PERTURBATIVE AND NONPERTURBATIVE HIGGS SIGNALS*

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We discuss the current picture of the standard Higgs sector at strong coupling and the phenomenological implications for direct searches at the LHC.

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Recently, considerable progress has been made in understanding the nature of the standard Higgs sector when its coupling becomes strong. Technically, computations on a lattice in the Higgs sector still have a long way to go to attain a precision useful phenomenologically, for instance when applied to LHC processes. Meanwhile, a new higher-order nonperturbative $1/N$ approach proved able to match the precision of two-loop perturbative results at low coupling, while its validity extends into the strong coupling zone as well. The availability of this nonperturbative approach opens up the perspective to explore in a reliable way exciting ideas such as the possibility of a Higgs boson coupled strongly to the vector bosons and to itself, and the formation of a spectrum of bound states at a higher scale. Such possibilities were proposed in the past. However, they could not be worked out from first principles because a nonperturbative solution was missing.

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From the experimental point of view, should a resonance similar to a Higgs boson be discovered at the LHC, it is crucial that its properties be understood sufficiently well theoretically, so that a standard Higgs can be distinguished from a nonminimal version. This can indeed be a serious issue if everything we know is perturbation theory, as it will become clear from the following example.

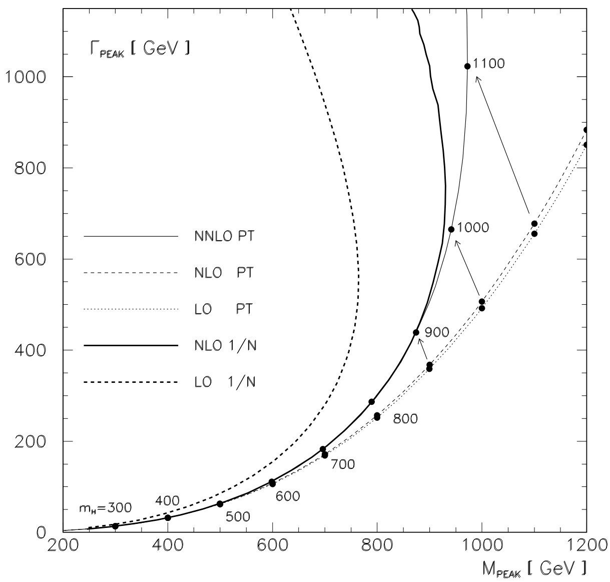

Fig. 1. The current knowledge of the Higgs width at strong coupling. The mass and width parameters M_{PEAK} and Γ_{PEAK} are extracted from the position and the height of the Higgs resonance in fermion scattering as if the resonance was of Breit-Wigner type. We give the relation between M_{PEAK} and Γ_{PEAK} in perturbation theory (LO, NLO and NNLO) and in the nonperturbative $1/N$ expansion (LO and NLO). For the perturbation theory curves we give the corresponding values of the on-shell mass parameter m_H.

Figure 1 summarizes the evolution of our knowledge of the width of the Higgs boson. Here we are concerned only with the width of a heavy Higgs boson, which decays dominantly into vector bosons. Quantum corrections at the one-loop level were first considered in Ref. [1]. There it was noticed that the computation of corrections of enhanced electroweak strength can be greatly simplified by using the equivalence theorem in Landau gauge. As one can see in this picture, the one-loop correction turns out to be fairly small. This suggested that perturbation theory is perfectly under control over the whole region of concern, even well above 1 TeV. There seemed to be little point in calculating higher-loop corrections.

However, at the same time another solution was known, which disagreed numerically quite strongly with perturbation theory. This approach attempted to calculate Green functions in the sigma model by expanding in $1/N$, where N is the number of degrees of freedom of the theory, instead of the coupling constant. This was proposed for the $\mathcal{O}(N)$–symmetric sigma model in refs. [2–4]. It was subsequently applied to the standard Higgs sector in refs. [5, 6]. The resulting leading order width does not appear to be numerically useful because it differs substantially from perturbation theory at low coupling, where perturbation theory is expected to be reliable. This large discrepancy raised doubts about the consistency of the $1/N$ approach.

