ABFST MODEL WITH VENEZIANO AMPLITUDES FOR $\pi\pi$ AND πp SCATTERING APPLIED TO THE REACTION $\pi^+ p \rightarrow 3\pi^+ 2\pi^- p$ AT 8 GeV/c

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The ABFST model with dual amplitudes taken to describe the off-mass shell $\pi\pi$ and πp scattering is applied to the reaction $\pi^+ p \rightarrow 3\pi^+ 2\pi^- p$ at 8 GeV/c. Previously the same process was studied in the framework of the ABFST model with $\pi\pi$ and πp phase shifts and the Dürr-Pilkuhn off-shell extrapolation. However this extrapolation is known to disagree with the existing data on the off-mass shell $\pi\pi$ scattering. By comparing the results of both calculations we study the dependence of the ABFST model description on a particular method of the off-mass shell extrapolation. We also compare the predictions of our model with experimental data using the Van Hove longitudinal phase-space method.

1. Introduction

Recently a revival of the early ABFST multiperipheral model [1] based on pion exchange mechanism (Fig. 1) is observed. The multiparticle amplitude in the ABFST model contains several two particles $\rightarrow$ two particles amplitudes which should provide a realistic description of $\pi\pi$ and πp scattering, particularly in low and intermediate energy region.

In the recent paper [3] the ABFST model was applied to the analysis of the channel $\pi^+ p \rightarrow 3\pi^+ 2\pi^- p$ at 8 GeV/c [2]. In Ref. [3] available $\pi\pi$ and πp phase shifts were

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used to describe $\pi\pi$ and πp vertices and off-shell corrections were introduced according to the method of Dürr and Pilkuhn [4].

It is worthwhile to notice the following points of the calculation presented in Ref. [3]:

1) predicted two-body invariant mass distributions are in good agreement with experimetal data,

2) the shapes of the longitudinal momentum distributions are in fair agreement with experimental results but the difference $\langle p^L_{\pi^+}\rangle - \langle p^L_{\pi^-}\rangle$ has a sign opposite to that found in the experiment,

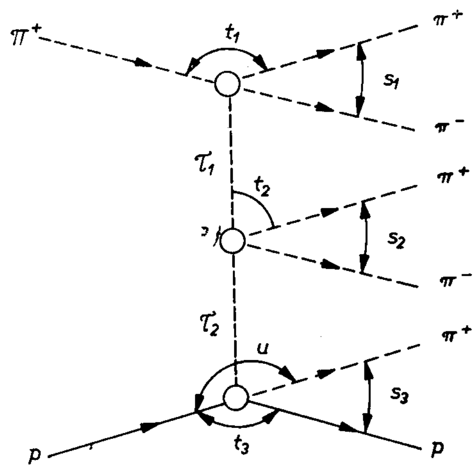

Fig. 1. Illustration of kinematical variables used in this paper

3) there is an important interference between different graphs obtained by Bose--Einstein symmetrization,

4) GGLP effect is well described as the result of Bose-Einstein symmetrization,

5) the total reaction cross-section obtained from the double peripheral graphs with pion exchanges is by factor of five too low.

The least certain point of the ABFST model calculation presented in Ref. [3] is the assumption of the Dürr-Pilkuhn off-mass-shell continuation of the elastic $\pi\pi$ and πp scattering amplitude. It is known that in the region of low $\pi\pi$ invariant mass the off-mass-shell $\pi\pi$ amplitude obtained with the Dürr-Pilkuhn method is not correct. This is mainly due to the fact that the Dürr-Pilkuhn parametrization does not predict any off-shell dependence for the S-wave, contrary to the experimental facts.

Thus we believe that it is interesting to investigate the problem to what extent conclusions (1)–(5) depend on the particular method of the off-mass-shell continuation chosen in Ref. [3]. In order to shed some light on this point we discuss again the reaction $\pi^+p \to 3\pi^+2\pi^-p$ at 8 GeV/c in the framework of the ABFST model but now with Veneziano dual functions assumed for the $\pi\pi$ and πp amplitudes. The off-mass-shell dependence of the Veneziano amplitude is different from the dependence introduced by Dürr-Pilkuhn form-factors and has been verified by experimental data for $\pi\pi$ scattering [5, 6].

