lippman Schwinger equation for a given partial wave

$$T(E) = V+VG(E)\,T(E)$$

with all the indices suppressed. In the simplest case of spin independent forces we assume the potential to be a matrix in the channel indices of the form.

$$V_{ij}(k, k') = V_i(k)\ \lambda_{ij} V_j(k') \tag{1}$$

where i, j stand for the channel indices and k is the relative momentum. The conditions of hermicity and time reversal invariance make the form-factors $V(k)$ to be real functions and the matric λ to be real and symmetric. In order to simulate more physical meson exchange potentials we require all possible singularities of the form-factors in the complex momentum plane to be concentrated on the imaginary positive semi-axis. We assume the potential to be short ranged in space and exclude an infinite number of oscillations. This means that $V(k)$ is expected to be continuous on the real positive semi-axis.

The solution for the scattering matrix T is of the form

$$T_{ij}(kk'E) = V_i(k)t_{ij}(E)V_j(k') \tag{2}$$

where

$$t^{-1}(E)_{ij} = \lambda_{ij}^{-1} - R_i(E)\delta_{ij} \tag{3}$$

$$R_i(E) = \int \frac{V_i^2(k)N_i^{-2}}{E-Q_i-\dfrac{k^2}{2\mu_i}+i\varepsilon}\, d_3k. \tag{4}$$

In the last formula Q_i is the threshold energy for a given channel i, and N_i is the normalization factor for the plane waves in the CM system. This is chosen to be

$$\langle ik|k'j\rangle = N_i^2\delta_{ij}\delta(k-k') = \delta_{ij}\delta(k-k')\,(2\pi)^2\mu_i \tag{5}$$

where μ_i is the reduced mass of a channel. The form-factors are assumed to fall off rapidly enough to provide convergence of the integral (4). For the specified analytical form of the form-factors the function R has the form

$$R_i(E) = P_i(E) - iq_i V_i^2(q) \tag{6}$$

where $P(E)$ is a real (on the real axis) function and

$$q_i = \sqrt{2\mu_i(E-Q_i)} \tag{7}$$

is the on shell value of the relative momentum. From the definition it is easy to see that $R(E)$ is a monotonic function of energy for $E < Q$.

If we choose the standing wave boundary condition that is the contour of integration which splits into two paths passing around singularity, in the formula (4), on both sides, the corresponding reaction matrix is

$$K_{ij}(kk'E) = V_i(k)k_{ij}(E)V_j(k')$$
$$k(E)_{ij}^{-1} = \lambda_{ij}^{-1} - P_i(E)\delta_{ij}. \tag{8}$$

The obvious relation

$$t_{ij}^{-1} = K_{ij}^{-1} + iq_i V_i^2 \delta_{ij} \tag{9}$$

is the Heitler equation for our case. Eq. (1) is the simplest possible form of the potential but it proves to be sufficient to describe the scanty data available.

It is well known that a one channel separable potential with one term, *i.e.* of the form (1), may produce at the most one bound state but a number of resonances may occur. This follows from the formula (4) which shows that on the real axis $R(E)$ is a monotonic function of energy below the threshold and may oscillate above the threshold, if $k^2V^2(k)$ is an oscillating function. The following statement is the counterpart in the two channel case: there may be a number of resonances in the open channel below the threshold of the closed channel but at the most one of them corresponds to a virtual bound state in the closed channel. To see this, let us specify the situation in the case of ΛN and ΣN scattering. If we denote

$$\begin{array}{cc} & \begin{array}{cc}\Lambda N & \Sigma N\end{array} \\ \hat{k}^{-1}(E) = & \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix} \begin{array}{c}\Lambda N \\ \Sigma N\end{array} \end{array} \tag{10}$$

then we obtain from Eq. (9)

$$\hat{t}(E) = \begin{pmatrix} \beta + iq_\Sigma V_\Sigma^2 & -\gamma \\ -\gamma & \alpha + iq_\Lambda V_\Lambda^2 \end{pmatrix} \frac{1}{(\beta + iq_\Sigma V_\Sigma^2)(\alpha + iq_\Lambda V_\Lambda^2) - \gamma^2}. \tag{11}$$

