ON THE ANGULAR DISTRIBUTION OF PRODUCTS OF DIRECT NUCLEAR REACTION $A(a, b)B$

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In the paper formulae for the angular distribution and its asymmetry are presented. The expansion of the angular distribution and its asymmetry into Legendre polynomials provides information about the contribution of the selected partial amplitudes to the asymmetry of the angular distribution. The restriction on the asymmetry coefficient $A(\theta)$ which follows from isospin conservation in the reaction $A(a, b)B$ is also presented.

Let us consider the direct transfer reaction $A(a, b)B$. When the spin-orbit terms in entrance and exit channels are neglected, the angular distribution of the momenta of the particle b is described by the formula [1],

$$\frac{d\sigma}{d\omega} = \frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{2J_B+1}{(2J_A+1)(2S_a+1)}\sum_{lsjm}|\beta_{sj}^{lm}|^2. \tag{1}$$

Here, k_b and k_a denote the momenta of particles b and a, respectively; J_B, J_A and S_a the spins of nuclei B, A and a; l and m the orbital momentum of the transferred particle and its projection on the axis of quantization, and s and j denote the spin of the transferred particle and its total angular momentum.

In the standard analysis of experimental data the angular distribution (1) is computed by means of numerical Code (Julie, DWUCK). The computed reduced amplitude β_{sj}^{lm} is given by the formula.

$$\beta_{sj}^{lm} = 4\pi\sum_{\substack{L_a,L_b\\M_aM_b}}\langle L_blM_b-m|L_aM_a\rangle\times$$

$$\times\langle L_bl00|L_a0\rangle\hat{L}_b\hat{L}_a^{-1}i^{L_a-L_b-l}Y_{M_b}^{L_b}(\hat{k}_b)\bar{Y}_{m-M_b}^{L_a}(\hat{k}_a)f_{L_b,L_a}^{lsj}. \tag{2}$$

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In this formula the partial amplitudes $f^{lsj}_{L_b,L_a}$ are given by the suitable integrals:

$$f^{lsj}_{L_b,L_a} = \frac{(4\pi)^{\frac{1}{2}}}{k_a k_b} \frac{B}{A} \int \chi_{L_b}\left(k_b, \frac{A}{B} r\right) G_{lsj}(r) \chi_{L_a}(k_a r) dr \tag{3}$$

where A and B denote the masses of the nuclei A and B.

Formula (3) as written, is known as the zero range approximation [1]. χ_{L_a} and χ_{L_b} are the partial waves of the distorted waves in entrance and exit channels. $G_{lsj}(r)$ describes the bound state of paritcle $(a-b)$ in the nucleus A when $A(a, b)B$ is a stripping reaction.

The computed angular distribution itself cannot give any information about the contribution of the selected partial amplitudes, but its expansion into Legendre polynomials can. It is the aim of this paper to find the formulae which describe the expansion of $\frac{d\sigma}{d\omega}$ into Legendre polynomials.

The coordinate system in which we will describe the reaction is shown in the Fig. 1. The momentum of the particle a, k_a, is parallel to the z-axis, and the y-axis is perpendicular to the reaction plane. In this coordinate system we obtain [2]

$$Y^{L_a}_{m-M_b}(\hat{k}_a) = \hat{L}_a(4\pi)^{-1/2}\delta_{m-M_b,0}. \tag{4}$$

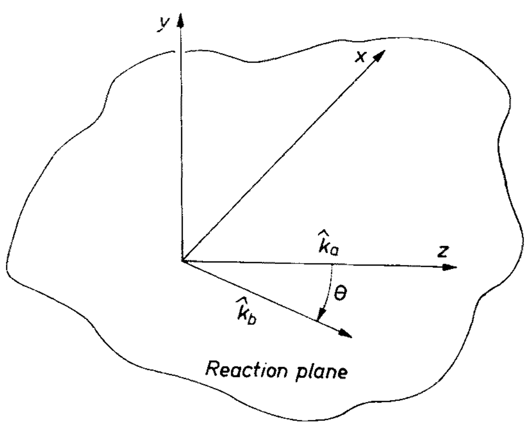

Fig. 1. Coordinate system for the reaction $A(a, b)B$

The square of the reduced amplitude can be written as follows:

$$|\beta^{lm}_{sj}|^2 = (4\pi)^{1/2} \sum_{\substack{L_a L_b \\ L'_a L'_b}} \sum_{\Lambda} \langle L_b l m - m | L_a 0\rangle \times$$

$$\times \langle L'_b l m - m | L'_a 0\rangle \langle L_b l 0 0 | L_a 0\rangle \langle L'_b l 0 0 | L'_a 0\rangle \times$$

$$\times \hat{L}^2_b (\hat{L}'_b)^2 i^{L_a - L_b - l} \hat{\Lambda}^{-1} (-1)^m (\overline{i^{L'_a - L'_b - l}}) f^{lsj}_{L_b,L_a} \bar{f}^{lsj}_{L'_b L'_a} Y^{\Lambda}_0(\hat{k}_b). \tag{5}$$

Formula (5) was obtained by adopting the well-known formula for spherical harmonics [2],

$$Y_m^{L_b}(\hat{k}_b)\overline{Y}_m^{L_b}(\hat{k}_b) = (-1)^m \sum_{\Lambda} \hat{L}_b\hat{L}'_b\hat{\Lambda}^{-1}(4\pi)^{-1/2} \times$$
$$\times \langle L_b L'_b m-m|\Lambda 0\rangle \langle L_b L'_b 00|\Lambda 0\rangle Y_0^{\Lambda}(\hat{k}_b). \qquad (6)$$

The sum over index m can be written as

$$\sum_m (-1)^m \langle L_b l m-m|L_a 0\rangle \langle L'_b l m-m|L'_a 0\rangle \times$$
$$\times \langle L_b L'_b m-m|\Lambda 0\rangle = (-1)^{L_b+L'_b+l-L'_a}\hat{L}_a\hat{l}^{-1} \times$$
$$\times \sum_m \langle L_a L_b 0-m|l-m\rangle \langle l L'_b -mm|L'_a 0\rangle \times$$
$$\times \langle L_b L'_b m-m|\Lambda 0\rangle. \qquad (7)$$

In order to obtain formula (7), the following symmetry property of the Clebsch-Gordan coefficients was used:

$$\langle L_b l m-m|L_a 0\rangle = (-1)^{L_b-m}\hat{L}_a\hat{l}^{-1} \times$$
$$\times \langle L_a L_b 0-m|l-m\rangle. \qquad (8)$$

Bearing in mind the relationship between the Glebsch-Gordan coefficients and the Racah coefficients, formula (5) can be rewritten into [2].

$$|\beta_{sj}^{lm}|^2 = (4\pi)^{1/2} \sum_{\substack{L_a L_b\\ L'_a L'_b}} \sum_{\Lambda} i^{L_a-L_b-l}(\overline{i^{L'_a-L'_b-l}})\hat{L}_b^2(\hat{L}'_b)^2 \times$$
$$\times \hat{\Lambda}^{-1}\hat{L}_a\hat{L}'_a(-1)^l W(L_a L_b L'_a L'_b, l\Lambda) \langle L'_a L_a 00|\Lambda 0\rangle \times$$
$$\times \langle L_b l 00|L_a 0\rangle \cdot \langle L'_b l 00|L'_a 0\rangle \langle L_b L'_b 00|\Lambda 0\rangle \times$$
$$\times f_{L_b,L_a}^{lsj}\overline{f}_{L'_b,L'_a}^{lsj} Y_0^{\Lambda}(\hat{k}_b) = (4\pi)^{1/2} \sum_{\substack{L_a L_b\\ L'_a L'_b}} \sum_{\Lambda,\Gamma} i^{L_a-L_b-l}(\overline{i^{L'_a-L'_b-l}}) \times$$
$$\times \hat{L}_b^2(\hat{L}'_b)^2\hat{L}_a^2(\hat{L}'_a)^2\hat{\Lambda}^{-1}(-1)^{L'_b-L'_a+L_b+L_a} W(L_a L_b L'_a L'_b, l\Lambda) \times$$
$$\times \langle L'_a L_a 00|\Lambda 0\rangle W(L_a L_b L'_a L'_b, l\Gamma) \langle L_b L'_b 00|\Gamma 0\rangle \times$$
$$\times \langle L'_a L_a 00|\Gamma 0\rangle \langle L_b L'_b 00|\Lambda 0\rangle f_{L_b L_a}^{lsj}\overline{f}_{L'_b L'_a}^{lsj} Y_0^{\Lambda}(\hat{k}_b). \qquad (9)$$

