REGGEON PERTURBATION THEORY IN A ZACHARIASEN--LIKE MODEL*

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Gribov's Reggeon graph technique is used to construct a simple Zachariasen-like mode with singularities in the j-plane. Decomposing the scattering amplitude into its one-Reggeon-irreducible and reducible parts yields a natural definition of the propagator, the vertex function, the formfactor and the renormalization constants of an intermediate Reggeon.

1. *Introduction*

During the last years a great many articles have been written concerning the connection between compositeness and vanishing renormalization constants of elementary particles [1] using to a large extent ideas put forward by Salam in 1962 [2]. In this kind of problem the decomposition of a partial wave scattering amplitude into its one particle-reducible and irreducible parts is a very useful tool in defining field-theoretical quantities of stable or unstable particles such as the propagator, the vertex function or the renormalization constants by means of on-shell quantities like the denominator function D of an N/D representation of the amplitude [3, 4].

Using Gribov's Reggeon graph technique [5] we construct in this note a simple Zachariasen-like model for a Reggeon scattering amplitude with a Reggeon coupling of the Yukawa- and Fermi-type. The decomposition of this amplitude into its one-Reggeon irreducible and reducible parts in the j-plane yields, then, a natural definition of the propagator, the vertex function, formfactor and renormalization constants of an intermediate Reggeon, *i.e.* of stable or unstable particles lying on a Regge trajectory. This model automatically takes into account also some amount of inelasticity by considering two-body intermediate states of higher spins.

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2. *Definition of the model*

Let us first remind the Reggeon graph technique developed by Gribov in studying the high energy behaviour of Feynman graphs. The Gribov rules read:

i) Reggeons are described as nonrelativistic particles n two dimensional momentum space with a free propagator $[j-\alpha(k^2)]^{-1}$.

ii) At each vertex the two-dimensional momenta k are conserved. A n-Reggeon intermediate state of spin j contributes a factor $[j-\sum_{i=1}^{n} j_i+(n-1)]^{-1}$.*

iii) Integrate over the two-dimensional momentum k and the angular momentum j of each intermediate Reggeon.

To be definite we consider now a simple model of identical Reggeons interacting *via* a Yukawa- and Fermi-coupling with a Lagrangian

$$L_{\text{int}} = g[\psi^+\psi^+\psi+\psi\psi\psi^+]+\lambda[\psi^+\psi^+\psi\psi] \tag{1}$$

where

$$\psi(r) = \sum_k a_k e^{ikr} \tag{2}$$

is the nonrelativistic field operator in two-dimensional space and g, λ are coupling functions which will be specified later.

The perturbation graphs for the Reggeon-Reggeon scattering amplitude $a_j(t;\, t_1t_2t_3t_4)$ with the external Reggeons on their angular momentum shell $j_i = \alpha(t_i)$ as given in Fig. 1

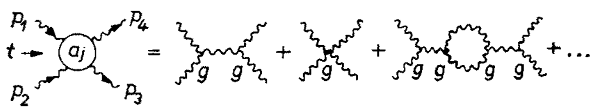

Fig. 1. Perturbation series of the four-Reggeon scattering amplitude a_j

$(t = (p_1+p_2)^2 = (p_3+p_4)^2;\ t_i = p_i^2)$ can then easily be summed up by means of an equation of the Bethe-Salpeter-type according to Fig. 2.

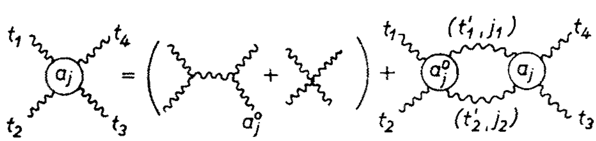

Fig. 2. Graphical representation of the Bethe-Salpeter equation of a_j

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* One can also introduce the equivalent factor

$$\delta(j-\sum_{i=1}^{p} j_i+(n-1))$$

corresponding to a conservation of $j-1$ at each vertex.