The condition for a resonance to occur in the open ΛN channel below the threshold for the closed ΣN channel is

$$\alpha(\beta - |q_\Sigma| V_\Sigma^2) - \gamma^2 = 0. \tag{12}$$

In the simplest case of uncoupled channels ($\gamma^2 = 0$) this condition describes two cases:

a) $\alpha = 0$, *i.e.* a resonance in the open channel. As was shown above there may be a number of them.

b) $\tilde{\beta} \equiv \beta - |q_\Sigma| V_\Sigma^2 = 0$, that is a bound state in the closed channel ΣN. There may be only one bound state under the assumption (1).

Relaxing the condition for γ we obtain a family of resonance trajectories depending on the strength of coupling. We can distinguish the trajectories, at least for some distance from the point $\gamma^2 = 0$, what allows for a physical interpretation of a possible resonance.

From the formula (11), with the condition (12), it may be seen that the width of the resonance is

$$\Gamma/2 = \frac{q_\Lambda V_\Lambda^2}{\alpha' + \frac{\alpha}{\beta}\tilde{\beta}'} \tag{13}$$

where the primes denote derivatives with respect to energy. This formula shows that the width of the b type resonance tends to zero ($\tilde{\beta} \to 0$) when the coupling tends to zero ($\gamma \to 0$) *i.e.* the resonance becomes a stable bound state.

There is a problem to which category there may belong a resonance observed experimentally below the upper threshold. This may be answered if we have a definite model (that is V_i in our case) or all the functions α, β, $\gamma(E)$ between the thresholds. There is another criterion, however, if the resonance is accompanied by a zero in the cross-section. Inspection od Eq. (11) shows that the only condition for zero in the scattering matrix $T_{\Lambda\Lambda}$ is $\tilde{\beta} = 0$ *i.e.* the existence of a virtual bound state. The other condition $\alpha = \infty$ is excluded by our assumption of continuity of $V(k)$ on the real axis. The actual position of zero is shifted from the position of resonance up or down according to the sign of α as it is clear from Eq. (12). This interpretation of the zero in the cross-section is, in fact, model independent and follows directly from the Heitler equation but the actual position of zero depends on the form of potential.

3. The hyperon nucleon scattering

The scattering is a two channel, ΛN and ΣN, problem. Region of interest begins at the ΛN threshold and reaches the low energy ΣN scattering. The actual experimental situation permits rather crude estimation of parameters. Thus we neglect effects due to the isospin symmetry breaking, *i.e.* Coulomb interaction and mass differences within multiplets. This simplification is the basis of phenomenological analysis of Ref. [5].

The following data are taken into account:

1) The lambda nucleon scattering lengths and effective ranges for the S wave spin singlet and triplet scattering. The various results are taken from Refs [4, 7, 8, 9, 10, 11].

2) Results of the low energy $\Sigma^\pm p$ scattering (110–170 MeV/c lab momentum) into elastic and inelastic $\Sigma^0 n$, Λn channels as summarized and described by the complex scattering lengths in Ref. [5]. This determines only a part of the reaction matrix leaving some parameters related to the ΛN scattering undetermined.

3) In order to explain the binding energies of the odd A hypernuclei (in particular ${}^5_\Lambda$He) it is necessary to introduce some suppresion of the ΣN channel in comparison with the free case, [2, 6]. The required property, then, is that if the $V_{\Sigma\Lambda}$ triplet potential is reduced in strength the triplet ΛN scattering length becomes smaller and the effective range becomes bigger, [2, 6]. Another suggestion from hypernuclear physics (binding energy of ${}^4_\Lambda$He) is a relation of the scattering lengths $|a_s| > |a_t|$, [2].

