ANALYSIS OF DECAYS OF ${}^{7}_{\Lambda}$Li

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The distributions of the p-α and d-α relative energy of the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow p+d+\alpha+\pi^-$ decay products are calculated with the use of the $\Phi(d-\alpha-\Lambda)$ cluster wave function of ${}^{7}_{\Lambda}$Li. For ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow {}^{4}\mathrm{He}+{}^{3}\mathrm{He}+\pi^-$ decay the distributions of the ${}^{3}\mathrm{He}-{}^{4}\mathrm{He}$ relative energy and angular distribution of the ^{3}He are calculated with the use of the $\Phi(d-\alpha-\Lambda)$ and $\Phi({}^{3}\mathrm{He}-t-\Lambda)$ cluster wave functions for the description of the ${}^{7}_{\Lambda}$Li. It is shown that the mentioned angular distribution strongly depends on the structure of the ${}^{7}_{\Lambda}$Li. The energy distributions for both models of the ${}^{7}_{\Lambda}$Li decay agree with the experimental data.

1. Introduction

The aim of this paper is to calculate the energy and angular distributions of the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow p+d+\alpha+\pi^-$ and ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow {}^{4}\mathrm{He}+{}^{3}\mathrm{He}+\pi^-$ decay products. The experimental data on these modes of the decay of the ${}^{7}_{\Lambda}$Li hypernucleus has been given in [1]. The analysis of the energy and angular distributions of hypernuclear decay products can be used to deduce the structure of hypernuclei, the interactions in final states and in some cases to determine spins of hypernuclei. In our work we shall estimate the influence of a hypernuclear wave function on the discussed distributions in the decay of the ${}^{7}_{\Lambda}$Li hypernucleus. The results can serve to test the wave function and the model used for the description of the ${}^{7}_{\Lambda}$Li hypernucleus. To describe the ${}^{7}_{\Lambda}$Li hypernucleus we use two functions: one corresponding to the $d-\alpha-\Lambda$ cluster structure and another-to the ${}^{3}\mathrm{He}-t-\Lambda$ structure. We hope that the latter mode of clustering of nucleons in the ${}^{7}_{\Lambda}$Li hypernucleus can play an important role in the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow {}^{4}\mathrm{He}+{}^{3}\mathrm{He}+\pi^-$ decay. The angular distribution of the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow {}^{4}\mathrm{He}+{}^{3}\mathrm{He}+\pi^-$ decay products strongly depends on the model which is assumed for the ${}^{7}_{\Lambda}$Li hypernucleus. In the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow p+d+\alpha+\pi^-$ decay we calculate the $p-\alpha$ and $d-\alpha$ relative energy distributions. The general shape of the theoretical curves agrees with the experimental data.

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2. ${}^{7}_{\Lambda}Li \rightarrow p+d+\alpha+\pi^{-}$ *decay mode*

In [2] the ${}^{7}_{\Lambda}$Li hypernucleus has been studied with a three-body model consisting of an α-particle, a deuteron and a Λ-particle. The reasons for such a model are that a two--body $\alpha-d$ model gives a good description of the ground state of the ${}^{7}_{\Lambda}$Li nucleus and because of the weak binding energy of the Λ-particle in the ${}^{7}_{\Lambda}$Li hypernucleus. The wave function corresponding to the $d-\alpha-\Lambda$ model can be written in the form:

$$\psi_{Hf} = g_{\alpha d}(r_{\alpha d})g_{\alpha\Lambda}(r_{\alpha\Lambda})g_{d\Lambda}(r_{d\Lambda}), \tag{1}$$

where $r_{\alpha d}$, $r_{\alpha\Lambda}$ and $r_{d\Lambda}$ denote the $\alpha-d$, $\alpha-\Lambda$ and $d-\Lambda$ separations, respectively. In [2] the more general form of the ${}^{7}_{\Lambda}$Li wave function has been considered:

$$\psi_{Hf} = g(r_{\alpha\Lambda}, r_{\alpha d})g_{d\Lambda}(r_{d\Lambda}), \tag{2}$$