The comparison of our results with experimental data is performed along the same lines as in Ref. [3]. In addition we shall also present a Van Hove longitudinal phase-space plot. Such an analysis of experimental data has been recently attempted for the considered reaction [7].

2. *ABFST model with dual* $2 \to 2$ *amplitudes*

Our description of the reaction $\pi^+p \to 3\pi^+2\pi^-p$ in the framework of the ABFST model is based on several simplifying assumptions. As in Ref. [3] we assume that only pions are exchanged between the final particles grouped in pairs. This strong assumption reflects our inability to include into the ABFST formalism other possible contributions like vector meson or baryon exchange. Furthermore we neglect in our calculation all the double one-pion exchange graphs except those with neutral pairs of pions (Fig. 1). It seems that such a reduction of the number of graphs can at most lead to small modifications of the final results. This is indeed true in the case of the previous calculations [3] and follows from the fact that $\pi^+\pi^-$ and π^+p cross-sections dominate over the $\pi^\pm\pi^\pm$ and π^-p cross-sections in the low energy region. Finally we neglect the spin structure of the π^+p scattering amplitude in the ABFST amplitude for the graph shown in Fig. 1. We believe that this simplification is not essential as long as only general features of the reaction are studied.

Our amplitude for the graph shown in Fig. 1 is as follows:

$$M = A_{\pi\pi}(s_1, t_1) \frac{e^{A\tau_1}}{\tau_1 - \mu^2} A_{\pi\pi}(s_2, t_2) \frac{e^{B\tau_2}}{\tau_2 - \mu^2} A_{\pi p}(s_3, t_3, u). \tag{1}$$

The functions $A_{\pi\pi}$ and $A_{\pi p}$ are supposed to describe the off-mass-shell $\pi^+\pi^-$ and π^+p scattering. For the $\pi^+\pi^-$ amplitude we take the dual Lovelace-Veneziano form [8]:

$$A_{\pi\pi}(s, t) = \frac{\Gamma(1-\alpha_\varrho(s))\Gamma(1-\alpha_\varrho(t))}{\Gamma(1-\alpha_\varrho(t)-\alpha_\varrho(s))}. \tag{2}$$

Amplitude (2) with linear Regge trajectories predicts zero total widths for resonances. Therefore, it has to be modified by hand in order to give finite total widths to resonances. It was pointed our in Ref. [5] that amplitude (2) describes correctly experimental data only if such a procedure conserves unitarity at least in the sense $\Gamma_{el} \leqslant \Gamma_{tot}$ [1]. In the reaction studied, the $\pi^+\pi^-$ system has usually a low invariant mass (Fig. 6). Therefore we can approximate amplitude (2) by the first three terms (corresponding to the ϱ, f and g mesons and their daughters) of its pole expansion as follows:

$$A_{\pi\pi}(s, t) = (1-\alpha_\varrho(s)-\alpha_\varrho(t)) \sum_{n=0}^{2} \binom{\alpha_\varrho(t)+n-1}{n} \frac{1}{n+1-\alpha_\varrho(s)}. \tag{3}$$

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[1] The ratio of elastic widths obtained from (2) with $\alpha_\varrho(s) = 0.483 + 0.885\, s$ is the following: $\Gamma_\varepsilon : \Gamma_\varrho = 9:2$; $\Gamma_{\varepsilon'} : \Gamma_{\varrho'} : \Gamma_f = 0:1:1$.

Now we can easily "unitarize" our amplitude by just replacing the sum of the poles (3) by the sum of the Breit-Wigner terms. Different total widths can be given in this way to parent and daughter resonances as required by the relation $\Gamma_{tot} \geqslant \Gamma_{el}$. To be specific we parametrize the total widths as follows: for parent resonances:

$$M\Gamma = 0.13\sqrt{s-4\mu^2}/0.885 \tag{4}$$

and for daughter resonances:

$$M\Gamma = 0.585\sqrt{s-4\mu^2}/0.885. \tag{5}$$

Expression (4) gives the observed width of the ϱ and f resonances whereas from (5) we get a purely elastic ε resonance and a strongly inelastic ϱ' resonance. Such an amplitude (apart from the possible multiplicative factors exponential in t) is used to describe the off-mass--shell $\pi\pi$ scattering with one and two virtual pions (upper and central vertex in Fig. 1, respectively). Fig. 2 illustrates the asymmetry of the $\pi\pi$ angular distribution predicted