4) An enhancement in the $p\Lambda$ interactions in the triplet state is observed, just below the $\Sigma^+ n$ threshold, in the reaction $K^- D \to \Lambda p \pi^-$, [3]. The width of the corresponding reso-

nance peak is about 10 MeV. Although, there is also an alternative interpretation of this process in terms of a two step reaction [1], the resonance appears in the 0 BE models of hyperon nucleon scattering, [2, 9].

5) In the forementioned boson exchange models of our reaction the σ^0 Ansatz with mass $\approx$ 400 MeV plays a prominent role. This is used as an indication of the range of the separable potentials.

We use the Yamaguhi form-factors $V = \kappa^2/(\kappa^2+k^2)$. The standard form of the scattering matrix $T_{\Lambda\Lambda}$ $(kk'E)$ on the energy shell is

$$-T_{\Lambda\Lambda}(E) = \left(\frac{1}{A_\Lambda} - iq_\Lambda\right)^{-1} = (q_\Lambda \cot \delta_\Lambda - iq_\Lambda)^{-1}$$

$$-\frac{1}{A} = \left(\alpha - \frac{\gamma^2}{\beta + iq_\Sigma V_\Sigma^2(q_\Sigma)}\right)\frac{1}{V_\Lambda^2(q_\Lambda)} \approx \frac{1}{a_\Lambda} - \frac{r_\Lambda q_\Lambda^2}{2} \tag{14}$$

where the last relation is valid around the threshold. Similar is the expression for the sigma nucleon scattering.

The case of isospin $T = 3/2$ is the simplest one and the results are given in Table I. There is not enough data to determine the two parameters λ and κ for the sigma scattering. We stick to the results of Ref. [5]. The solution denoted there (a) and (b), $\mu = 0$ is used

TABLE I

Parameters of the separable potential, $T = 3/2$

J	a_Σ[fm]	r_Σ[fm]	κ_Σ[fm^{-1}]	λ_Σ[fm]	Ref.
1	0	0	—	0	ours
1	0	0			[5]
1	0.33	−0.79			[12]
0	−3.2	2.0	1.84	−0.81	ours
0	−2.7±0.7	0			[5]
0	−6±1	2.1±0.3			[9]
0	−5	3.71			[13]

as it gives the signs of a_Λ, in agreement with the theoretical calculations [9, 13] and also fits better the $T = 1/2$ case. The effective range is set arbitrarily to be close to the calculations and to keep $a_\Sigma(E)$ within the quoted error in the whole region of investigated scattering we (110–170 MeV/c lab momentum).

In the isospin $T = 1/2$ case our problem is a two channel one. However, experiment shows that coupling of the channels in the spin singlet state is negligible, [5]. The results are given in Table II. In the sigma case only one experimental parameter is available and we used the procedure applied to the $T = 3/2$ case.

The spin triplet $T = 1/2$ case is the most interesting one. In Fig. 1 we give the behaviour of the ΛN scattering lengths. The two different curves were fitted to the results (a) and (b),

$\mu = 0$, of Ref. [5]. For the sake of comparison the singlet $1/A_\Lambda$ curve has also been drawn The rapid changes of the triplet scattering parameters around the ΣN threshold are due. to the other channel. Both results produce resonances at energies $E_R = 0.99$ MeV and

TABLE II

Potential parameters, $T = 1/2$, spin singlet

Y	a [fm]	r [fm]	κ [fm^{-1}]	λ [fm]	Ref.
Λ	-2.25	3.29	1.32	-0.90	[10]/ours
	-2.25	3.71			[2]
	-2.0	5.0			[8]
	-1.8	2.8			[7]
	-1.61	3.22			[11]
Σ	-0.9 ± 0.4	0			[5]
	-1.0	3.0	1.68	-0.54	[5]/ours