where

$$g(r_{\alpha\Lambda}, r_{\alpha d}) = [1-(p+me^{-cr_{\alpha\Lambda}})r_{\alpha d}]\,(e^{-a_1 r_{\alpha d}}+se^{-a_2 r_{\alpha d}}) \tag{3}$$

and

$$g_{d\Lambda}(r_{d\Lambda}) = e^{-c_1 r_{d\Lambda}}. \tag{4}$$

The parameters of these functions have been fitted to the binding energy of the ${}^{7}_{\Lambda}$Li hypernucleus [2]. In the form (2) the exclusion principle are included by the term containing the factor $[p+m\exp(-cr_{\alpha\Lambda})]\,r_{\alpha d}$.

We use the form (2) for the description of the ${}^{7}_{\Lambda}$Li hypernucleus in the calculations of energy distributions of the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow p+d+\alpha+\pi^{-}$ decay products. The martix element for this decay in the absence of final state interactions and neglecting the parity nonconserving term in the weak interaction Hamiltonian is:

$$\langle\psi_f|H_1|\psi_{Hf}\rangle \sim \int e^{i(\boldsymbol{p}_n\boldsymbol{r}_n+\boldsymbol{p}_d\boldsymbol{r}_d+\boldsymbol{p}_\alpha\boldsymbol{r}_\alpha+\boldsymbol{p}_\pi\boldsymbol{r}_\pi)}\varphi_{gr}(d)\varphi_{gr}(\alpha)\times$$
$$\times\,\delta(\boldsymbol{r}_\Lambda-\boldsymbol{r}_\pi)\delta(\boldsymbol{r}_n-\boldsymbol{r}_\pi)\psi_{Hf}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)d\boldsymbol{r}_n d\boldsymbol{r}_\alpha d\boldsymbol{r}_\pi d\boldsymbol{r}_\Lambda d\boldsymbol{r}_d, \tag{5}$$

where H_1 is the Hamiltonian of the weak interaction, ψ_f is a final state wave function which is a product of plane waves of emitted particles, ψ_{Hf} denotes the initial function of the ${}^{7}_{\Lambda}$Li hypernucleus. By $\boldsymbol{p}_n$, $\boldsymbol{p}_d$, $\boldsymbol{p}_\alpha$ and $\boldsymbol{p}_\pi$ we denote momenta of the proton, deuteron, α-particle and pion, respectively; $\boldsymbol{r}_n$, $\boldsymbol{r}_d$,-$\boldsymbol{r}_\alpha$, $\boldsymbol{r}_\pi$ and $\boldsymbol{r}_\Lambda$ are the respective proton, deuteron, α-particle, pion and Λ-particle coordinates.

The final state of the ${}^{7}_{\Lambda}\mathrm{Li} \rightarrow p+d+\alpha+\pi^{-}$ decay is a 4-body state and in the hypernucleus centre-of-mass system we may introduce the following systems of variables:

1. The p-α relative momentum, $\boldsymbol{p}_1$, the d-α relative momentum, $\boldsymbol{q}_1$, and the total momentum of the proton and the α-particle, $\boldsymbol{P}_1$, which is equal to the total momentum of the deuteron and the pion with the oposite sign.

$$\boldsymbol{p}_1 = \frac{1}{m_n+m_\alpha}(m_\alpha\boldsymbol{p}_n - m_n\boldsymbol{p}_\alpha), \tag{6a}$$

$$\boldsymbol{q}_1 = \frac{1}{m_d+m_\pi}(m_d\boldsymbol{p}_\pi - m_\pi\boldsymbol{p}_d), \tag{6b}$$

$$\boldsymbol{P}_1 = \boldsymbol{p}_n+\boldsymbol{p}_\alpha = -(\boldsymbol{p}_d+\boldsymbol{p}_\pi). \tag{6c}$$

The total energy expressed in these variables is:

$$\varepsilon = \frac{p_1^2}{2\mu_p^{(1)}} + \frac{q_1^2}{2\mu_q^{(1)}} + \frac{P_1^2}{2\mu^{(1)}} \tag{7}$$

where $\mu_p^{(1)}$ is the $p-\alpha$ reduced mass, $\mu_q^{(1)}$ is the $d-\pi$ reduced mass and $\mu^{(1)}$ is the $(p+\alpha)-$ $-(d+\pi)$ reduced mass.

