GLUON SQUEEZED STATES IN QCD JET

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We study evolution of colour gluons and prove the possibility of gluon squeezed states at the nonperturbative QCD jet stage. Angular and rapidity dependences of squeezed gluon second correlation function are studied. We demonstrate that the new gluon states can have both sub-poissonian and super-poissonian statistics corresponding to antibunching and bunching of gluons.

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1. Introduction

Analogies between multiple hadron and photon production in quantum optics (QO) were discussed long ago [1, 2].

In particular, Squeezed States (SS), introduced by Stoler [3] and named by Hollenhorst [4], provoke great interest. These states can have reduced uncertainties compared with coherent ones, sub-poissonian (for coincide phases) and super-poissonian (for antiphases) statistics corresponding to antibunching and bunching of photons, can decrease quantum noise [5]. The squeezed light is generated from coherent one by nonlinear devices and is pure quantum nonperturbative phenomenon [5–8].

In particle physics the study of SS was stimulated by sub-poissonian Multiplicity Distributions (MD) for low energy lepton-hadron, $e^+e^-, p\overline{p}$-collisions. There were also a number of phenomenological attempts to describe oscillatory behaviour of hadron MD for different high-energy processes by general squeezed state MD by analogy with QO [9–14]. Weakness of these approaches was in its isolation from QCD.

Studying correlations in subsystems of hadrons in different high-energy processes led to the conclusion that for the MD description we should take into account of both QCD perturbative stage which gives wide distribution with super-poissonian MD [15] and nonperturbative stage, which must have sub-poissonian MD [16] that is typical for the SS distribution.

Quark and gluon jets in e^+e^-, hadron–hadron, ep scattering processes give good possibility for experimental tests of both perturbative and non-perturbative QCD.

The necessity of taking into account of nonperturbative QCD stage was displayed also particularly in local parton–hadron duality, fragmentation part of Monte-Carlo generators, power corrections, instanton contributions.

QCD jet perturbative evolution prepares some field of gluons [17] which then selfinteracts nonperturbatively in consequence of nonlinearities of Hamiltonian. Our hint is that nonperturbative selfinteraction during jet evolution can be a source of gluon SS by analogy with nonlinear devices in QED for photon SS.

In this paper we study colour evolution of gluon states at the nonperturbative stage of QCD for the jet ring. We check the fulfilment of the condition of squeezing for evolved gluon state and study the dependences of the second correlation function on jet cone angle and rapidity. Some preliminary results were discussed in [18, 19].

2. The nonperturbative evolution of gluons

Let us consider QCD gluon jet. At the end of perturbative QCD cascade the factorial moments F_q in jet are close to those of a negative binomial distribution [20]. Therefore gluon multiplicity distribution is like to negative binomial one which corresponds to specific superposition of coherent states with poissonian multiplicity distribution [21]. Since at this moment there are gluons with different colours and vector components then at initial time there is superposition of products of gluon coherent states with different the colour index b and the vector component l of the next form $\prod_{b=1}^{8}\prod_{l=1}^{3}|\alpha_l^b(0)\rangle$.

Let us consider at first the time evolution of gluon coherent state $|\alpha_l^b(0)\rangle$ defined by the Schrödinger equation with the Hamiltonian $\hat{H}_g$ which has the standard QCD form [18, 22]

$$\hat{H}_g = \hat{H}_0 + \hat{V} = \int\Big\{\frac{1}{2}\Big(\hat{\vec{E}}_a\hat{\vec{E}}_a + \hat{\vec{B}}_a\hat{\vec{B}}_a\Big) - g\,\hat{\vec{E}}_a\,C_{abc}\,\hat{\vec{A}}_b\hat{A}_c^0 + \frac{g}{2}\,\hat{\vec{B}}_a\,C_{abc}\Big[\hat{\vec{A}}_b\hat{\vec{A}}_c\Big] + \frac{g^2}{2}\left(C_{abc}\,\hat{\vec{A}}_b\,\hat{A}_c^0\right)^2 + \frac{g^2}{2}\left(\frac{1}{2}C_{abc}\Big[\hat{\vec{A}}_b\hat{\vec{A}}_c\Big]\right)^2\Big\}\,d^3x\,, \qquad (1)$$