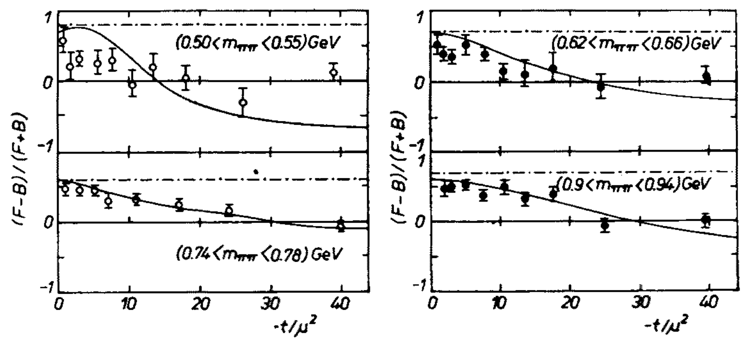

Fig. 2. Forward-backward asymmetry, $(F-B)/(F+B)$, in $\pi^+\pi^-$ c. m. system as a function of virtual pion mass squared for different values of $\pi^+\pi^-$ effective mass. Solid curves show the predictions of the dual $\pi\pi$ amplitude (3). Dashed-dotted lines show the predictions of the Dürr-Pilkuhn parametrization [13] with phase-shifts for on-shell scattering taken from Ref. [14]. The experimental data are taken from Ref. [6]

by our amplitude as a function of the virtual pion mass squared for several values of the $\pi\pi$ invariant mass. We observe an excellent agreement of our predictions with the experimental data obtained from the analysis of the reaction $\pi^-p \to n\pi^+\pi^-$. The shape of the angular distribution predicted by amplitude (3) gradually changes from forward-peaked on the mass shell to backward-peaked for large negative values of the virtual pion mass squared. This is in contrast to the predictions of the Dürr-Pilkuhn off-mass-shell continuation which are also shown in Fig. 2.

The amplitude $A_{\pi p}$ for the off-mass-shell π^+p scattering is more ambiguous. We do not know any experimental data similar to those on $\pi\pi$ scattering which could verify theoretical parametrizations of the off-mass-shell π^+p scattering. Nevertheless the experience with the $\pi\pi$ scattering makes the application of Dürr-Pilkuhn form-factors doubtful also in this case. Therefore we follow here our approach to the $\pi\pi$ reaction and use a dual Veneziano amplitude for the off-mass-shell π^+p scattering. Such an approach is now based on less firm grounds, also because in the low energy region the on-mass-shell πp dual amplitude with all the desired properties has not yet been proposed. Therefore we construct a simple approximate form of the Veneziano amplitude for π^+p scattering and study its off-mass-shell properties for strongly virtual pions. For simplicity we neglect the spin structure of the π^+p amplitude and furthermore we assume the dominance of the $\Delta(3/2^+)$ and $N(5/2^-)$ exchange degenerate trajectories[2]. In the s and u channels (for notation see Fig. 1) we can then write our amplitude as follows:

$$A_{\pi^+p}(s, t, u) = \frac{\Gamma(1-\alpha_\varrho(t))\Gamma(3/2-\alpha_\Delta(s))}{\Gamma(3/2-\alpha_\varrho(t)-\alpha_\Delta(s))} - \frac{\Gamma(3/2-\alpha_\Delta(s))\Gamma(3/2-\alpha_\Delta(u))}{\Gamma(2-\alpha_\Delta(s)-\alpha_\Delta(u))}. \quad (6)$$

The relative sign of the Veneziano terms is chosen in such a way that the parent resonances with spin $J = 5/2, 9/2, \ldots$ and isospin $T = 3/2$ do not appear in the amplitude. Simple amplitude (6) built only of the leading Veneziano terms contains some daughter resonances which are unwanted in the on-mass-shell scattering. We assume, however, that amplitude (6) has the correct partial wave content in the region of large negative mass squared, τ, of the virtual pion[3]. Some of our final results seem to support the description of the off-mass-shell π^+p scattering in terms of amplitude (6).

Finally, in amplitude (1) the exponential factors $e^{A\tau_1}$ and $e^{B\tau_2}$ appear in addition to pion propagators. These factors are necessary to cancel the blowing-up of Veneziano terms for large negative mass-squared of virtual pions. The exponents A and B are free parameters within our model and have to be determined from experimental data.