2. The $d-\alpha$ relative momentum, $\boldsymbol{p}_2$, the $p-\pi$ relative momentum, q_2, and the total momentum of the deuteron and α-particle, P_2, which is equal to the total momentum of the proton and the pion with the oposite sign.

$$\boldsymbol{p}_2 = \frac{1}{m_d+m_\alpha}(m_\alpha \boldsymbol{p}_d - m_d \boldsymbol{p}_\alpha), \tag{8a}$$

$$\boldsymbol{q}_2 = \frac{1}{m_n+m_\pi}(m_n \boldsymbol{p}_\pi - m_\pi \boldsymbol{p}_n), \tag{8b}$$

$$\boldsymbol{P}_2 = \boldsymbol{p}_d + \boldsymbol{p}_\alpha = -(\boldsymbol{p}_n + \boldsymbol{p}_\pi). \tag{8c}$$

The total energy is:

$$\varepsilon = \frac{p_2^2}{2\mu_p^{(2)}} + \frac{q_2^2}{2\mu_q^{(2)}} + \frac{P_2^2}{2\mu^{(2)}} \tag{9}$$

here $\mu_p^{(2)}$ is the $d-\alpha$ reduced mass, $\mu_q^{(2)}$ is the $p-\pi$ reduced mass and $\mu^{(2)}$ is the $(d+\alpha)-$ $-(\mathrm{p}+\pi)$ reduced mass. By E_1 and E_2 we will denote the $p-\alpha$ and $d-\alpha$ relative energies, respectively. The differential decay rate, for the ${}^7_\Lambda\mathrm{Li} \to p+d+\alpha+\pi^-$ decay in which E_i $(i = 1, 2)$ is within the interval dE_i is given by:

$$\frac{d\sigma}{dE_i} \sim \sqrt{E_i} \int |\langle \psi_f | H_1 | \psi_{Hf} \rangle|^2 \delta\left(\Delta - \frac{p_i^2}{2\mu_p^{(i)}} - \frac{q_i^2}{2\mu_q^{(i)}} - \frac{P_i^2}{2\mu^{(i)}}\right) d\boldsymbol{q}_i d\boldsymbol{P}_i, \tag{10}$$

where Δ is the energy of the ${}^7_\Lambda\mathrm{Li} \to p+d+\alpha+\pi^-$ decay ($\Delta = 30.69$ MeV).

To calculate the matrix element (5) we use the variables $\boldsymbol{r}_i$, $\boldsymbol{\varrho}_i$ and R $(i = 1, 2)$ which are connected with the coordinates of the proton, the deuteron and the α-particle by the formula:

$$\boldsymbol{r}_1 = \boldsymbol{r}_n - \boldsymbol{r}_d, \tag{11a}$$

$$\boldsymbol{\varrho}_1 = \boldsymbol{r}_\alpha - \frac{m_n \boldsymbol{r}_n + m_d \boldsymbol{r}_d}{m_n + m_d} \tag{11b}$$

$$\boldsymbol{R} = \frac{m_n \boldsymbol{r}_n + m_d \boldsymbol{r}_d + m_\alpha \boldsymbol{r}_\alpha}{m_n + m_d + m_\alpha} \tag{11c}$$

and

$$\boldsymbol{r}_2 = \boldsymbol{r}_n - \boldsymbol{r}_t \tag{12a}$$

$$\boldsymbol{\varrho}_2 = \boldsymbol{r}_{HC} - \frac{m_n \boldsymbol{r}_n + m_t \boldsymbol{r}_t}{m_n + m_t}. \tag{12b}$$