where $\hat{H}_0 = \dfrac{1}{2}\displaystyle\int\Big\{\Big(\hat{\vec{E}}_a\hat{\vec{E}}_a + \hat{\vec{B}}_a\hat{\vec{B}}_a\Big)\Big\}\,d^3x$ is the Hamiltonian of the "free" gluons, $\hat{\vec{E}}_a = -\vec{\nabla}\hat{A}^0_a - \dfrac{\partial \hat{\vec{A}}_a}{\partial t}$, $\hat{\vec{B}}_a = \left[\vec{\nabla}\hat{\vec{A}}_a\right]$, $\hat{\vec{A}}_a$ — vector potential of the gluon field, C_{abc} are structure constants of the $\mathrm{SU}_c(3)$ group.

Let the X-axis coincides with the jet axis and the origin of coordinates is at the beginning of the perturbative cascade. Then gluon momentum has next spherical coordinates: $\vec{k} = (|\vec{k}|\cos\theta, |\vec{k}|\sin\theta\,\sin\varphi, |\vec{k}|\sin\theta\,\cos\varphi)$, where $0 \le \theta \le \theta_{\max}$ is the angle between $\vec{k}$ and jet axis, $\theta_{\max}$ is a half of the jet cone angle; φ is the azimuth angle ($0 \le \varphi \le 2\pi$). Assume for simplicity that the gluons at the end of the perturbative stage of jet evolution have close energies. This assumption does not change the results.

Then it is easy to find that the Hamiltonian of gluon selfinteraction for the jet ring with cone angle θ in momentum representation (with taking into account of the Lorenz gauge condition[1]) is equal

$$\hat{V} = \frac{k_0^4}{4(2\pi)^3}\left(1-\frac{q_0^2}{k_0^2}\right)^{3/2} g^2\pi C_{abc}C_{adf}\left\{\left(2-\frac{q_0^2}{k_0^2}\right)\left[a^{bcdf}_{1212}+a^{bcdf}_{1313}\right]+a^{bcdf}_{2323}\right.$$
$$\left.+\frac{\sin^2\theta}{2}\left(1-\frac{q_0^2}{k_0^2}\right)\left[2a^{bcdf}_{2323} - a^{bcdf}_{1212} - a^{bcdf}_{1313}\right]\right\}\sin\theta\,. \qquad (2)$$

Here $a^{bcdf}_{ijkj} = \hat{a}^{b+}_i\hat{a}^{c+}_j\hat{a}^d_k\hat{a}^f_j + \hat{a}^{b+}_i\hat{a}^c_j\hat{a}^{d+}_k\hat{a}^f_j + \hat{a}^b_i\hat{a}^{c+}_j\hat{a}^{d+}_k\hat{a}^f_j + c.c.$, $\hat{a}^b_i\left(\hat{a}^{b+}_i\right)$ are annihilation (production) operators of gluons, k_0 and q_0 are gluon energy and virtuality at the end of perturbative cascade. Integration over θ gives the total jet cone Hamiltonian.

The solution of the Schrödinger evolution equation for small time has the evident form

$$|\alpha^b_l(t)\rangle \simeq |\alpha^b_l(0)\rangle - i\hat{H}_g|\alpha^b_l(0)\rangle t\,. \qquad (3)$$

It gives possibility to study colour evolution of the gluon field within a short time. As an example consider the evolution of the gluon coherent state with the colour index b=1 and the vector component $l=1$

$$\begin{aligned}
&|\alpha^1_1(t)\rangle \simeq \left\{1 - 2it\pi\sin\theta\left(u_3 + u_4|\alpha^1_1|^2\right)\right\}|\alpha^1_1(0)\rangle \\
&- 2it\pi\sin\theta\, u_4\,\alpha^1_1\hat{D}(\alpha^1_1)|a^1_1(0)\rangle \\
&- 2it\pi\, u_2(1+u_1)\sin\theta\,\left(\alpha^1_1\right)^2 {\sum_{k=2}^{7}}' \left(|\alpha^1_1(0), 2a^k_2\rangle + |\alpha^1_1(0), 2a^k_3\rangle\right) \\
&+ it\pi\, u_2u_1\sin^3\theta\,\left(\alpha^1_1\right)^2 {\sum_{k=2}^{7}}' \left(|\alpha^1_1(0), 2a^k_2\rangle + |\alpha^1_1(0), 2a^k_3\rangle\right), \qquad (4)
\end{aligned}$$