Evaluating the j_1 and j_2 integration, we get explicitly

$$a_j(t;\,t_1t_2t_3t_4) = R(t;\,t_1t_2,\,t_3t_4)\prod_{i=1}^{4} g(t_i)\left\{\Lambda+\frac{g^2(t)}{j-\alpha(t)}\right\}+$$

$$+\int\frac{dt_1'dt_2'\theta(-\tau)}{(-\tau)^{1/2}}\,\frac{a_j^0(t;\,t_1t_2t_1't_2')a_j(t;\,t_1't_2't_3t_4)}{j-(\alpha(t_1')+\alpha(t_2')-1)} \tag{3}$$

where τ is the triangle function

$$\tau(t,\,t_1,\,t_2) = t^2+t_1^2+t_2^2-2tt_1-2tt_2-2t_1t_2,$$

$$R(t;\,t_1t_2,\,t_3t_4) = q^{j-\alpha(t_1)-\alpha(t_2)}(t,\,t_1t_2)\cdot q^{j-\alpha(t_3)-\alpha(t_4)}(t,\,t_3t_4) \tag{4}$$

and q is the incoming or outgoing 3-momentum.

In Eq. (3) we have taken into account a threshold factor R as it is given by coupling particles with spin according to $j_1+j_2\to j\to j_3+j_4$ with the external Reggeons on their angular momentum shell $j_i = \alpha(t_i)$. Further more we have assumed a factorizable dependence on external masses of the 3- and 4-Reggeon coupling functions

$$g \equiv g(t_1)g(t_2)g(t),\ (g(t_3)g(t_4)g(t))$$

$$\lambda \equiv \prod_{i=1}^{4} g(t_i)\Lambda \tag{5}$$

resulting in a neglection of Toller angle dependences. Finally, $\theta(-\tau)(-\tau)^{-1/2}$ is the Jacobian for the transformation $d^2k\to dt_1'dt_2'$.

Due to the factorizable kernel, Eq. (3) can be solved algebraically with the solution

$$a_j(t;\,t_1t_2t_3t_4) = \frac{N_j}{D_j} = \frac{R(t;\,t_1t_2,\,t_3t_4)\prod_{i=1}^{4} g(t_i)\left\{\Lambda+\frac{g^2(t)}{j-\alpha(t)}\right\}}{1-H_j(t)\left\{\Lambda+\frac{g^2(t)}{j-\alpha(t)}\right\}} \tag{6}$$

where

$$H_j(t) = \int\frac{dt_1'dt_2'\theta(-\tau)}{(-\tau)^{1/2}}\,\frac{R(t;\,t_1't_2',\,t_1't_2')g^2(t_1')g^2(t_2')}{j-(\alpha(t_1')+\alpha(t_2')-1)} \tag{7}$$

is the contribution of a two Reggeon bubble.

Eq. (6) is completely analogous to the usual combined Zachariasen-model [6] with the replacement of the particle poles and the unitarity cuts in the energy plane by the Regge pole and the AFS-cuts [7] in the angular momentum plane j.

3. *Field-theoretical decomposition in the j-plane*

As can be seen, with some algebra, Eq. (6) admits a field-theoretical decomposition into its one-Reggeon reducible and irreducible parts according to

$$a_j(t;\, t_1t_2t_3t_4) = R(t;\, t_1t_2,\, t_3t_4)g^2(t) \prod_{i=1}^{4} g(t_i)\Gamma_j(t)\Delta'_j(t)\Gamma_j(t) +$$

$$+b_j(t;\, t_1t_2t_3t_4) \tag{8}$$

where the vertex function $\Gamma_j(t)$, the propagator $\Delta'_j(t)$ and the one-Reggeon irreducible amplitude b are defined by

$$\Gamma_j(t) = (1-H_j(t)\Lambda)^{-1}$$

$$\Delta'_j(t) = \frac{1}{j-\alpha(t)}\left(1-g^2(t)\Sigma_j(t)\frac{1}{j-\alpha(t)}\right)^{-1}$$