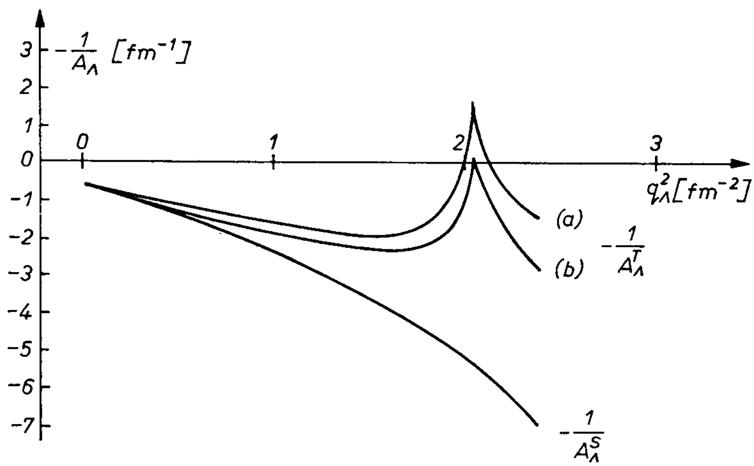

Fig. 1. Plot of $(-1/A_\Lambda)$ for the spin singlet and triplet cases. The curves (*a*) and (*b*) correspond to different parameters of Ref. [5]. Above the ΣN threshold Re $(-1/A_\Lambda)$ is given

0.014 MeV below the ΣN channel threshold. For the last the average value 2131.9 MeV was used. The parameters are

$$\hat{\lambda}_a = \begin{pmatrix} -0.552, & \pm 0.884 \\ , & -1.53 \end{pmatrix}, \quad \hat{\lambda}_b = \begin{pmatrix} -0.655, & \pm 0.720 \\ , & -1.26 \end{pmatrix} \text{fm} \tag{15}$$

and $\kappa_\Sigma = \kappa_\Lambda = 1.60$ fm^{-1} in the both cases. The scattering parameters generated by these potentials are given in the Table III. The set (a) gives a slightly greater attraction and this is the reason why the resonance is more distinctly separated from the threshold. An interesting pattern is displayed by the cross-section in this case, as it is shown Fig. 2. The double peak is due to the proximity of resonance and the ΣN threshold. The positive answer to

TABLE III

Scattering parameters, $T = 1/2$, spin triplet, $A_\Sigma = a_\Sigma + i b_\Sigma$

a_Λ [fm]	r_Λ	a_Σ	b_Σ	Ref.
-2.21	2.30	-2.52	5.5	ours (a)
		-2.3 ± 0.7	4.0 ± 1.3	[5] (a)
-2.21	2.56	$+3.0$	4.5	ours (b)
		$+2.3 \pm 0.7$	4.0 ± 1.3	[5] (b)
-2.2	3.5			[8]
-2.12	3.31			[2]
-1.6	3.3			[7]
-1.61	3.62			[11]
-1.5	2.0			[4]