In these varibles, the wave function (2) may be expanded into the series of Legendre polynomials:

$$\psi_{Hf}(\boldsymbol{r}_i, \boldsymbol{\varrho}_i) = \sum_l f_l(r_i, \varrho_i) P_l(\hat{r}_i \hat{\varrho}_i), \tag{13}$$

where

$$f_l(r_i, \varrho_i) = \frac{2l+1}{2} \int \psi_{Hf}(\boldsymbol{r}_i, \boldsymbol{\varrho}_i) P_l(\hat{r}_i \hat{\varrho}_i) d(\hat{r}_i \hat{\varrho}_i) \tag{14}$$

The plain wave in the formula (5) is expanded into the partial waves:

$$e^{ip_i r_i + iP_i \varrho_i} = (4\pi)^2 \sum_{lm} \sum_{l'm'} i^{l+l'} j_l(r_i p_i) j_{l'}(\varrho_i P_i) Y^*_{lm}(\hat{r}_i) Y_{lm}(\hat{p}_i) Y^*_{l'm'}(\hat{\varrho}_i) Y_{l'm'}(\hat{P}_i) \tag{15}$$

Substituting (13) and (15) into (5) and integrating over directions of the vectors $\boldsymbol{r}_i$ and $\boldsymbol{\varrho}_i$ we obtain:

$$\langle \psi_f | H_1 | \psi_{Hf} \rangle \sim \sum_l (-)^l \int \varrho_i^2 r_i^2 j_l(\varrho_i P_i) j_l(r_i p_i) f_l(r_i, \varrho_i) P_l(\cos \varphi_i) dr_i d\varrho_i, \tag{16}$$

where

$$\cos \varphi_i = \hat{p}_i \hat{P}_i. \tag{16a}$$

In numerical calculations partial waves with $1 > 0$ were found to be unimportant. The calculated spectra of the relative $p-\alpha$ and $d-\alpha$ energy are presented in Figures 1 and 2 together with the experimental curves. We see that the general shape of the theoretical

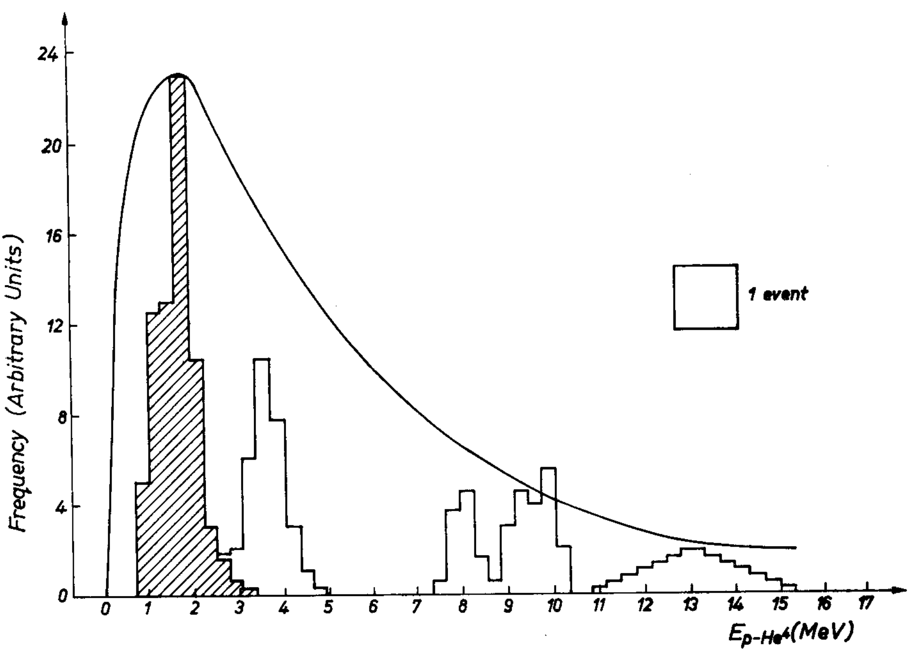

Fig. 1. Distribution of the p-α relative energy in the $^7_\Lambda\text{Li} \rightarrow p+d+\alpha+\pi^-$ decay

curves agrees with the experimental data. Obviously, final $p-\alpha$ and $d-\alpha$ interactions, neglected here, should be important for the $p-\alpha$ and $d-\alpha$ relative energy distributions. It seems that the experimental data are not yet sufficiently exact for the detailed analysis.