The full amplitude for the process $\pi^+p \to 3\pi^+2\pi^-p$ is obtained from amplitude (1) by symmetrization in the variables of identical pions. We obtain in this way a sum of 12 terms each of them of the form (1). The calculation has been performed with all interference effects taken into account but we have also studied separately the contribution of the interference terms.

3. *Results*

Numerical calculation have been performed by the Monte Carlo method with the help of the FOWL program. The histograms obtained have been smoothed by hand and then compared with experimental distributions.

The free parameters A and B were not determined by a fitting procedure but instead they were adjusted to give reasonable results for single particle distributions and two--body mass distributions. We tried several sets of parameters A and B and observed some regularities in the dependence of final results on the values of these parameters. The acceptable values of parameter B have to lie in a small interval, between 1.1 GeV^{-2} and 1.5 GeV^{-2},

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[2] Exchange degeneracy of these two trajectories was used in Ref. [9] in the construction of a πN Veneziano amplitude.

[3] The predicted asymmetry of π^+p angular distribution as a function of τ is shown in Fig. 3c, d for several values of invariant mass. In the calculation the imaginary part of trajectory was introduced according to the formula: $\alpha_\Delta(s) = 0.13+0.9\,s+i0.22\,\sqrt{s-1.16}$.

otherwise the two-body mass distributions predicted by the model clearly disagree with experimental data[4].

The results of our calculation are less sensitive to the value of parameter A. Although we obtain reasonable distributions for values of A, say, from 1.8 GeV^{-2} to 3.5 GeV^{-2},

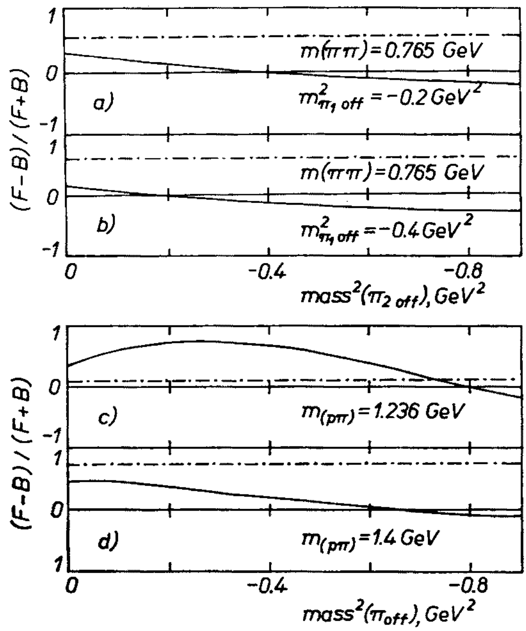

Fig. 3. Forward-backward asymmetry, $(F-B)/(F+B)$, as a function of virtual pion mass squared: a, b): in $\pi^+\pi^-$ c. m. system for $\pi^+\pi^-$ scattering of two virtual pions, for $m_{\pi\pi} = 0.765$ GeV; c, d): in π^+p c. m. system. The solid curves show the predictions of amplitude (3) for a, b) and amplitude (6) for c, d). Dashed-dotted lines show the predictions of $\pi^+\pi^-$ and π^+p amplitudes with Dürr-Pilkuhn formfactors [13]

the best agreement of our results with experimental data is observed when $A \approx 2B$. The large value of A as compared to B which is required to describe correctly the experimental data means that four final pions have a tendency to be produced with small relative momenta. This is also reflected in the mean values of four-momentum transfers-squared τ_1 and τ_2 which can be easily calculated with our amplitude if interference terms are neglected. For instance for $A = 2.6$ GeV^{-2} and $B = 1.3$ GeV^{-2} we obtain $\tau_1 = -0.35$ GeV^2 and $\tau_2 = -0.7$ GeV^2.[5]

[4] It is interesting to notice that when $B \approx 1.3$ GeV^{-2}, the dependence of the central $\pi\pi$ vertex and of the πp vertex on the virtual pion mass τ_2 are cancelled by the exponential function $e^{B\tau_2}$ (for an average value of τ_1). The only τ_2 dependence which is left in our amplitude (after performing the integration over angular variables) arises from the pion propagator.