However, we calculated one order higher in both expansions [7–9], and it turns out that the discrepancy between perturbation theory and the nonperturbative $1/N$ expansion is reduced dramatically. As can be seen in Fig. 1, the two expansions appear to be nicely converging towards a common solution. The next-to-leading $1/N$ solution and two-loop perturbation theory are in a remarkable agreement up to such high values of the Higgs mass as 800–900 GeV.

Also it can be seen from Fig. 1 that the true value of the decay width can differ considerably from the tree and one-loop level calculations, which so far were used widely for phenomenological studies for the LHC. As the coupling in the Higgs sector increases, an interesting saturation of the mass takes places, where only the total decay width grows. Should a standard Higgs resonance be discovered at the LHC somewhere in the zone where the saturation effect comes into place, the low-order perturbative analysis would suggest that its coupling to the vector bosons is too strong to be compatible with the standard model.

The two-loop perturbative analysis is based, just as the one-loop calculation, on the use of the equivalence theorem in Landau gauge. The main difficulty at the two-loop level is that it involves the evaluation of massive two-loop Feynman graphs at finite external momentum. This is known to be a difficult problem because the scalar integrals in the general kinematic case are usually unknown analytic functions. One particular case where the special functions involved were identified is the so–called sunset self-energy topology. This was shown to be related to the Lauricella functions [10]. It turns out that even in this case the diagram is most efficiently evaluated by means of integral representations. For this reason, we developed a general approach which is based entirely on integral representations when the external momentum of the graph is finite [7, 11]. The case with zero external momentum can always be treated analytically and the general solution has been known for a long time [12]. Our general solution is numerical. However, due to the use of deterministic adaptative algorithms combined with an optimized complex integration path defined in terms of spline functions, the solution is fast and accurate. It was already used for the calculation of several physical processes of phenomenological interest [7, 13–15]. The two-loop result shown in Fig. 1 was obtained with this method, and was also reproduced in Ref. [8] with different numerical methods.

Regarding the use of numerical versus analytical methods for this type of calculations, we would like to make the following remark. The nontrivial two-point functions involved in this calculation were first calculated numerically [7, 8], but later on an analytical solution was obtained for them when the external momentum is on-shell [16]. This was possible because the calculation is in essence a one-scale problem if treated in Landau gauge. This simplifies the problem considerably. For the three-point case, because of the complexity of the diagrams, an analytical solution was not found so far. Still, the existing numerical solution is accurate enough for any practical purpose.

Our nonperturbative solution to this problem is based on considering an $\mathcal{O}(N)$-symmetric sigma model, which recovers the standard model for $N = 4$. At an intermediate stage, a double expansion is performed, both in the coupling constant and in $1/N$. Because of the combinatorial structure of this theory, it is possible to calculate and to sum up the Feynman graphs of all orders which are generated for a given order in $1/N$. This procedure works in principle for any Green function, but the complexity of the problem increases with the number of external legs.

The leading order of the $1/N$ expansion is simply the well–known geometric series of bubble self-energy one-loop graphs and was known for a long time [2–4]. However, the next-to-leading order is much more difficult to calculate. The first problem encountered is to identify the relevant diagrams in all loop orders. This is most elegantly solved by a combinatorial trick proposed by Coleman, Jackiw and Politzer [2]. Their idea consists of adding a nondynamical piece to the Lagrangian, which contains an unphysical auxiliary field. As a result, the dynamics of the theory remains unchanged, but the Feynman rules are modified, and there are no quartic vertices left. This leads to a rearrangement of Feynman graphs in the higher orders.

Actually, this idea was used originally only at leading order, where it does not really simplify the problem. The real power of this rearrangement is apparent only in higher orders. For the next-to-leading order calculation the combinatorial rearrangement of Feynman graphs is practically unavoidable.

After identifying the relevant graphs in all loop orders, a method is needed for evaluating them. Barring a few trivial cases of limited applicability, an analytic solution is not available. We developed a highly efficient numerical approach based on the work of Ref. [17] on three–loop massive graphs. In Ref. [9] we applied this to two-point functions. Meanwhile it was also extended to three-point functions [18]. Most probably these methods can be extended to more complicated processes.

