LETTERS TO THE EDITOR

ON THE FISHBANE CONJECTURE IN THE GALILEAN INVARIANT LEE MODEL

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In this letter a conjecture of Fishbane (P. M. Fishbane, *Phys. Rev.*, **177(II)**, 2323 (1969)) is investigated in the framework of Gali-Lee model. It is found that this conjecture is true.

In a recent paper [1] it was shown that the mass shift δm_V and the scattering phase shifts in the Lee model [2] are related by

$$\delta m_V = \frac{1}{2\pi i}\int_{\mu}^{\infty} d\omega \,\mathrm{Tr}\ln S, \tag{1}$$

where S is that submatrix of the S-matix which connects channels of the same quantum numbers as the mass-shifted (V) particle.

In the simplest nontrivial sector of the model, this relation is

$$\delta m_V = \frac{1}{\pi}\int_{\mu}^{\infty} \delta(\omega) d\omega \tag{2}$$

where $\delta(\omega)$ is the phase shift of $N-\theta$ scattering. This formula is very remarkable because it connects mass-shifts ("unphysical quantities") with a simple integral over physical phase shifts.

The aim of this paper is to examine this conjecture in the Galilean invariant version of the Lee model [3–5]. We restrict ourselves to the $N-\theta$ sector and consider that the V, N and θ particles are spinless.

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In this sector the V-particle propagator is

$$\Delta_V(\Omega) = \left[\Omega - U_0 - \lambda_0^2 \int d^3q \, \frac{f^2(\omega)}{\Omega - \omega + i\varepsilon}\right]^{-1}, \tag{3}$$

where U_0 is the "bare" energy of V, λ_0 is the unrenormalized coupling constant, $f(\omega)$ is the cut-off function. $q^2 = 2\mu\omega$ with $\mu = m_1 m_2/m_3$, where $m_1 = m_N$, $m_2 = m_\theta$ and $m_3 = m_V$ $(= m_1 + m_2)$ so that μ is the reduced mass.

The calculation of the $N-\theta$ scattering is straightforward, due to the close connection between the $N-\theta$ scattering amplitude and the V-particle propagator. We have

$$S(\omega) = \exp(2i\delta(\omega)) = \Delta_V(\omega)/\Delta_V^*(\omega), \tag{4}$$

i. e.

$$\delta(\omega) = \frac{1}{2i} \ln \frac{\Delta_V(\omega)}{\Delta_V^*(\omega)}. \tag{5}$$

In [3], for the energy shift the following expression is stated

$$\delta U = U - U_0 = -\lambda_0^2 \int d^3q \, \frac{f^2(\omega)}{\omega - U}, \tag{6}$$

with $U < 0$, the physical energy.

By means of Eq. (6) we obtain

$$\Delta_V^{-1}(\Omega) = (\Omega - U)\left[1 - \lambda_0^2 \int d^3q \, \frac{f^2(\omega)}{(\omega - U)(\Omega - \omega + i\varepsilon)}\right]. \tag{7}$$

If we denote

$$D(\Omega) = \left[1 - \lambda_0^2 \int d^3q \, \frac{f^2(\omega)}{(\omega - U)(\Omega - \omega + i\varepsilon)}\right] \tag{8}$$

we observe that

$$\delta(\omega) = \frac{1}{2i} \ln \frac{D(\omega)}{D^*(\omega)}. \tag{9}$$

The following feature in the asymptotic behaviour is remarkable

$$D^{-1}(\Omega) \underset{\Omega \to \infty}{\to} 1 - \frac{\lambda_0^2}{\Omega} \int d^3q \, \frac{f^2(\omega)}{\omega - U} = 1 + \frac{\delta U}{\Omega} \tag{10}$$

i. e.

$$D(\Omega) \underset{\Omega \to \infty}{\to} 1 - \frac{\delta U}{\Omega}. \tag{11}$$

We now write the identity (see Fig. 1)

$$\oint_{C_t + C_R(R \to \infty)} \ln D(\omega) d\omega = 0 = \int_{C_t} \ln D(\omega) d\omega + \int_{C_R(R \to \infty)} \ln D(\omega) \, d\omega \tag{12}$$

and observe that

$$\int_{C_1} \ln D(\omega)d\omega = 2i\int_{\mu}^{\infty} \delta(\omega)d\omega \tag{13}$$

correspondingly

$$\int_{C_R(R\to\infty)} \ln D(\omega)d\omega = -2\pi i\delta U. \tag{14}$$

Then

$$\delta U = \frac{1}{\pi}\int_{\mu}^{\infty} \delta(\omega)d\omega. \tag{15}$$

This is a relation of the same kind as Eq. (2) and we conclude that the Fishbane conjecture is true in the Gali-Lee model as in the original Lee model. Of course this is a surprising result, probably peculiar to these (Lee) models.

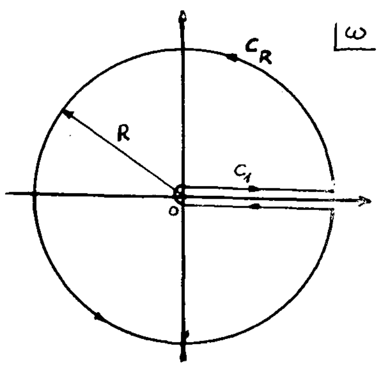

Fig. 1

Not long ago by the effort of Hagen [6], the Galilean invariant Lee model was generalized to the case in which all particles are allowed to have arbitrary spins and parities. The results are qualitatively identical with those of the spinless case and therefore we do not insist on a detailed treatment of this situation. We mention only that the Fishbane conjecture is also true as in the spinless case.

Finally we wish to stress the fact that for more realistic theories it is important to see if such properties are present and eventually with what degree of precision.

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