Taking into account the relationship between the spherical harmonics and Legendre polynomials [2]

$$Y_0^{\Lambda}(\hat{k}_b) = \hat{\Lambda}(4\pi)^{-1/2}P_{\Lambda}(\hat{k}_b) \qquad (10)$$

and that the $L_b+L_b'-\Lambda$ is an even number, we obtain for the sum of the squares of the reduced amplitudes

$$\sum_m |\beta_{sj}^{lm}|^2 = \sum_{\substack{L_aL_b \\ L'_aL'_b}} \sum_{\Lambda,\Gamma} i^{L_a-L_b-l}(\overline{i^{L'_a-L'_b-l}}) \times$$

$$\times \hat{L}_a^2(\hat{L}_a')^2\hat{L}_b^2(\hat{L}_b')^2 W(L_aL_bL_a'L_b', l\Lambda) \times$$

$$\times W(L_aL_bL_a'L_b', l\Gamma) \langle L_bL_b'00|\Gamma 0\rangle \langle L_a'L_a00|\Gamma 0\rangle \times$$

$$\times \langle L_bL_b'00|\Lambda 0\rangle \langle L_a'L_a00|\Lambda 0\rangle f_{L_b,L_a}^{lsj} \bar{f}_{L'_b,L'_a}^{lsj} P_\Lambda(\hat{k}_b). \tag{11}$$

Formula (11) can be written in a more compact from when we use the so-called $\bar{Z}$-coefficients [3],

$$\bar{Z}(a, b, c\, d; ef) = \hat{a}\hat{b}\hat{c}\hat{d}\ \langle ac00|f0\rangle\ W(abcd, ef). \tag{12}$$

Bearing in mind formulae (11) and (12), the angular distribution for the momenta of the particle b can be written as follows:

$$\frac{d\sigma}{d\omega} = \frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{2J_B+1}{(2J_A+1)(2S_a+1)} \times$$

$$\times \sum_\Lambda a_\Lambda P_\Lambda(\hat{k}_b)$$

$$a_\Lambda = \sum_{lsj}\sum_{L_bL'_b}\sum_{L_aL'_a}\sum_{\Gamma}(\overline{i^{L'_a-L'_b-l}})i^{L_a-L_b-l}\langle L_bL_b'00|\Gamma 0\rangle \times$$

$$\times \langle L_bL_b'00|\Lambda 0\rangle \bar{Z}(L_aL_bL_a'L_b', l\Lambda)\bar{Z}(L_aL_bL_a'L_b', l\Gamma) f_{L_bL_a}^{lsj}\bar{f}_{L'_bL'_a}^{lsj}. \tag{13}$$

We define the asymmetry of the angular distribution $A(\theta)$ as

$$A(\theta) = \frac{1}{2}\left\{\left[\frac{d\sigma}{d\omega}\right]_\theta - \left[\frac{d\sigma}{d\omega}\right]_{\pi-\theta}\right\}. \tag{14}$$