[1] Gluon squeezing is due to the Hamiltonian nonliniarities and does not depend on gauge fixing.

where $|a_l^b(0)\rangle$ is a single gluon vector, $\hat{D}(\alpha) = \exp\{\alpha\hat{a}^+ - \alpha^*\hat{a}\}$ is the displacement operator of amplitude α, the explicit forms of the constants u_1, u_2, u_3, u_4 and $\sum_{k=2}^{7}{}'(\,)$ are given in the Appendix.

Analogously we can also investigate the evolution of coherent gluon states with any other colour charges and vector components.

As the result the following conclusion has been obtained:

1) for the initial vectors with the colour indexes $b = 1, 2, 3$ the vectors with another colour indexes $k = \overline{4, 7}$ appear;

2) if the initial vectors have the colour indexes $b = \overline{4, 7}$ then the new vectors with colour indexes $k = 1, 2, 3, 8$ and the vectors with the combination of the colour indexes 3,8: $|\alpha_l^b, a^3, a^8\rangle$ appear;

3) as the result of the evolution of colour coherent state with $b = 8$ the mixed colour states with colour indexes 4,5,6,7 appear.

It is clear that namely the difference among the structure constants of the $SU_c(3)$-group for different colour indexes leads to the different evolution of the corresponding colours.

3. Gluon squeezed state

To prove that the evolved state $|f\rangle$ is the Gluon Squeezed State (GSS) we must check by analogy with QO the fulfilment of the squeezing condition which has in particular the form [8]

$$\left\langle N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle = \left\langle\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle - \frac{1}{4} < 0\,, \tag{5}$$

where $\left\langle\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle = \left\langle\left((\hat{X}_l^b)_{\frac{1}{2}} - \left\langle(\hat{X}_l^b)_{\frac{1}{2}}\right\rangle\right)^2\right\rangle$, averaging is made over $|f\rangle$, real and imaginary components of the complex amplitude of the gluon field are defined by the operators $(\hat{X}_l^b)_1 = [\hat{a}_l^b + (\hat{a}_l^b)^+]/2$ and $(\hat{X}_l^b)_2 = [\hat{a}_l^b - (\hat{a}_l^b)^+]/2i$, the operator of normal ordering N is

$$\left\langle N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle = \frac{1}{4}\Bigg\{\pm\left[\left\langle\left(\hat{a}_l^b\right)^2\right\rangle - \left\langle\hat{a}_l^b\right\rangle^2\right] \pm \left[\left\langle\left(\hat{a}_l^{b+}\right)^2\right\rangle - \left\langle\hat{a}_l^{b+}\right\rangle^2\right] + 2\left[\left\langle\hat{a}_l^{b+}\hat{a}_l^b\right\rangle - \left\langle\hat{a}_l^{b+}\right\rangle\left\langle\hat{a}_l^b\right\rangle\right]\Bigg\}\,. \tag{6}$$

