

PARTIAL TRANSITIONS IN MUON CAPTURE BY COMPLEX NUCLEI. I. THE CAPTURE RATE

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The partial capture rates in muon capture by some light nuclei (^9Be , ^{10}B , ^{13}C , ^{14}N , ^{15}N and ^{19}F) are calculated. The dependence of capture rates on nuclear structure is investigated. The possibility of deriving muon-nucleon coupling constants from data on light nuclei is discussed.

1. Introduction

At the present time one cannot obtain a complete information on muon-nucleon coupling constants from the experimental data on hydrogen. Therefore one is forced to extract the necessary information from experiments on complex nuclei. For this purpose the partial transitions (transitions to the definite state of the final nuclei) in light nuclei seem to be of principal interest to date. Other processes are connected to a greater extent with nuclear problems and their interpretation depends essentially on the assumptions on the nuclear structure. However, even in the case of partial transitions, one fails to solve the nuclear part of the problem reliably. This can be clearly seen on the example of the transitions in ^3He and ^{12}C . The structure of these nuclei is known sufficiently well. Nevertheless one cannot, as yet, determine all components of their wave functions sufficiently accurately in order to extract reliably additional information on muon-nucleon coupling constants. As to the other nuclei, the accuracy of calculation of the capture rate is hardly comparable with the appropriate one in ^3He and ^{12}C . Therefore it is of great importance to find such characteristics of the transitions which are not so sensitive to the details of the nuclear structure. It is clear that such properties will be inherent most likely in the characteristics depending on the ratio of the nuclear matrix elements rather than on their absolute value. These are the ratio of the capture rates from two states of the hyperfine

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doublet of mesoatoms (A_+/A_-), asymmetry of the recoil nuclei relative to the muon polarization vector, γ - ν correlation, *etc.*

In the present paper we discuss the capture rates from two states of the hyperfine doublet of mesoatoms only and investigate their dependence on nuclear structure. We deal essentially with the allowed transitions, where the present theory provides the calculation of the nuclear matrix elements rather reliably. We confine ourselves to $1p$ -shell nuclei from ${}^9\text{Be}$ to ${}^{14}\text{N}$ and consider only one $(2s-1d)$ -shell nucleus, namely ${}^{19}\text{F}$. The first forbidden transitions are discussed on the example of ${}^{15}\text{N}$. For heavier nuclei the density of the excited states of the daughter nuclei essentially increases and the separation of the partial transitions becomes very complicated. It should be noted that among other transitions only the correlation measurements in the transition ${}^{20}\text{Ne}(O^+) - {}^{20}\text{F}(1^+)$ seem to be of interest.

2. The general expression for capture rates from the hyperfine state of the mesoatom

The general expression for the capture rate from the definite state of the hyperfine doublet has the following form

$$A_F = 4(\alpha Z m_\mu)^3 (Z_{\text{eff}}/Z)^4 \frac{2I_f + 1}{2I_i + 1} q^2 A_F \tag{1}$$

where for the allowed transitions

$$A_+^{(0)} = (2F_+ + 1)^{-1} \{ [\sqrt{(I_i + I_f + 2)(I_i - I_f + 1)} \mu_0(-1) - \sqrt{(I_f - I_i + 1)(I_f + I_i)} \mu_1(-1)]^2 + \\ + [\sqrt{\frac{1}{2}(I_f - I_i + 2)(I_f + I_i - 1)} \mu_2(2) - \sqrt{\frac{1}{2}(I_i - I_f + 2)(I_i + I_f + 3)} \mu_1(2)]^2 \} \tag{2a}$$

$$A_-^{(0)} = (2F_- + 1)^{-1} \{ [\sqrt{(I_f + I_i)(I_f - I_i + 1)} \mu_0(-1) + \sqrt{(I_f + I_i + 2)(I_i - I_f + 1)} \mu_1(-1)]^2 + \\ + [\sqrt{\frac{1}{2}(I_f + I_i + 3)(I_i - I_f + 2)} \mu_2(2) + \sqrt{\frac{1}{2}(I_f + I_i - 1)(I_f - I_i + 2)} \mu_1(2)]^2 \} \tag{2b}$$

TABLE I

The numerical values of $(Z_{\text{eff}}/Z)^4$					
Nucleus	${}^9\text{Be}$	${}^{10}\text{B}$	${}^{13}\text{C}$	${}^{14}\text{N}$ and ${}^{15}\text{N}$	${}^{19}\text{F}$
$(Z_{\text{eff}}/Z)^4$	0.927	0.889	0.885	0.829	0.730