[5] The fact that pions tend to be grouped together has already been found in the so-called $F(t)$ model. In this model one assumes that the matrix element depends only on the four momentum transfer from the initial to final nucleon and that the pion emission is described by the relativistic phase space. It was found that the phenomenological $F(t)$ model reproduces the main features of particle emission in the considered reaction [10].

One can raise an objection against trying to obtain a detailed agreement of our results with the experimental data. The reason is, that as in Ref. [3], our amplitude for the values of parameters A and B discussed above gives too small a value for the cross-section[6]. One may argue that if the ABFST model describes only a part of the data, then it should not be treated too seriously in other possible tests. The invalidity of ABFST model can follow from the absence of other important contributions such as *e. g.* multibaryon exchange. Also the approximation of the amplitude by the nearby singularities may not be correct for the multiparticle reaction, which is characterized by large mean values of momentum transfer. We think that an investigation of the energy dependence of the cross-section could shed some light on this problem. In spite of the small value of the reaction cross-section we present here the results obtained with the "best" values of parameters A and B namely, $A = 2.6\ \mathrm{GeV}^{-2}$ and $B = 1.3\ \mathrm{GeV}^{-2}$. We hope that the terms absent in the ABFST approach may show a similar behaviour. In any case our results can serve as a basis for a discussion of the role of the off-mass-shell continuation in the framework of the ABFST model.

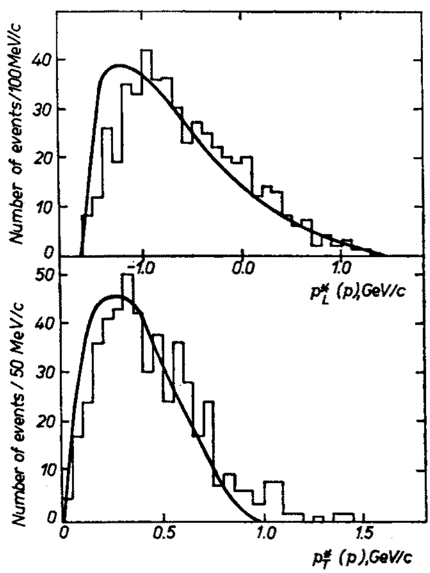

Fig. 4. Experimental proton c. m. longitudinal momentum and transverse momentum distributions compared with the predictions of the model

We start the comparison between our results and experimental data with the discussion of single particle distributions. Proton longitudinal and transverse momenta distributions are shown in Fig. 4. We observe, as in Ref. [3], a too strong backward peaking of the proton. It is also reflected in the average value of the computed proton longitudinal momen-

[6] We estimate the reaction cross-section to be (5—15) % of the experimental cross-section. The number is not precise since the normalization of the on-mass-shell π^+p amplitude is not too meaningful due to the oversimplified form of this amplitude.

tum which is too low by 100 MeV (Table I). Of course the proton longitudinal momentum distribution depends on the value of A and B. However, we were not able to find values of A and B which would give a better distribution for the proton longitudinal momentum without drastic undesired changes in other distributions. We feel therefore that we can

TABLE I

The average values of the longitudinal momenta

Particle	$\langle p_L^* \rangle$, GeV/c	
	Model	Experiment
p	−0.65	-0.57 ± 0.03
π^+	0.07	0.10 ± 0.01
π^-	0.22	0.13 ± 0.01

repeat the comment from Ref. [3] that the observed disagreement is probably due to the neglected contributions such as multibaryon exchange.

In Fig. 5 we present the longitudinal and transverse momentum distributions of pions and the average values $\langle p_L \rangle$ are collected in Table I. We observe that the difference between π^+ and π^- is qualitatively reproduced by the model, namely, the longitudinal