To check the squeezing of the final evolved state it is sufficient to make averaging of the $N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2$ over the vector $\prod_{b=1}^{8}\prod_{l=1}^{3}|\alpha_l^b(t)\rangle$. In this case it is easy to see that there can be $\left\langle N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle \neq 0$. From the explicit form of $\left\langle N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle$ for the colour index $b=1$ and an arbitrary vector component l

$$\left\langle N\left(\triangle(\hat{X}_l^1)_{\frac{1}{2}}\right)^2\right\rangle = \pm 4\pi\, u_2\, t\, d\theta\Big\{(1+u_1)\sin\theta\Big[\delta_{l1}(Z_{33}+Z_{22})+(1-\delta_{l1})Z_{11}\Big]$$
$$+(1-\delta_{l1})\sin\theta\Big[\delta_{l2}Z_{33}+\delta_{l3}Z_{22}\Big]+u_1\sin^3\theta$$
$$\times\Big[-\tfrac{1}{2}\delta_{l1}(Z_{22}+Z_{33})+\delta_{l2}\,(Z_{33}-\tfrac{1}{2}Z_{11})+\delta_{l3}\,(Z_{22}-\tfrac{1}{2}Z_{11})\Big]\Big\}, \qquad (7)$$

($Z_{mn} = \sum_{k=2}^{7}{}' \langle(\hat{X}_m^k)_1\rangle\langle(\hat{X}_n^k)_2\rangle$, $m,n = 1,2,3$) one can see that the squeezing condition (5) is fulfiled with uncertainties $\triangle(\hat{X}_l^1)_2 < \frac{1}{4} < \triangle(\hat{X}_l^1)_1$ under the next conditions: $\langle(\hat{X}_l^1)_1\rangle < 0$, $\langle(\hat{X}_1^1)_2\rangle < 0$ or $\langle(\hat{X}_l^1)_1\rangle > 0$, $\langle(\hat{X}_l^1)_2\rangle > 0$ and with uncertainties $\triangle(\hat{X}_l^1)_1 < \frac{1}{4} < \triangle(\hat{X}_l^1)_2$ under the next conditions: $\langle(\hat{X}_l^1)_1\rangle > 0$, $\langle(\hat{X}_l^1)_2\rangle < 0$ or $\langle(\hat{X}_l^1)_1\rangle < 0$, $\langle(\hat{X}_l^1)_2\rangle > 0$.

Thus the evolved vector $\prod_{b=1}^{8}\prod_{l=1}^{3}|\alpha_l^b(t)\rangle$ or its combinations can describe the GSS.

Note that if we make averaging over the superposition of the gluon coherent states with fixed colour and vector components, or the superposition of the gluon coherent states with fixed colour, or fixed vector component, or superposition of the single gluon states, then we can see that the quantity $\left\langle N\left(\triangle(\hat{X}_l^b)_{\frac{1}{2}}\right)^2\right\rangle$ is equal zero, the condition (5) is not fulfiled and these states are not GSS.

4. Two gluon correlation function

What could be an experimental indication on GSS? To answer this question, we can study the angular dependence of squeezed gluon second correlation function by well-known methods of QO.

By analogy with QO we can write the second normalized correlation function of gluons in the form

$$K^b_{l(2)}(\theta_1,\theta_2) \;=\; \frac{\left\langle \hat{a}^{b+}_l\,\hat{a}^{b+}_l\,\hat{a}^b_l\,\hat{a}^b_l\right\rangle}{\left\langle \hat{a}^{b+}_l\,\hat{a}^b_l\right\rangle^2} - 1\,. \tag{8}$$

The averaging here, as it must, is carried out over the state vector

$$\prod_{b=1}^{8}\prod_{l=1}^{3}|\alpha^b_l(\theta_1,t),\alpha^b_l(\theta_2,t)\rangle$$

at the moment t. If $K^b_{l(2)}>0$ then bunching of gluons takes place and the gluon antibunching can occur in the case $K^b_{l(2)}<0$. For a coherent field with a poissonian distribution of gluons $K^b_{l(2)}$ is equal 0. At the beginning of the nonperturbative region $K^b_{l(2)}=0$ because the gluon state vector at the initial moment is the product of the gluon coherent states.