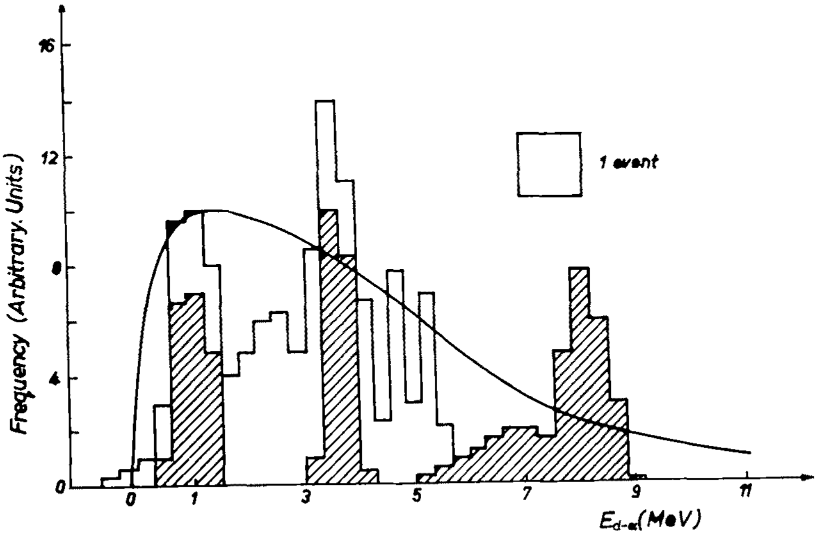

Fig. 2. Distribution of the $d-\alpha$ relative energy in the ${}^7_\Lambda$Li $\to p+d+\alpha+\pi^-$ decay

3. ${}^7_\Lambda Li \to {}^4\mathrm{He}+{}^3\mathrm{He}+\pi^-$ *decay mode*

In this case, for a description of the ${}^7_\Lambda$Li hypernucleus we use, beside the function (2), a function which corresponds to the ${}^3\mathrm{He}-t-\Lambda$ cluster structure of the ${}^7_\Lambda$Li hypernucleus [3]. As it has been discussed in [4] the ^{6}Li nucleus displays pronouncedly both the $d-\alpha$ and ${}^3\mathrm{He}-t$ cluster structure. Thus, for the ${}^7_\Lambda$Li hypernucleus we ought also to consider these two possibilities of the clustering of nucleons. It seems that the ${}^3\mathrm{He}-t$ structure can play an important role in the ${}^7_\Lambda$Li $\to {}^4\mathrm{He}+{}^3\mathrm{He}+\pi$ decay. We calculate angular and energy distributions for this decay using the function (2) and the function which has the following form [3]:

$$\psi_{Hf} = \Phi({}^3\mathrm{He}-t)\, \psi(\Lambda-\mathrm{core}), \tag{17}$$

where

$$\Phi({}^3\mathrm{He}-t) = N\varphi_{gr}({}^3\mathrm{He})\varphi_{gr}(t)\,(1-\tfrac{8}{9}\beta_0 R^2)e^{-2/3\beta_0 R^2} \tag{18}$$

and

$$\psi(\Lambda-\mathrm{core}) = e^{-az^2}\ (a = 0.06\ \mathrm{fm}^{-2}). \tag{19}$$

In the formula (18) N is a normalization factor, $\varphi_{gr}({}^3\mathrm{He})$ and $\varphi_{gr}(t)$ describe the internal motion of the ^{3}He and the triton. The parameter $\beta_0 = 0.433\ \mathrm{fm}^{-2}$ corresponds to the size of the α-particle [4]. The function $\Phi({}^3\mathrm{He}-t)$ describes the internal motion of the core (^{6}Li nucleus) of the ${}^7_\Lambda$Li hypernucleus. In [4] it has been shown that the ${}^3\mathrm{He}-t$ relative motion wave function of the ^{6}Li nucleus is close to the $2s$ shell model oscilator wave function, which is used in the formula (18). The Gaussian form of the function $\psi(\Lambda\text{-core})$ in (19) is taken from Ref. [5] where the parameter a has been fitted to the binding energy of the ${}^{7}_{\Lambda}\mathrm{Li}$ hypernucleus in the case of a simple ${}^{6}\mathrm{Li}-\Lambda$ model.