Then, formulae (13) and (14) give

$$A(\theta) = \frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{2J_B+1}{(2J_A+1)(2S_a+1)}\sum_{\Lambda_{\text{odd}}} a_\Lambda P_\Lambda(\hat{k}_b) \tag{15}$$

with

$$a_\Lambda = \sum_{lsj}\sum_{L_bL'_b}\sum_{L_aL'_a}\sum_{\Gamma}(\overline{i^{L'_a-L'_b-l}})i^{L_a-L_b-l} \times$$

$$\times \bar{Z}(L_aL_bL_a'L_b', l\Lambda)\bar{Z}(L_aL_bL_a'L_b', l\Gamma) \times$$

$$\times \langle L_bL_b'00|\Lambda 0\rangle \langle L_bL_b'00|\Gamma 0\rangle f_{L_bL_a}^{lsj}\bar{f}_{L'_bL'_a}^{lsj}.$$

In formula (15) the sums over $L_aL'_a$, $L_bL'_b$ obey the following restrictions (which follow from symmetry properties of the Clebsch-Gordan coefficients)

$$\begin{aligned} L_b+L'_b &= \text{odd number} \\ L_a+L'_a &= \text{odd number} \\ \Gamma &= \text{odd number.} \end{aligned} \tag{16}$$

When there is no transfer of angular momentum $l = 0$, during the $A(a, b)B$ reaction formula (13) can be written in the simpler form

$$\frac{d\sigma}{d\omega} = \frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{(2J_B+1)}{(2J_A+1)(2S_a+1)}\sum_{\Lambda} a_\Lambda P_\Lambda(\hat{k}_b)$$

with

$$a_\Lambda = \sum_{L_a}\sum_{L_a'} \hat{L}_a^2(\hat{L}'_a)^2\langle L_aL'_a00|\Lambda 0\rangle f^j_{L_aL_a}\bar{f}^j_{L'_aL'_a}$$

$$f_{L_aL_a} = \frac{(4\pi)^{1/2}}{k_ak_b}\frac{B}{A}\int \chi_{L_a}(r)G_{0jj}(r)\chi_{L_a}(r)dr \tag{17}$$

$$f_{L'_aL'_a} = \frac{(4\pi)^{1/2}}{k_ak_b}\frac{B}{A}\int \chi_{L'_a}(r)G_{0jj}(r)\chi_{L'_a}(r)dr.$$

In many experimental papers on direct nuclear reactions the asymmetry of angular distributions is treated as a fundamental property of these reactions. It can be shown, however, that under special circumstances the angular distributions of the products of direct two-body reactions must be symmetric. Let us consider a reaction $A(a, b)B$ which proceeds with isospin conservation. Suppose that the particles B and b belong to the same isospin multiplet I_1. Let one of the particles, A or a, have isospin equal zero. Then, if isospin is conserved in the reaction $A(a, b)B$, the following selection rule must be fulfilled [4]:

$$I+2I_1+L_b+\sigma = \text{odd.} \tag{18}$$

In formula (18) I is the total isospin and σ is the total spin of particles B and b. From the selection rule (18) now find that $L_b+\sigma$ is odd(even) when $I+2I_1$ is even (odd). This means that when selection rule (18) is fulfilled, then (16) cannot be satisfied and a_Λ (odd) is equal zero. The selection rule (18) can also be used to verify isospin conservation in nuclear reactions [5].

The obtained formuale (13) and (14) give the expansion of the angular distribution and asymmetry of this angular distribution into Legendre polynomials. In many analyses of the mechanism of the nuclear reactions the expansion of the experimental angular distribution into Legendre polynomials is included. A comparison of experimentally obtained a_Λ with their theoretical values may decide which direct mechanism dominates in the reaction $A(a, b)B$. All information about the selected mechanism of the reaction $A(a, b)B$ is included in the integrals (3).

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