In (1), Z_{eff} is the effective charge of the nucleus. Their values are given in Table I. We shall take into account also the strong matrix elements of the next (second) order of forbiddenness because their contribution is sufficiently important:

$$A_+^{(2)} \approx (2F_+ + 1)^{-1} \{ [\sqrt{\frac{1}{3}(I_i + I_f + 4)(I_i - I_f + 3)} \mu_2(-3) - \\ - \sqrt{\frac{1}{3}(I_f - I_i + 3)(I_f + I_i - 2)} \mu_3(-3)]^2 + \frac{1}{4}(I_i - I_f + 4)(I_i + I_f + 5) \mu_3^2(4) \} \tag{3a}$$

$$A_{-}^{(2)} \approx (2F_{-} + 1)^{-1} \{ [\sqrt{\frac{1}{3}} (I_f + I_i - 2) (I_f - I_i + 3) \mu_2(-3) + \\ + \sqrt{\frac{1}{3}} (I_f + I_i + 4) (I_i - I_f + 3) \mu_3(-3)]^2 + \frac{1}{4} (I_f + I_i - 3) (I_f - I_i + 4) \mu_3^2(4) \}. \quad (3b)$$

In (2) and (3) I_i and I_f are spins of the initial and final nuclei, $F_{\pm} = I_i \pm 1/2$, $\mu_u(k)$ are the combinations of the nuclear matrix elements and coupling constants [1].

3. Calculation of nuclear matrix elements

The shell model wave functions with an intermediate coupling scheme were used to calculate the nuclear matrix elements. The parameters are chosen to describe the low-lying states of light nuclei. The wave functions for nuclei $A = 9, 10, 13$ and 14 are taken from [2], for $A = 15$ from [3], and for $A = 19$ from [4]. The results are mostly sensitive to the parameter “ k ” of an intermediate coupling scheme. At present one cannot determine this parameter uniquely. Therefore we shall vary it within the range of its optimal values. Such a variation enables one to investigate the dependence of the capture rate on this parameter.

The ratio $[12u]/[12i]$ for some transitions is given in Table II. From the calculation it follows that we must take into account all $[12u]$ matrix elements from both allowed $[12i]$ and second forbidden a — $[122]$ and $[123]$ cases. This fact was indicated in pa-

TABLE II

The ratio of matrix elements $[12u]/[12i]$ in some transitions

	${}^9\text{Be} \rightarrow {}^9\text{Li}$				${}^{10}\text{B} \rightarrow {}^{10}\text{Be}$		${}^{11}\text{B} \rightarrow {}^{11}\text{Be}$	${}^{13}\text{C} \rightarrow {}^{13}\text{B}$	${}^{14}\text{N} \rightarrow {}^{14}\text{C}$	${}^{19}\text{F} \rightarrow {}^{19}\text{O}$
	$(3/2)_1$	$(3/2)_2$	$5/2$	$1/2$	$(2^+)_1$	$(2^+)_2$	$1/2$	$3/2$	2^+	$3/2^+$
$[123]/[12i]$	9.45	0.012	-4.41	—	-15.72	15.11	—	—	0.24	—
$[122]/[12i]$	-2.57	-0.68	9.55	-1.41	-0.33	-1.64	-13.64	3.66	2.27	-0.53

pers [5, 6] when investigating the transitions in ${}^{11}\text{Be}$ and ${}^{14}\text{N}$ nuclei. The obtained results enables one to generalize such a conclusion to all transitions in light nuclei. Therefore in all calculations we take into account the strong matrix elements of higher degree of forbiddenness.

4. Partial transitions in light nuclei

1. Transitions ${}^9\text{Be}(3/2^-) \rightarrow {}^9\text{Li}$. The ground state spin of ${}^9\text{Be}$ is $3/2^-$. Allowed are the transitions to $I = 1/2^-$, $3/2^-$ and $5/2^-$ levels of ${}^9\text{Li}$. The theory predicts such bound states of ${}^9\text{Li}$ (Fig. 1a). The experimental information on ${}^9\text{Li}$ is very poor. It is known only that this nucleus has bound states. Their quantum numbers are undetermined.

