BORN–INFELD NONLINEAR ELECTRODYNAMICS*

Iwo Białynicki-Birula

Centre for Theoretical Physics, Polish Academy of Sciences
Al. Lotników 32/46, 02–668 Warsaw, Poland

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This is only a summary of a lecture delivered at the Infeld Centenial Meeting. In the lecture the history of the Born–Infeld nonlinear electrodynamics was presented and some general features of the theory were discussed.

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1. Summary of the lecture

Sixty-five years ago Leopold Infeld went to England as a Fellow of the Rockefeller Foundation. In Cambridge he met Rutherford and Dirac and entered into the collaboration with Max Born, who has just arrived in England. The result of this collaboration was the Born–Infeld electrodynamics.

The origins of the nonlinear electrodynamics of Born and Infeld can be traced to the work of Gustav Mie, who made the first attempt to construct a purely electromagnetic theory of charged particles. The question, why a charged particle does not explode under the repulsion of the Coulomb forces acting between its constituents, had been asked by a number of physicists before Mie. With the advent of the theory of relativity, this question has ballooned into a whole problem known under the name: the theory of the electron. Many a great physicists, including Einstein, Lorentz, Planck, Poincare, Sommerfeld, and von Laue have contributed to the discussion on this subject.

In the theory of the electron before Mie, the electron was not treated as a purely electromagnetic entity, but it was also "made of other stuff", like, for example, Poincare stresses and the mechanical mass. Mie wanted only the electromagnetic field to be responsible for all the properties of

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the electron. In particular, he wanted the electromagnetic current to be made of electromagnetism. In order to achieve this goal, Mie assumed that the potential four-vector enters directly into the Lagrangian and not only through the field strength. The generation of the current has been achieved in this manner, but the price was very high. The potentials acquired a physical meaning and the gauge invariance was lost. This property has been found unacceptable by other physicists and the theory of Mie has been shelved for two decades.

In May 1933 Born, after his dismissal from the University of Goettingen by the Nazis, decided to leave Germany. To make his leaving the country less conspicuous, he first took vacation in the Italian Dolomites. As he has later written in his memoirs, the idea to modify the Maxwell theory has come to him during this vacation. After arriving in Cambridge, where he was offered a stipend from the Stokes Foundation, Born started to work on this idea and soon he was joined by Infeld. Born and Infeld revived Mie's theory in a somewhat different form and in a series of papers [1] they proposed not only a specific, very interesting model, but also developed a theory of fully relativistic and gauge invariant nonlinear electrodynamics [2].

There are two ways to arrive at the nonlinear electrodynamics of Born and Infeld. One way was followed by the original discoverers and it relies on the idea of simplicity. The Born–Infeld theory is derived from the "simplest possible" Lagrangian: the square root of the determinant of a second rank covariant tensor. Such a structure automatically guarantees the invariance of the theory under arbitrary coordinate transformations. Taking as this second rank tensor the sum of the metric tensor and the electromagnetic field tensor (multiplied by a dimensional constant to equalize the dimensions), one obtains the Born-Infeld theory. There is also a second, more sophisticated method based on the unique property of Born–Infeld electrodynamics. As has been shown by Boillat and Plebański [3,4], one can arrive at the Born–Infeld theory by imposing "no-birefringence" condition. It turns out that this theory is the only nonlinear theory of the electromagnetic field in which the speed of light does not depend on the polarization. The standard theory of Maxwell has obviously the same property, but among nonlinear theories one only has the Born–Infeld electrodynamics. There was no indication for many years that the Born–Infeld electrodynamics describes the real world of electromagnetism and this theory remained for many years just a curiosity — an interesting model of a nonlinear theory.

However, more recently there has been a renewed interest in this theory in connection with the modern theory of strings [5] and p-branes [6]. In turns out that determinantal structures very much like the Born–Infeld Lagrangian frequently appear in these theories. So maybe, the last word about the Born-Infeld theory has not yet been said.

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