$$\Sigma_j(t) = H_j(t)(1-H_j(t)\Lambda)^{-1}$$

$$b_j(t;\, t_1t_2t_3t_4) = R(t;\, t_1t_2,\, t_3t_4)\prod_{i=1}^{4} g(t_i)\frac{\Lambda}{1-H_j(t)\Lambda} =$$

$$= R(t;\, t_1t_2,\, t_3t_4)\prod_{i=1}^{4} g(t_i)\Lambda\Gamma_j(t). \tag{9}$$

One can further introduce a Reggeon form factor (improper vertex) by

$$F_j(t) = D_j^{-1}(t) \tag{10}$$

satisfying the relation

$$F_j(t)\cdot\frac{1}{j-\alpha(t)} = \Gamma_j(t)\Delta'_j(t) \tag{11}$$

which is the j-plane analogue of a corresponding formula defined in the energy plane [4].

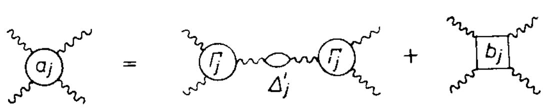

Fig. 3. The diagrams representing the decomposition (8)

The diagram version of Eqs (8), (9), (10) is given in Fig. 3 and Fig. 4. The renormalized pole position is now given according to Eq. (9) by the solution of the equation

$$\alpha_r(t)-\alpha(t) = g^2(t)\Sigma_{\alpha_r(t)}(t) \tag{12}$$

or, because of Eq. (8), by the vanishing of the denominator function of Eq. (6). Depending on the sign of $D_j(t)$ at the position of the branch point $j_{\mathrm{AFS}} = 2\alpha(t/4)-1$ of the AFS-cut we obtain then, at most, two out-put Regge poles (if $D_{j_{\mathrm{AFS}}} < 0$) lying below and above the unrenormalized input Regge pole $\alpha(t)$.

Let us next perform the renormalization of the leading Regge pole by extending the usual renormalization prescriptions of the energy plane to the j-plane. Thus, we define

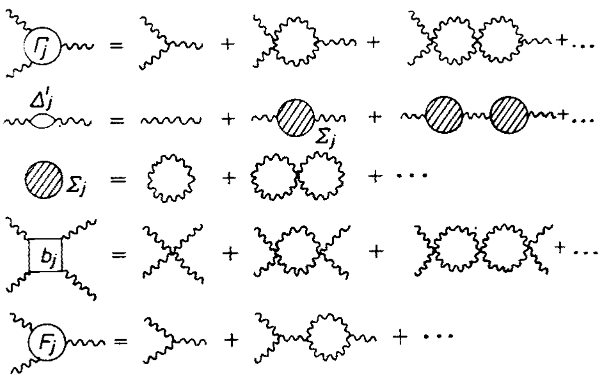

Fig. 4. Perturbation series of the vertex function, the propagator, the self-energy part, the amplitude b and the formfactor

the "wave function renormalization constant" $Z_3(t)$ of the Reggeon by the residue of the unrenormalized propagator, *i.e.*

$$Z_3(t) = \left(1 - g^2(t)\frac{\partial}{\partial j}\Sigma_j(t)|_{j=\alpha_r(t)}\right)^{-1} \tag{13}$$

and the "vertex renormalization constant" $Z_1(t)$ by the reciprocal value of the unrenormalized vertex at the renormalized pole position

$$Z_1(t) = 1 - H_{\alpha_r(t)}(t)\cdot \Lambda. \tag{14}$$

If we introduce renormalized coupling functions by*

$$g_r^2(t) = (Z_3(t)/Z_1^2(t))g^2(t); \quad (Z_2(t)\equiv 1)$$

$$\Lambda_r(t) = \Lambda/Z_1(t) \tag{15}$$

the renormalization constants can be expressed by renormalized quantities according to

$$Z_3(t) = 1 + g_r^2(t)\left.\frac{\partial H_j(t)}{\partial j}\right|_{j=\alpha_r(t)}$$

$$Z_1(t) = (1 + H_{\alpha_r(t)}(t)\Lambda_r(t))^{-1}. \tag{16}$$

* The renormalization of external Reggeons will not be considered here.