The matrix element for the ${}^{7}_{\Lambda}\mathrm{Li} \to {}^{4}\mathrm{He} + {}^{3}\mathrm{He} + \pi^{-}$ decay is calculated using the method described in the previous chapter. With the plane waves in the final state we obtain the following expression for the decay rate:

$$\frac{d\sigma}{dEd\vartheta} \sim [E(\Delta - E)]^{1/2} \left| \sum_{l} (-)^{l} \int \varrho^{2} r^{2} j_{l}(p_{3}\varrho) j_{l}(p_{4}r) f_{l}(r, \varrho) \xi(r) P_{l}(\hat{p}_{3}\hat{p}_{4}) dr d\varrho \right|^{2}. \tag{20}$$

Here E is the ${}^{3}\mathrm{He}-{}^{4}\mathrm{He}$ relative energy, ϑ is the angle between the ${}^{3}\mathrm{He}-{}^{4}\mathrm{He}$ relative momentum and the pion momentum in the centre-of-mass system of the ${}^{7}_{\Lambda}\mathrm{Li}$. The functions $f_{l}(r, \varrho)$ are the coefficients of the expansion of the ${}^{7}_{\Lambda}\mathrm{Li}$ wave function into the series of the Legendre polynomials.

In the case of the function $\Phi({}^{3}\mathrm{He}-t-\Lambda)$ (formula (17)) the function $\xi(r)$ describes the relative motion of the proton with respect to the triton in the α-particle. It is the coefficient of the lowest term (corresponding to the ground state of the triton) of the expansion of the α-internal wave function in terms of states of the triton. We use the function $\xi(r)$ given by Nadi and Riad [6] in the form of a curve which has been obtained from the experimental data on the (p, d) pick-up on ${}^{4}\mathrm{He}$. If we use the function $\Phi(d-\alpha-\Lambda)$ (formula (2)) for the description the initial state then $\xi(r)$ is the relative wave function of the proton with respect to the deuteron in the ${}^{3}\mathrm{He}$ nucleus and it is the coefficient of the lowest term of the expansion of the ${}^{3}\mathrm{He}$-internal wave function in terms of states of the deuteron. In this case we take $\xi(r) = \exp(-br)/r$ ($b = 0.6\ \mathrm{fm}^{-1}$) [6].

Other denotations in the formula (20) are

$$p_{3} = p - \frac{m_{t}}{m_{\alpha} + m_{\mathrm{He}}} p_{\pi}, \tag{21a}$$

$$p_{4} = \frac{m_{t}}{m_{\alpha}} p_{\pi}, \tag{21b}$$

$$p = \frac{1}{m_{\alpha} + m_{\mathrm{He}}} (m_{\alpha} p_{\mathrm{He}} - m_{\mathrm{He}} p_{\alpha}), \tag{21c}$$

if we use the function $\Phi({}^{3}\mathrm{He}-t-\Lambda)$ and

$$p_{3} = p + \frac{m_{\alpha}}{m_{\alpha} + m_{\mathrm{He}}} p_{\pi}, \tag{22a}$$

$$p_{4} = \frac{m_{d}}{m_{\mathrm{He}}} p_{\pi}, \tag{22b}$$

$$p = \frac{1}{m_{\alpha} + m_{\mathrm{He}}} (m_{\alpha} p_{\mathrm{He}} - m_{\mathrm{He}} p_{\alpha}), \tag{22c}$$

if we use the function $\Phi(d-\alpha-\Lambda)$.