Averaging over the evolved vector $\prod\limits_{b=1}^{8}\prod\limits_{l=1}^{3}|\alpha^b_l(\theta_1,t),\alpha^b_l(\theta_2,t)\rangle\!\rangle$ which also describes gluon squeezed state, we obtain

$$K^b_{l(2)}(\theta_1,\theta_2) \;=\; -\,\frac{M_1(\theta_1,\theta_2)}{|\alpha^b_l|^4-2|\alpha^b_l|^2M_1(\theta_1,\theta_2)+M_2(\theta_1,\theta_2)}\,, \tag{9}$$

where for the colour 1 and an arbitrary vector component l

$$\begin{aligned} M_1(\theta_1,\theta_2) \;=\;& 24\,t\,u_2\,\pi|\alpha|^2|\beta|^2\sin(2\delta+\pi/2)\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1}) \\ &\times(\sin\theta_1+\sin\theta_2)-\frac{1}{2}u_1(3\delta_{l1}-1)(\sin^3\theta_1+\sin^3\theta_2)\Big\}, \end{aligned} \tag{10}$$

$$\begin{aligned} M_2(\theta_1,\theta_2) \;=\;& 80\,t\,u_2\,\pi|\alpha|^3|\beta|^3\sin(\delta+\pi/4)\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1}) \\ &\times(\sin\theta_1+\sin\theta_2)-\frac{1}{2}\,u_1(3\delta_{l1}-1)(\sin^3\theta_1+\sin^3\theta_2)\Big\}. \end{aligned} \tag{11}$$

Here for simplicity we supposed that $\alpha^1_l=|\alpha|\,e^{i\gamma_1}$ for $\forall l$ and $\alpha^b_l=|\beta|\,e^{i\gamma_2}$, when $b\neq 1$, for $\forall\,l$, $\gamma_1-\gamma_2=\delta+\dfrac{\pi}{4}$ (phase δ defines the direction of squeezing maximum [5]).

At the same time for the squeezed state of photons the second normalized correlation function at $0 < r_l < \dfrac{1}{4}$ (r_l is squeezing parameter for component l) is [5, 8]

$$K_{l(2)} = -\frac{r_l[\alpha_l^2\,\mathrm{e}^{-2i\delta} + (\alpha_l^*)^2\,\mathrm{e}^{2i\delta}]}{|\alpha_l|^4 - 2r_l|\alpha_l|^2[\alpha_l^2\,\mathrm{e}^{-2i\delta} + (\alpha_l^*)^2\,\mathrm{e}^{2i\delta}]}\,. \tag{12}$$

It can be both less than 0 in the case of phase $\delta = 0$ (coincide phases), and more than 0 in the case of phase $\delta = \dfrac{\pi}{2}$ (antiphases) corresponding to antibunching (sub-poissonian MD) and bunching (super-poissonian MD) of photons [5].

Unlike corresponding expression (12) in QO $K^b_{l(2)}(\theta_1, \theta_2)$ for GSS (9) includes also function $M_2(\theta_1, \theta_2)$ which appears due to the different colours and vector components of gluons in the Hamiltonian.

We can write correlation function in the terms of the rapidity $K^b_{l(2)}(y_1, y_2)$ by transformation

$$\sin\theta = \sqrt{1 - \frac{\tanh^2 y}{u_1}}, \quad d\theta = -\frac{dy}{\cosh^2 y\sqrt{u_1 - \tanh^2 y}}\,. \tag{13}$$

The angle and the rapidity dependences of squeezed gluon correlation function are plotted for $b = 1$ at the time $t = 0.001$, $\theta_2 = 0$ and $y_2 = 0$ (Fig. 1) under some reasonable parameters: $g = 1$, that corresponds to the bound between perturbative and nonperturbative regions; $q_0^2 = 1\,\mathrm{GeV}^2$ that corresponds to the gluon virtuality at the beginning of the nonperturbative stage; $k_0 = \dfrac{\sqrt{s}}{2\langle n_{\mathrm{gluon}}\rangle}$ corresponds to a gluon energy; $\sqrt{s} = 91$ GeV and $\langle n_{\mathrm{gluon}}\rangle = 10$.

From Fig. 1 we notice that angle correlations have singularity at $\theta_1 \sim 10^{-9}$ and then with increasing θ_1 they decrease and approach to the value 0.522. Besides $K^1_{l(2)}(0, 0) = 0$ at the beginning of the nonperturbative stage ($\theta_1 = \theta_2 = 0$) because the gluon state vector at the initial moment is the product of the gluon coherent states.