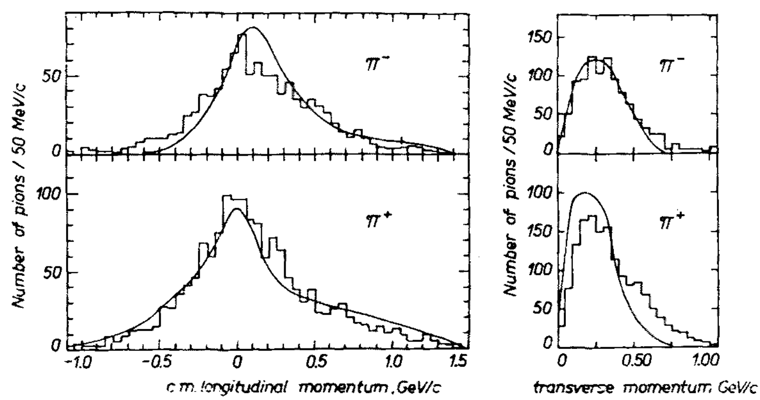

Fig. 5. Longitudinal and transverse momentum distributions of pions compared with the predictions of the model

momenta of π^- are predicted to be higher than those of π^+. This is in contrast to the results obtained in Ref. [3]. It follows from the already discussed fact that in our model the asymmetry of $\pi\pi$ angular distribution becomes negative for large negative mass squared of virtual pion (Fig. 2, 3a, b).

Quantitatively, the difference between the average longitudinal momenta of π^- and π^+ predicted by the model is too large by 120 MeV which is mainly due to the momentum of π^- being too high. It seems to us that the behaviour of π^- which follows from our model can mean that the $\pi\pi$ scattering with two virtual pions (central vertex in Fig. 1) is only approximately described by the dual amplitude (3). Amplitude (3) was checked by comparison with independent experimental data on scattering with one virtual pion only[7].

Our previous discussion is also supported by the two-body mass distributions which are shown in Fig. 6 and Fig. 7. We observe a serious discrepancy between the calculated

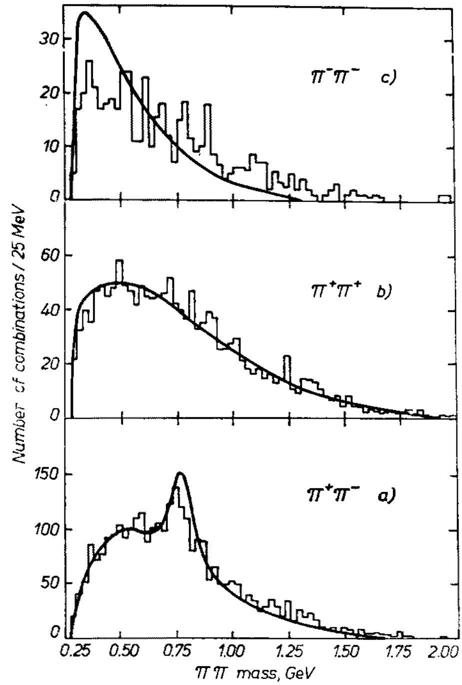

Fig. 6. Experimental mass distributions compared with the predictions of the model for: a) $\pi^+\pi^-$, b) $\pi^+\pi^+$, c) $\pi^-\pi^-$

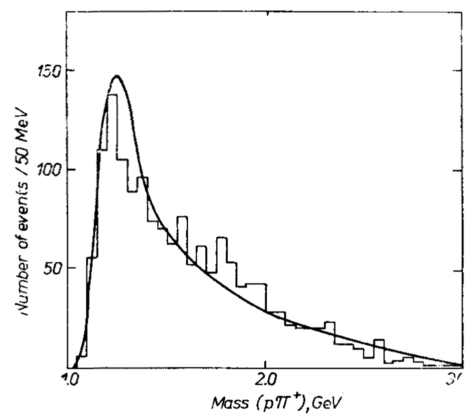

Fig. 7. Experimental mass distribution of π^+p compared with the predictions of the model

[7] Of course if the description of the central vertex has some uncorrect features, they are reflected mainly on the π^- distributions, since there are only two π^- mesons in the final state.

and experimental distributions of the $\pi^-\pi^-$ mass which is in our model too strongly peaked at low values. This tendency is understood if we notice that for average values of momentum transfers τ_1 and τ_2 the $\pi\pi$ asymmetry in the upper vertex is small positive whereas it is negative in the central vertex. Therefore the two mesons are preferably produced with small relative momenta, since the pions from the upper vertex move more forward than the pions from the central vertex. The remaining two-body mass distributions predicted by the model are in good agreement with experimental data. In particular we describe correctly the resonance production both in the meson-meson and meson-baryon systems. The π^+p mass distribution, which is very well reproduced by the model, supports our amplitude (6) as a reasonable description of the off-mass-shell π^+p scattering.