The results of calculation of the capture rate to the ground and three excited states are given in Table III. Together with the wave function of [2] we used the functions of

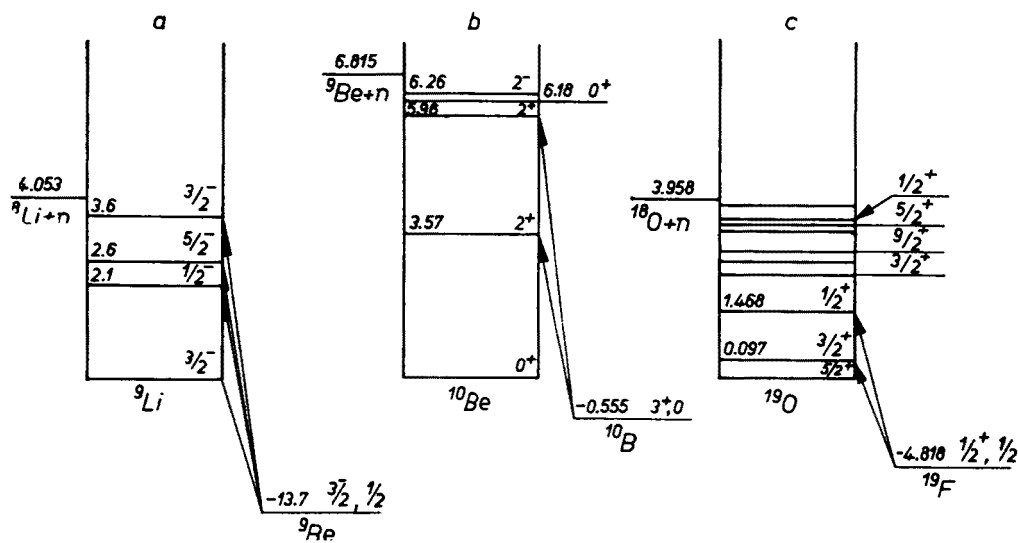


Fig. 1. Partial transitions scheme in muon capture by ${}^9\text{Be}$, ${}^{10}\text{B}$ and ${}^{19}\text{F}$

TABLE III

The capture rate in $\mu^- + {}^9\text{Be} \rightarrow {}^9\text{Li} + \nu$ (in sec^{-1})

g_P/g_A	$k = -1.0 \text{ MeV}$			$k = -1.2 \text{ MeV}$			$k = -1.4 \text{ MeV}$			Barcer [7]			[101] approximation		
	A_+	A_+/A_-	A_{stat}	A^+	A_+/A_-	A_{stat}	A^+	A_+/A_-	A_{stat}	A_+	A_+/A_-	A_{stat}	A_+	A_+/A_-	A_{stat}
Ground state ($3/2^-$) of ${}^9\text{Li}$															
4	62	0.306	114	40	0.283	78	27	0.257	56	40	0.289	77	34	0.372	55
7	60	0.317	109	39	0.294	74	27	0.269	54	39	0.297	74	32	0.387	52
12	61	0.351	103	40	0.329	71	28	0.305	52	39	0.326	70	32	0.431	47
Level ($1/2^-$) at $E = 2.1 \text{ MeV}$															
4	8	0.046	72	9	0.052	67	9	0.059	61	7	0.075	41	2	0.011	62
7	12	0.075	68	12	0.083	63	12	0.091	57	10	0.110	39	4	0.025	58
12	21	0.158	63	21	0.170	58	20	0.184	53	15	0.206	36	8	0.066	53
Level ($3/2^-$) at $E = 3.6 \text{ MeV}$															
4	50	0.420	76	40	0.426	60	33	0.436	50	48	0.426	73	28	0.371	46
7	51	0.435	76	40	0.440	60	34	0.447	50	49	0.441	73	27	0.385	43
12	53	0.463	76	42	0.464	60	35	0.468	50	51	0.470	73	26	0.425	40
Level ($5/2^-$) at $E = 2.6 \text{ MeV}$															
4	88	1.752	74	53	1.282	49	33	0.937	34	87	1.888	72	104	55.75	66
7	77	1.258	71	45	0.920	47	28	0.670	33	76	1.338	68	96	24.60	62
12	60	0.739	68	34	0.532	45	20	0.378	32	59	0.771	65	85	9.44	56

paper [7], where the level scheme and transitions in nuclei $A = 9$ were investigated in detail. Both sets of functions are close to each other when the parameter is at its optimal value ($k = -1.2$ MeV).

The capture rate to all bound levels is very small. This is due to the suppression of the principal matrix element [101] resulting from the Yang scheme selection rule. As was mentioned in [8], the same holds for the inverse process of beta decay of ${}^9\text{Li}$. Since the principal matrix element is small, the effect of all others increases. This is easily seen from the comparison of the results of Table III. It should be noted that in the last three rows (the so-called [101] approximation) the results are obtained by taking into account only the principal matrix element [101], the latter being calculated for the optimum value of parameter k .