Defining now

$$\Gamma_j^r(t) \equiv Z_1(t)\Gamma_j(t) = \left(1-(j-\alpha_r(t))\left[\frac{H_j(t)-H_{\alpha_r}(t)}{j-\alpha_r(t)}\right]\Lambda_r(t)\right)^{-1}$$

$$\Delta_j'^r(t) \equiv Z_3^{-1}(t)\Delta_j'(t) \tag{17}$$

we can rewrite Eq. (8) by renormalized quantities as follows

$$a_j(t;\, t_1t_2t_3t_4) = R(t;\, t_1t_2,\, t_3t_4)\,[g_r^2(t)\prod_{i=1}^{4} g(t_i)\Gamma_j^r(t)\Delta_j'^r(t)\Gamma_j^r(t)+$$

$$+\prod_{i=1}^{4} g(t_i)\Lambda_r(t)\Gamma_j^r(t)]. \tag{18}$$

Note that

$$\Delta_j'(t) \underset{j\to\infty}{\to} j^{-1} \tag{19}$$

With Eqs (16) and (17) the renormalization constant $Z_3(t)$ can also be expressed by the renormalized function $\Delta_j'^r(t)$ as

$$Z_3^{-1}(t) = \lim_{j\to\infty}(j\Delta_j'^r(t)). \tag{16'}$$

We remark that the usual s-wave pseudopoles of the Zachariasen-model [1] which appear in the vertex function and the b_j-amplitude are now transformed into pseudo-Regge poles. It is easily seen that such poles cancel each other in Eq. (8), as it should be, since a_j is free of them. It is worth mentioning that the Regge pole parameters $g_r^2(t)$ and $\alpha_r(t)$ were up to now entirely arbitrary.

Although being analytic in the j-plane (absence of Kronecker δ_{jJ}-factors) the intermediate Reggeon should be interpreted therefore as an elementary object. Performing now the limit

$$Z_3(t) = 1+g_r^2(t)\left.\frac{\partial H_j(t)}{\partial j}\right|_{j=\alpha_r(t)} \to O, \quad \left(\left.\frac{\partial H}{\partial j}\right|_{j\to\alpha_r} < 0\right) \tag{20}$$

one obtains a connection between $g_r(t)$ and $\alpha_r(t)$ and thus a reduction of free parameters. In this limit we have a composite Reggeon.*

4. Conclusion

We have shown in a simple Zachariasen-like model that the scattering amplitude possesses a field-theoretical decomposition in the j-plane and that renormalization of Regge poles can be defined along the line of usual particle renormalization. Since two-

* Note that $Z_3(t)$ and $Z_1(t)$ possess two-particle cuts. The continuation through such a cut to a complex value $t = m_J^2 \equiv \alpha_r^{-1}(j)$ yields complex constants $Z_3(m_J^2)$, $Z_1(m_J^2)$ for resonances which correspond to the Z_1^J and Z^J introduced in Ref. [4].

-Reggeon intermediate states correspond to two-body states with high spin particles, this model takes als into account, automatically, some amount of inelasticity. Expressions similar to Eq. (6) ($\lambda \equiv O$) are expected to occur also in the dual perturbation theory by summing up multiloop diagrams. Considering the work of Neveu and Scherk [8] we should replace then $H_j(t)$ by the following self-energy-type expression of the dual resonance model

$$\Sigma(\alpha_t) = 4\pi^2 \int_0^1 \frac{dx_2 dx_4}{\ln^2 x_2 x_4} (x_2 x_4)^{-\alpha_0 - 1}(1-x_2)^{-1+\alpha_0}(1-x_4)^{-1+\alpha_0} \times$$

$$\times e^{\alpha_0 h_1} q_0^{-4}[(x_2 x_4)^{1/2}] e^{\alpha_t H(x_2, x_4)} \theta(2 - \varepsilon - x_2 - x_4) \qquad (21)$$

where the functions h_1, H, q_0 are defined in [8]. This formula also contains daughter contributions which were not considered in our simple model.

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