At the same parameters the rapidity correlation function has two maximums $K^1_{l(2)}(y_1, y_2 = 0) = -2\cdot 10^{-5}$ at $y_1 = \pm 1.98$ and minimum $K^1_{l(2)}(y_1 = 0, y_2 = 0) = -7.295\cdot 10^{-5}$. Since the rapidity correlations fall in the negative region then we have antibunching of gluons at the nonperturbative stage of QCD jet.

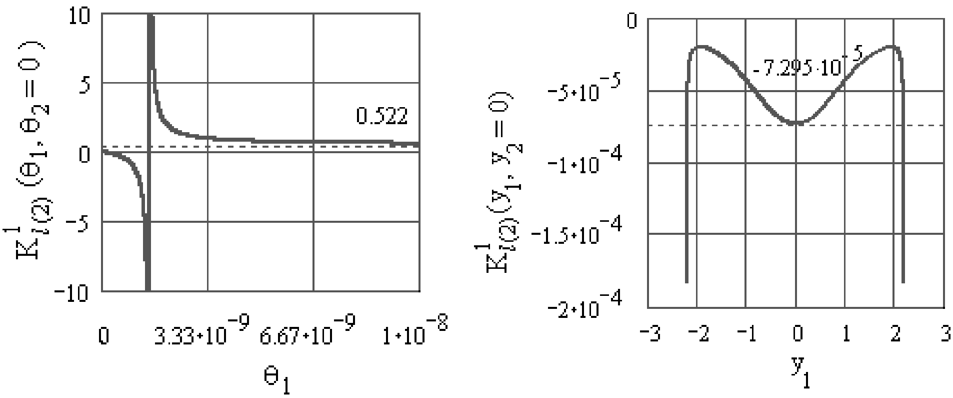

Fig. 1. The angular and rapidity dependences of the squeezed gluon correlation function ($K^1_{l(2)}(\theta_1)$ at $\theta_2 = 0$ and $K^1_{l(2)}(y_1)$ at $y_2 = 0$), $K^1_{l(2)}(0,0) = 0$.

5. Conclusion

Thus nonperturbative quantum evolution of gluon state prepared by perturbative cascade stage in jets can lead at least in a small time to quantum qluon states–squeezed states.

The two gluon correlation function in GSS is calculated by analogy with QO. It was demonstrated that the angle correlation function $K^1_{l(2)}(\theta_1, \theta_2 = 0)$ has singularity at some angle and then with increasing θ_1 it decreases and approaches to the constant value. At the same time the rapidity correlations fall into the negative region and have minimum at $y_1 = 0$. This behaviour corresponds to antibunching of gluons this is a sub-poissonian multiplicity distribution at the nonperturbative stage of QCD jet.

Such behaviour of the correlation function could be the sign of gluon squeezed states. This effect can be searched experimentally at LEP and other facilities where jets are clearly seen under adequate taking into account the perturbative cascade correlations and hadronization.

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Appendix

$$\sum_{k=2}^{7}{}'(\,) = \sum_{k=2}^{3}(\,) + \frac{1}{4}\sum_{k=4}^{7}(\,),\ u_1 = \left(1 - \frac{q_0^2}{k_0^2}\right),\ u_2 = \frac{k_0^4}{4(2\pi)^3}\frac{g^2}{2}(u_1)^{\frac{3}{2}},$$

$$u_3 = \frac{k_0^3}{2}(u_1)^{\frac{1}{2}}\,15\left(1 + u_1 + \frac{q_0^4}{2k_0^4}\right) + 24\,u_2(3 + 2u_1),$$

$$u_4 = \frac{k_0^3}{2}(u_1)^{\frac{1}{2}}\left\{\frac{q_0^4}{k_0^4} + \left(1 + u_1 - \frac{q_0^4}{k_0^4}\right)\sin^2\theta\right\} + 6\,u_2[2(1 + u_1) - u_1\sin^2\theta]\,.$$

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