Taking into account the strong dependence of the capture rates and their ratio on the parameter " k " one concludes that the muon capture transitions in ${}^9\text{Be}$ are not favourable for studying the muon-nucleon coupling constants.

2. Transitions ${}^{10}\text{B}(3^+) \rightarrow {}^{10}\text{Be}(2^+)$. Transitions to the bound excited states of ${}^{10}\text{Be}$ (Fig. 1b) with $I = 2^+$, $E = 3.37$ MeV and $E = 5.96$ MeV are allowed. The calculated values for the capture rates are given in Table IV. Since the principal matrix element

TABLE IV
The capture rate to 2^+ levels in ${}^{10}\text{B}$ (in 10^3 sec^{-1})

g_F/g_A	$k = -1.0$ MeV			$k = -1.2$ MeV			$k = -1.4$ MeV			[101] approximation		
	A_+	A_+/A_-	A_{stat}	A_-	A_+/A_-	A_{stat}	A_-	A_+/A_-	A_{stat}	A_-	A_+/A_-	A_{stat}
Level at $E = 3.37$ MeV												
0	3.63	0.169	1.90	4.53	0.146	2.32	5.53	0.128	2.75	0.52	0.002	0.22
7	2.71	0.227	1.51	3.42	0.199	1.86	4.20	0.178	2.23	0.42	0.035	0.19
12	2.16	0.332	1.34	2.75	0.297	1.64	3.39	0.271	1.98	0.35	0.095	0.17
Level at $E = 5.96$ MeV												
0	16.64	0.019	7.31	17.45	0.020	7.68	18.15	0.021	7.99	16.12	0.002	6.93
7	13.42	0.065	6.25	14.04	0.066	6.55	14.58	0.066	6.80	12.96	0.035	5.81
12	11.43	0.138	5.80	11.94	0.139	6.07	12.38	0.140	6.29	10.98	0.095	5.30

[101] for the transition to the level at $E = 3.37$ MeV is suppressed, the capture rate is small and sensitive to the parameter k .

The transition to the $E = 5.96$ MeV level is of significant interest. The capture rate is large and within the region $-1.0 \text{ MeV} < k < -1.4 \text{ MeV}$ it changes only slightly (about 10%). This means that A_+ and A_- are not so sensitive to the parameter k . Finally, the ratio A_+/A_- is practically independent of k (Fig. 2a). Indeed, the curve 1 of Fig. 2a represents this ratio practically for all values of k . Although the ratio itself is slightly dependent on k , it is very essential that all the matrix elements, including the second

forbidden ones, are taken into account. In the [101] approximation (curve 2) the ratio is almost twice as small.

The weak dependence of Λ_+/Λ_- on k enables one to hope to obtain some new information on muon-nucleon coupling constants provided that the experimental data are available.

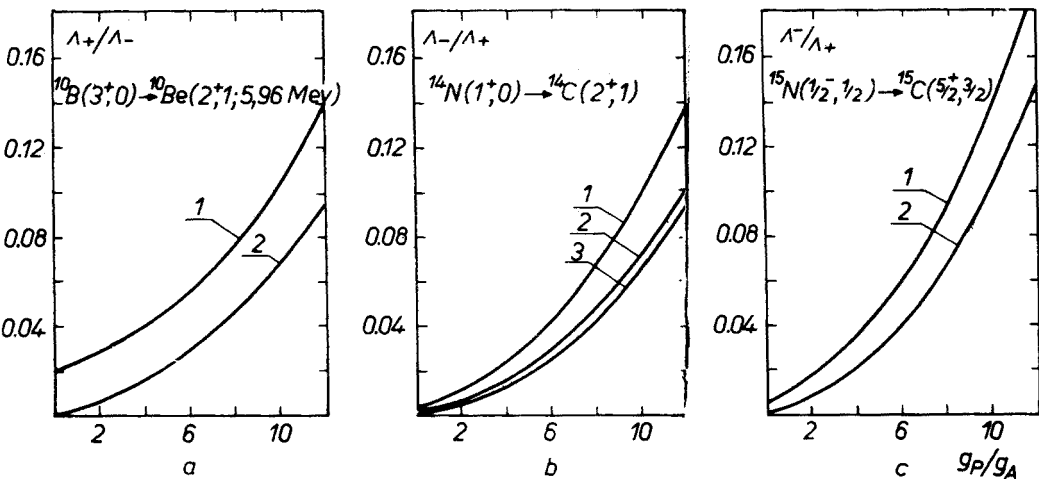


Fig. 2. The ratio of the capture rates from two states of the hyperfine doublet as a function of g_P/g_A