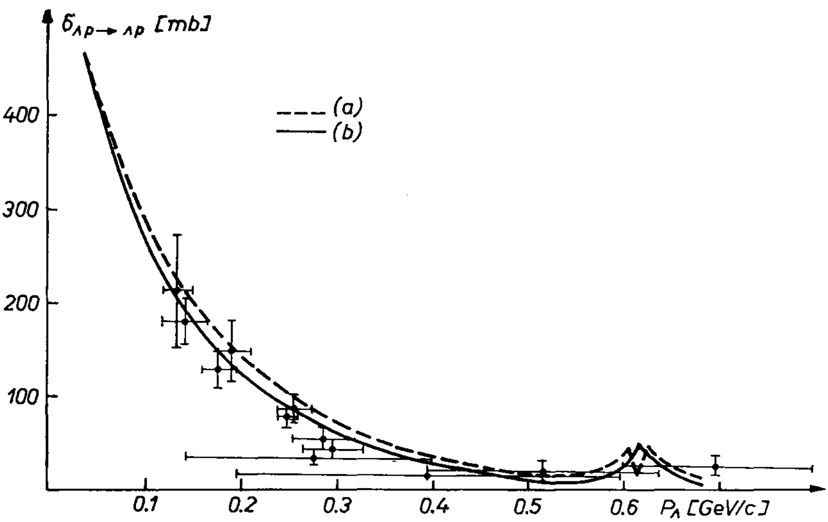

Fig. 2. Singlet and triplet lambda nucleon cross-sections. See captions to Fig. 1

the question whether the resonance really occurs (that is $1/A_\Lambda(E_r) = 0$) or not may be given by detailed knowledge of the cross-section around the threshold, in particular, by the fact, whether there are two or one peak. By small changes of parameters in the case (b) we could get rid of the resonance but in all the cases an enhancement in the cross-section would appear. As was discussed in Section 2 the way to see the nature of the resonance is the limiting procedure $\lambda_{\Sigma\Lambda} \to 0$. If we do this in the solution (15) the resonance position moves closer to the ΣN threshold. In the limit we obtain the pole in the expression $T_{\Sigma\Sigma}$ corresponding to a stable bound state with the energy $E_B = 0.73$ MeV in the case (a) and $E_B = 0.0017$ MeV in the case (b). Thus the resonance is of the type b according to the discussion of the previous section. The reverse reaction matrix element $\alpha(E)$ is a regular negative function of energy in all the region. This causes the zero in the cross-section (see Eq. (12)) to be pushed upwards with respect to the resonance. However, because of the proximity of the threshold, this zero does not occur. In the limit of the uncoupled channels the ΛN triplet scattering length is reduced, as it is required by the assumption 3. This particular condition make a fit to the parameters (c) and (d) of Ref. [5] rather difficult.

In conclusion let us mention that although the characteristic parameters of the virtual bound state are model dependent the fact of its existence (or nonexistence) follows from the general formalism of the reaction matrix. It would be very interesting to have more accurate data to settle the problem. There is also a hope that a detailed analysis of three body hyperonic reactions will be also helpful.

REFERENCES

[1] G. Alexander, *Proceedings of International Conference on Hypernuclear Physics*, Argonne 1969, p. 5.

[2] B. W. Downs, *Proceedings of International Conference on Hypernuclear Physics*, Argonne 1969, p. 51.

[3] D. Cline, R. Laumann, J. Mapp, *Proceedings of International Conference on Hypernuclear Physics*, Argonne 1969, p. 92.

[4] G. Fast, J. J. de Swart, *Proceedings of International Conference on Hypernuclear Physics*, Argonne 1969, p. 814.

[5] Y. Gell, G. Alexander, I. Stumer, *Nuclear Phys.*, **B22**, 583 (1970).

[6] A. R. Bodmer, *Phys. Rev.*, **141**, 1387 (1966); A. R. Bodmer, D. M. Rote, *Nuclear Phys.*, **A169**, 1 (1971).

[7] G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, W. Lughofer, *Phys. Rev.*, **173**, 1452 (1968).

[8] B. Sechi-Zorn, B. Kehoe, J. Twitty, R. A. Burnstein, *Phys. Rev.*, **175**, 1735 (1968).

[9] G. Fast, Z. C. Helder, J. J. de Swart, *Phys. Rev. Letters*, **22**, 1453 (1969).

[10] R. C. Herndon, Y. C. Tang, *Phys. Rev.*, **159**, 853 (1967).

[11] A. Deloff, *Nuclear Phys.*, **84**, 585 (1968).

[12] J. J. de Swart, C. Dullemond, *Amer. J. Phys.*, **19**, 458 (1962).

[13] H. G. Dosh, V. F. Müller, *Phys. Letters*, **19**, 320 (1965).