In Fig. 3 the $^3He-{}^4He$ energy distributions obtained with the use of the functions $\Phi(^3He-t-\Lambda)$ and $\Phi(d-\alpha-\Lambda)$ are presented. The general shape of both the curves agrees with the experimental data [1]. Formula (20) when integrated over E gives the angular distribution presented in the Fig. 4. We see that the angular distribution calculated with the use of the $\Phi(^3He-t-\Lambda)$ wave function displays a significant assymetry in the forward direction, while the $d-\alpha-\Lambda$ model favours the oposite direction. This is a conse-

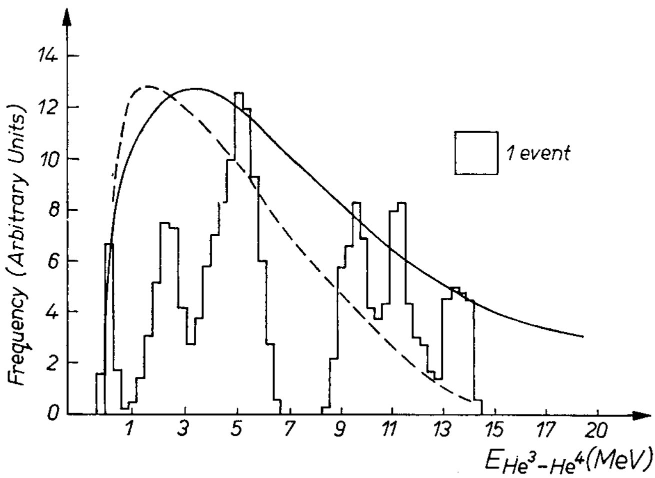

Fig. 3. Distribution of the $^3He-{}^4He$ relative energy in the $^7_\Lambda Li \to {}^4He + {}^3He + \pi^-$ decay. The solid and broken lines are obtained with the use the $\Phi(d-\alpha-\Lambda)$ and $\Phi(^3He-t-\Lambda)$ functions, respectively

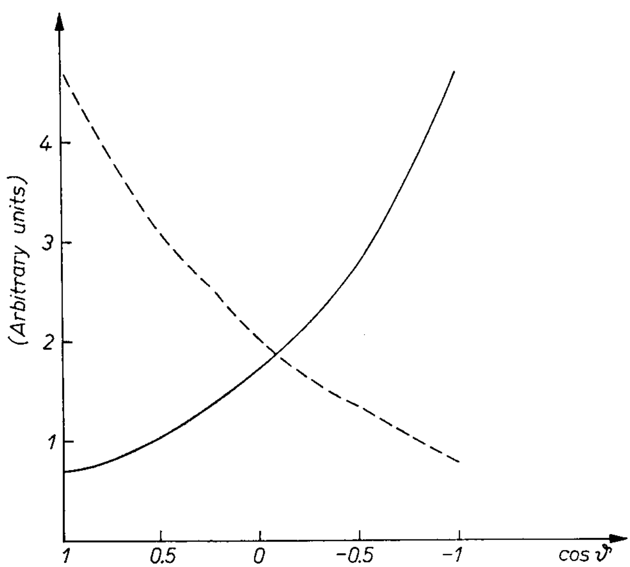

Fig. 4. Angular distribution of the $^7_\Lambda Li \to {}^4He + {}^3He + \pi^-$ decay products, ϑ is the angle between the $^3He-{}^4He$ relative momentum and the pion momentum in the centre-of-mass system of the $^7_\Lambda Li$. The solid line corresponds to the function $\Phi(d-\alpha-\Lambda)$ and the broken one to the function $\Phi(^3He-t-\Lambda)$

quence of the fact that in the $d-\alpha-\Lambda$ model the α-particle plays the role of the spectator while in the $^3He-t-\Lambda$ "model" the triton is the spectator in the process of the capture of a proton from the Λ-particle decay. Thus, the angular distribution strongly depends on the kind of clusters contained in the $^7_\Lambda Li$ hypernucleus and could give informations about the structure of the $^7_\Lambda Li$ and about the mechanism of the $^7_\Lambda Li \rightarrow {}^4He+{}^3He+\pi$ decay. Experimental data would be very necessary for such informations. Unfortunately, no experimental angular distributions for the decay of the $^7_\Lambda Li$ have been published so far.

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