Because an analytical solution is not available, the ultraviolet $1/\epsilon$ poles cannot be isolated from the graphs as usual and absorbed into the $1/N$ counterterms. A few remarks about renormalization are in order here. First, in contrast with perturbation theory, the choice of renormalization scheme is of no relevance whatsoever. After summing up the complete perturbative series of the $1/N$ coefficients no residual scheme dependence is left. One obtains precisely the same physical result by working in any intermediate renormalization scheme. This freedom can be best exploited for simplifying to some extent the calculation. Second, the wave function renormalization constants turn out to be finite, as they should be in a nonperturbative solution. Only the coupling constant counterterms are truly ultraviolet divergent. Since it is complicated to extract the $1/\epsilon$ poles explicitly, we performed the intermediate renormalization in a nonstandard way, similar to the BPHZ procedure.

At the fundamental level of the theory, there is the problem of treating the leading-order tachyons of the $\mathcal{O}(N)$-symmetric sigma model in the $1/N$ expansion. The sigma model is widely believed to be trivial, although a rigorous proof does not exist yet. Within perturbation theory, an indication of triviality is the existence of the Landau pole. Similarly, in the $1/N$ expansion there is a tachyon in the Green functions. In perturbation theory the Landau pole is generated in a region where the beta function is not obtained reliably. Thus the Landau pole can be considered at most an indication of triviality. In the $1/N$ expansion the validity of the result depends only on the value of N, and not on how strong the coupling is. So the previous argument does not apply. Not much is known about the convergence properties of the $1/N$ expansion, and it was even suggested that a nonuniform convergence may explain the occurrence of the tachyon. Independently from what happens in higher orders, the tachyon cannot be considered a prediction of the theory in the usual derivation of the $1/N$ expansion. Normally the $1/N$ solution for the Green functions is obtained by summing up its perturbative expansion. The final result is thus determined only up to an arbitrary function which vanishes in perturbation theory. This freedom can be used to preserve causality. The residuum of the tachyon pole is precisely such a function. As such, it can be subtracted at its pole without upsetting the original information from the Feynman diagrams. Our tachyonic regularization simply subtracts the tachyon pole from the leading-order two-point functions. This procedure can be repeated consistently in higher orders if necessary.

It is interesting to note that the saturation effect is actually within the direct production reach of the Large Hadron Collider. We only considered the standard Higgs production by gluon fusion. The other production mechanism, the vector boson scattering, is not yet available nonperturbatively or at two-loop order. It is only known at one-loop [19]. Studies which were performed at tree level indicate the gluon fusion to dominate up to about 1 TeV [20].

The gluon fusion process at hadron colliders was studied in detail at leading order [21]. We included the correction of enhanced electroweak strength at NNLO, as a first approximation for the nonperturbative $1/N$ result [22]. This is because the three-point function is not available yet in the $1/N$ expansion at NLO. The use of the two-loop result is justified up to about 1.1 TeV because the two-loop Higgs width agrees well with the nonperturbative result. To simplify the analysis and to avoid the need for precise detector details, such as actual energy and angular resolutions, we confined our analysis to the nonhadronic decay channels. We considered a 100 fb^{-1} sample and we asked for a 5σ effect. Then, the four charged lepton channel can reach up to an on-shell Higgs mass of about 830 GeV [22]. The two charged lepton and missing transverse momentum channels can reach up to about 1030 GeV. As one can see in Fig. 1, this value is well within the saturation zone. It is possible that the hadronic channels may allow one to go even deeper in the saturation zone, due to a higher branching ratio. The analysis is complicated by the presence of a heavy QCD background. How well can the QCD background be separated from the signal is a matter of detector energy and angle resolution. This is a study which still needs to be done to assess the full potential of the LHC.

In conclusion, we now have the tools for calculating both two-loop corrections and NLO nonperturbative $1/N$ expansions in the Higgs sector. By combining the two expansions we were able to elucidate the strong coupling behaviour, and to establish the presence of a mass saturation effect at about 930 GeV. We treated the Higgs resonance in gluon fusion with these methods, and we established that the mass saturation effect is within the reach of the LHC even by conservatively considering only purely leptonic channels.

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