Another distribution very sensitive to the assumed parametrization of the off-mass--shell π^+p scattering (particularly to the asymmetry of the π^+p angular distribution predicted for the off-mass-shell scattering) is the distribution of the decay (Jackson) angle of the

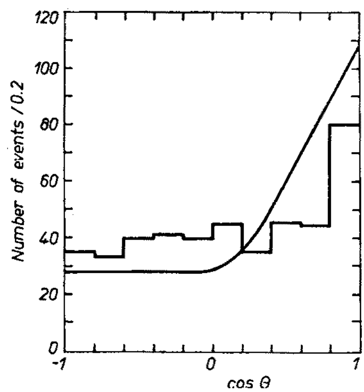

Fig. 8. Experimental angular distribution of the proton in the π^+p system (Jackson frame) in the mass region (1.15—1.35) GeV compared with the predictions of the model

π^+p system. For $\Delta^{++}(1236)$ mass band this distribution is shown in Fig. 8 together with the experimental histogram.

An interesting phenomenon observed in many pion systems is the angular correlation of pion pairs (GGLP effect [11]). The opening angles between momentum vectors of like--charge pions have a tendency to be smaller than those of unlike charge pions. The effect is usually expressed in terms of the γ coefficients which are defined as the ratios of the number of pions emitted with opening angles larger than 90° to the number of pairs emitted with angles smaller than 90°:

$$\gamma = \frac{N(\theta > 90^\circ)}{N(\theta < 90^\circ)}.$$

The observed angular correlation of pion pairs means that the following inequality of the γ coefficients holds:

$$\gamma^{\text{like}} < \gamma^{\text{unlike}}.$$

It has also been observed that the effect is stronger in the c. m. system of pions than in the overall c. m. system. Furthermore, the difference between γ^{like} and γ^{unlike} depends on the mass of the pionic system and decreases with the increase of pionic mass. The factors responsible for the GGLP effect have been discussed in many papers. Originally it was suggested [11] that the effect is due to the Bose-Einstein symmetrization, which tends to enhance the number of $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs with small opening angles. Later works tried to explain the effect as a result of different $\pi^+\pi^-$ and $\pi^\pm\pi^\pm$ interaction, in particular, the possible role of resonances in the observed effect was widely discussed.

In Ref. [3] the GGLP effect was discussed in connection with the ABFST model. This discussion supports the idea that the GGLP effect is due to the Bose-Einstein symmetrization. Indeed, the inequality $\gamma^{\text{like}} < \gamma^{\text{unlike}}$ was obtained in Ref. [3] only when interference terms between different graphs obtained from symmetrization were included into the calculation. One should also mention that the success was only partial, namely, the γ^{like} coefficients calculated separately for $\pi^+\pi^+$ and $\pi^-\pi^-$ satisfied the relation $\gamma^{++} < \gamma^{--}$ whereas the contrary is found in experiment.

TABLE II

Angular correlation coefficients calculated in the rest frame of all pions

	Coefficient	Experiment	Model	Model without interference terms
$p^*(p) \leqslant$ $\leqslant 1$ GeV/c	γ^{++}	1.80 ± 0.13	1.7	1.5
	γ^{--}	1.50 ± 0.18	0.4	0.55
	γ^{+-}	1.66 ± 0.08	1.7	1.8
$p^*(p) >$ > 1 GeV/c	γ^{++}	1.32 ± 0.11	1.55	1.55
	γ^{--}	1.24 ± 0.17	0.75	1.0
	γ^{+-}	2.07 ± 0.12	2.0	2.0
TOTAL	γ^{++}	1.58 ± 0.08	1.6	1.55
	γ^{--}	1.38 ± 0.13	0.6	0.8
	γ^{+-}	1.82 ± 0.08	1.9	1.9

We have calculated the γ coefficients in the framework of our version of the ABFST model. The results are summarized in Table II. It is interesting to compare them with the results of Ref. [3]. We have found that

1) the GGLP effect is in good agreement with experimental data even if interference terms are neglected. Although the contribution from interference terms to the cross-section is not negligible (35%) it does not affect appreciably neither the single particle distributions nor the angular correlation of pion pairs;

2) the inequality $\gamma^{--} < \gamma^{++}$ is in agreement with experimental data. Unfortunately, the value of the γ^{--} predicted by the model is too low. This reflects again our known difficul-

ties with the π^- mesons. We should notice, however, that the experimental values of γ^{++} and γ^{+-} are correctly reproduced by the model;

3) the dependence of the GGLP effect on the mass of pionic system is correct.