3. Transitions $^{13}\text{C}(1/2^-) \rightarrow ^{13}\text{B}$. Allowed are transitions to the levels at about $E = 3.7$ MeV and to the ground state of ^{13}B . The calculated values for the capture rate are given in Table V. Both the capture rates and the ratio Λ_-/Λ_+ to the ground state of

TABLE V

The capture rate in $\mu^- + ^{13}\text{C} \rightarrow ^{13}\text{B} + \nu$ (in 10^3 sec^{-1} for the transition to the ground state and in sec^{-1} for $E = 3.7 \text{ MeV}$ one)

g_P/g_A	$k = -1.0 \text{ MeV}$			$k = -1.2 \text{ MeV}$			$k = -1.4 \text{ MeV}$			[101] approximation		
	Λ_+	Λ_-/Λ_+	Δ_{stat}	Λ_+	Λ_-/Λ_+	Δ_{stat}	Λ_+	Λ_-/Λ_+	Δ_{stat}	Λ_+	Λ_-/Λ_+	Δ_{stat}
Ground state (3/2-) of ^{13}B												
0	12.88	0.005	9.67	10.15	0.007	7.63	7.87	0.009	5.92	10.86	0.003	8.16
7	11.45	0.007	8.61	9.06	0.005	6.81	7.05	0.003	5.30	9.04	0.052	6.90
12	10.76	0.039	8.17	8.55	0.034	6.48	6.68	0.028	5.06	8.04	0.135	6.30
Level (1/2-) at $E = 3.7 \text{ MeV}$												
	Λ_+/Λ_-			Λ_+/Λ_-			Λ_+/Λ_-			Λ_+/Λ_-		
0	50	0.193	103	69	0.207	136	86	0.224	161	27	0.112	80
7	54	0.226	100	74	0.239	133	91	0.253	159	26	0.131	68
12	57	0.254	99	78	0.265	132	95	0.276	158	27	0.164	62

^{13}B are very sensitive to g_P/g_A and at the same time to the parameter k . The latter would make the interpretation of the experimental data difficult. Therefore the transition to the ground state of ^{13}B is less favourable in comparison with transition in ^{10}B .

4. Transition $^{14}\text{N}(1^+) \rightarrow ^{14}\text{C}(2^+)$. Transition to the 2^+ level at $E = 7.01$ MeV is of interest. The peculiarity of this level is the large admixture of $2\hbar\omega$ excitation. From the experimental data it follows that the wave function of $E = 7.01$ MeV state has the following structure:

$$\Psi(2^+) = \alpha_1 \Psi(1p^{10}) + \alpha_2 \Psi(1p^8[2s-1d]^2) \quad (4)$$

where $\Psi(1p^{10})$ is the wave function with configuration $1s^4 1p^{10}$. This function is taken from [2]. The optimal value is $k = -1.05$ MeV, $\Psi(1p^8[2s-1d]^2)$ is the wave function with $2\hbar\omega$ excitation. Its explicit form is unimportant because it does not contribute to the matrix elements. This is due to the smallness of such an admixture to the ground state of ^{14}N . According [9] to the experimental data, $|\alpha_2/\alpha_1| = 1.04 \pm 0.02$. The capture rate, calculated with this value of α is given in Table VI. At $k = -1.05$ MeV the theoretical value of Λ_{stat} is equal to $11.3 \times 10^3 \text{ sec}^{-1}$ and is in good agreement with the experi-

TABLE VI
The capture rate to 2^+ level in transition $\mu^- + ^{14}\text{N} - ^{14}\text{C} + \nu$ (in 10^3 sec^{-1})

g_P/g_A	$k = 0$			$k = -1.05 \text{ MeV}$			$k = -1.4 \text{ MeV}$			[101] approximation		
	Λ_+	Λ_-/Λ_+	Λ_{stat}	Λ_+	Λ_-/Λ_+	Λ_{stat}	Λ_+	Λ_-/Λ_+	Λ_{stat}	Λ_+	Λ_-/Λ_+	Λ_{stat}
0	20.71	0.001	13.81	18.86	0.002	12.59	18.44	0.003	12.31	19.87	0.003	13.27
7	17.82	0.034	12.09	16.33	0.038	11.10	15.95	0.039	10.84	16.28	0.053	11.14
12	16.30	0.096	11.39	14.99	0.101	10.50	14.64	0.102	10.26	14.26	0.139	10.16

mental one: $\Lambda_{\text{exp}} = (10 \pm 3) \cdot 10^3 \text{ sec}^{-1}$. The ratio Λ_-/Λ_+ is given in Fig. 2b, where curve 1 is the [101]-approximation, curve 2 includes all matrix elements calculated in the intermediate coupling scheme, and curve 3 in $j-j$ coupling.