Finally we should like to discuss the comparison of model predictions with experimental data performed in the spirit of the Van Hove longitudinal phase-space analysis. Recently an extension of the longitudinal phase-space analysis has been attempted in the case of the 6-body final state processes [7]. In order to reduce the complexity of the problem the events were chosen with at least two pions slow in the overall c. m. system. These events were analyzed by means of the longitudinal phase-space plot technique developed to study 4-body final state processes [12]. This analysis provides full information about the longitudinal momenta of all particles in the approximation in which the longitudinal

TABLE III

Percentage of the total number of events with slow $\pi\pi$ pairs*

$\pi\pi$ pair	Percentage	
	Model	Experiment
$\pi^+\pi^-$	26%	(25±3)%
$\pi^+\pi^+$	13%	(16±2)%
$\pi^-\pi^-$	5%	(5±1)%

* Among two different $\pi\pi$ pairs satisfying our criterion we chose slower one.

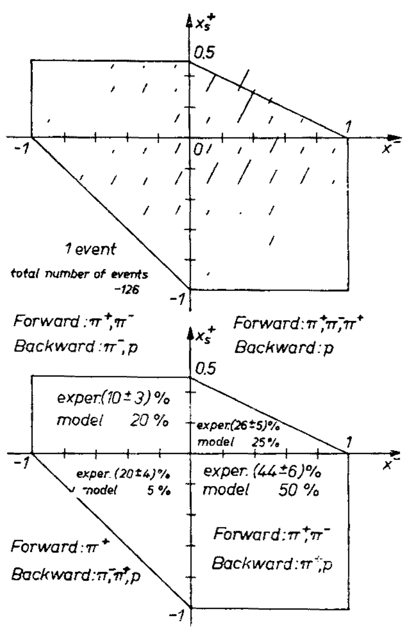

Fig. 9. Longitudinal phase-space distribution compared with the predictions of the model. For explanation see the text

momenta of slow pions are equal to zero. Slow pions were defined according to the longitudinal momentum criterion:

$$|p_L^*(\pi)| < 150 \text{ MeV}/c.$$

We have analyzed the model predictions in a similar way. In Table III we present the percentage of events which satisfy the longitudinal momenta criterion for different pion pairs. (We analyzed only the events with negative c. m. longitudinal momentum of proton.) The statistically most significant group of events with slow $\pi^+\pi^-$ system is further analyzed by means of a two-dimensional plot in x-variables which are defined as follows:

$$x_i = 2p_{Li}^* / \sum_{j=1}^{6} |p_{Lj}^*|.$$

The results of such an analysis are shown in Fig. 9 together with experimental data. The overall agreement of theoretical and experimental results is reasonable. It is better for $x^- > 0$ than for $x^- < 0$ as expected on the basis of the π^- longitudinal momentum distribution. The small number of events for the configuration $x_s^+ < 0$, $x^- < 0$ can be due to the absence in our model of pomeron exchange contribution.

4. *Conclusions*

We have applied the ABFST model with dual Veneziano amplitudes taken to describe the off-mass-shell $\pi^+\pi^-$ and π^+p scattering to the reaction $\pi^+p \to 3\pi^+2\pi^-p$ at 8 GeV/c. Previously the same process has been studied in the framework of the ABFST model with $\pi\pi$ and πp phase shifts and the Dürr-Pilkuhn off-shell extrapolation.

We have found that the total reaction cross-section depends weakly on which of the two methods of the off-mass-shell continuation is used. We confirm therefore the conclusion from Ref. [3] that the ABFST model with pion exchange predicts a too small cross-section for the considered reaction at 8 GeV/c. Proceeding, nevertheless, further on with the analysis of the two versions of the ABFST model we find several important differences. In particular the difference between π^+ and π^- longitudinal momenta distributions observed in the experiment is in our case qualitatively reproduced and the GGLP effect is obtained independently of the Bose-Einstein symmetrization.

We have completed the comparison od model predictions with experimental data by presenting the results of the Van Hove longitudinal phase-space-type analysis. Reasonable agreement with experimental data is observed also in this case.

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