As follows from Table VI and Fig. 2b, the capture rate depends weakly on parameter k , so does the ratio Λ_-/Λ_+ . At the same time they are both very sensitive to g_P/g_A : Therefore one can hope that this transition is favourable for the analysis of muon nucleon coupling constants.

5. Transition $^{19}\text{F}(1/2^+) \rightarrow ^{19}\text{O}$. The level scheme of ^{19}O formed in muon capture by ^{19}F is given on Fig. 1c. In ^{19}O there is a large number of bound states. However only

TABLE VII
The capture rate in $\mu + ^{19}\text{F} - ^{19}\text{O} + \nu$ (in sec^{-1}) at $g_P/g_A = 7$

	Λ_+	Λ_-	Λ_-/Λ_+	Λ_{stat}
$3/2^+$	1006	110	0.11	782
$1/2^+$	140	1168	8.35	397

some of them are identified. Therefore we consider only the transitions to the two low-lying levels: $3/2^+$ at $E = 0.097$ MeV and $1/2$ at $E = 1.46$ MeV. The calculated value for the capture rate is given in Table VII.

6. Transitions $^{15}\text{N}(1/2^-) \rightarrow ^{15}\text{C}$. In ^{15}C there are two bound states: the ground ($1/2^+$) and excited ($5/2^+$) ones. The calculated capture rate is given in Table VIII. The capture rate is sensitive to the parameter of intermediate coupling (in this case such

TABLE VIII
The capture rate in $\mu^- + ^{15}\text{N} - ^{15}\text{C} + \nu$ (in 10^3 sec^{-1})

	$V_0 = -50 \text{ MeV}$			$V_0 = -40 \text{ MeV}$			$V_0 = 0$		
g_P/g_A	A_+	A_-/A_+	A_{stat}	A_+	A_-/A_+	A_{stat}	A_+	A_-/A_+	A_{stat}
	Ground state ($1/2^+$) of ^{15}C								
0	3.82	0.100	2.96	4.11	0.122	3.21	5.53	0.316	4.59
7	3.32	0.046	2.53	3.59	0.061	2.75	4.90	0.220	3.94
12	3.04	0.017	2.29	3.28	0.027	2.49	4.54	0.153	3.58
	Level at ($5/2^+$) $E = 0.75 \text{ MeV}$								
0	5.76	0.012	4.32	7.38	0.002	5.54	15.82	0.005	11.89
7	4.60	0.051	3.50	5.88	0.056	4.49	12.57	0.075	9.66
12	3.96	0.150	3.12	5.07	0.160	4.00	10.81	0.195	8.64

a parameter is the amplitude V_0 of the residual interaction between nucleons in nuclei). The ratio A_-/A_+ is less sensitive to V_0 for the transition to the $5/2^+$ level, as is seen from Fig. 2c, where curve 1 is the result in the $j-j$ coupling scheme ($V_0 = 0$), and curve 2 in intermediate coupling with $V_0 = -50$ MeV. However, bearing in mind the situation [1] in partial transitions in ^{16}O , we should not overestimate the capability of the model. Some additional calculations are thus needed.

5. Conclusion

The transitions discussed in the present paper together with transitions in ^3He , ^6Li , ^{11}Be , ^{12}C and ^{16}O (which have been extensively discussed by many authors) exhaust practically all the transitions in the region of light nuclei, which appear to be of interest. Unfortunately almost in all cases one cannot avoid the model description of the nuclear structure when analysing the characteristics of the partial transitions. Some new possibilities arise when one investigates the ratio of the capture rates from two states of the hyperfine doublet of mesoatoms. In some cases, in transitions to the level $E = 5.96$ MeV in ^{10}Be and to $E = 7.01$ MeV in ^{14}C the ratio is practically independent of the parameter k of the shell model. One can hope that such cases are favourable for obtaining some new information on the muon-nucleon coupling constants.

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