

THREE FLAVOR NEUTRINO OSCILLATIONS AND APPLICATION TO LONG BASELINE EXPERIMENTS *

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Using the result of the three flavor analysis of the old Kamiokande data, the recent Superkamiokande data of atmospheric neutrinos and the CHOOZ reactor data, it is shown that the third mixing angle θ_{13} is small. It is proposed to determine the small value of θ_{13} and the CP violating phase δ in very long baseline experiments by measuring the appearance probability $P(\nu_\mu \rightarrow \nu_e)$ and the T violating effect $P(\nu_e \rightarrow \nu_\mu) - P(\nu_\mu \rightarrow \nu_e)$ which are enhanced by the matter effect of the Earth.

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1. Introduction

The solar neutrino data [1–5] and the atmospheric neutrino experiments [6–10] provide strong evidence for neutrino oscillations. In the framework of two flavor neutrinos, these experimental data are explained by two sets of the oscillation parameters ($\Delta m_{\odot}^2, \sin^2 2\theta_{\odot}$) $\simeq (\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(10^{-2}))$ (small angle MSW solution), ($\mathcal{O}(10^{-5}\text{eV}^2), \mathcal{O}(1)$) (large angle MSW solution), or ($\mathcal{O}(10^{-10}\text{eV}^2), \mathcal{O}(1)$) (vacuum oscillation solution), and ($\Delta m_{\text{atm}}^2, \sin^2 2\theta_{\text{atm}}$) $\simeq (10^{-2.5}\text{eV}^2, 1.0)$.

Without loss of generality we assume that $|\Delta m_{21}^2| < |\Delta m_{32}^2| < |\Delta m_{31}^2|$ where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. If both the solar neutrino deficit and the atmospheric neutrino anomaly are to be solved by energy dependent solutions, we have to have $\Delta m_{21}^2 \simeq \Delta m_{\odot}^2$ and $\Delta m_{32}^2 \simeq \Delta m_{\text{atm}}^2$, *i.e.*, we have mass hierarchy in this case. Therefore I will assume mass hierarchy in the three

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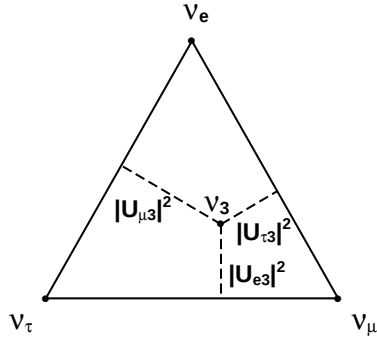


Fig. 1. Triangle of unit height for ν_3 . The vertical position of the state in the triangle graph is $|U_{e3}|^2$, which indicates deviation from the two flavor mixings.

flavor framework throughout this talk. I will adopt the triangle representation which has been introduced by Fogli, Lisi, and Scioscia [11]. Fig. 1, which will play a role in the analysis of atmospheric neutrino data, represents how the most massive state ν_3 mixes with three flavor eigenstates with the coefficients $|U_{\alpha 3}|^2$ ($\alpha = e, \mu, \tau$), where $|U_{\alpha 3}|^2$ are the elements of the MNS mixing matrix U [12]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \quad (1)$$

2. Constraints from atmospheric neutrino anomaly

The atmospheric neutrino data of Superkamiokande have been analyzed by [13, 14] in the three flavor framework, where smaller mass squared difference Δm_{21}^2 is ignored. The two flavor analysis of the most up-to-date data by the Superkamiokande group shows that the allowed region of the mass squared difference is $1 \times 10^{-3} \text{eV}^2 < \Delta m^2 < 7 \times 10^{-3} \text{eV}^2$ at 90% CL [8]. The analyses in [13, 14] are strictly speaking different from the original one in [8], since the full data which are binned with respect to the energy as well as the zenith angle are not used in [13, 14]. The analysis in [13] has been updated with the recent data for 850 days, where the upward going μ data [9] have also been incorporated. It was found that the region of the mass squared difference which is as small as $5 \times 10^{-4} \text{eV}^2$ is allowed at 90%CL (cf. Fig. 2).

On the other hand, the CHOOZ group has updated their result on $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ in the reactor disappearance experiment [15], and the mass squared difference is limited to $\Delta m^2 < 7 \times 10^{-4} \text{eV}^2$ for the maximum mixing. In our three flavor scheme with mass hierarchy, the disappearance

probability for the CHOOZ experiment is given by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E} \right), \quad (2)$$

so if $\Delta m_{32}^2 > 7 \times 10^{-4} \text{eV}^2$ then $\sin^2 2\theta_{13}$ has to be small. The allowed region which is obtained by the constraints of the Superkamiokande atmospheric neutrino data and the CHOOZ data is given in Fig. 2. As is seen in Fig. 2, relatively large θ_{13} is still allowed for $\Delta m_{32}^2 < 1 \times 10^{-3} \text{eV}^2$.

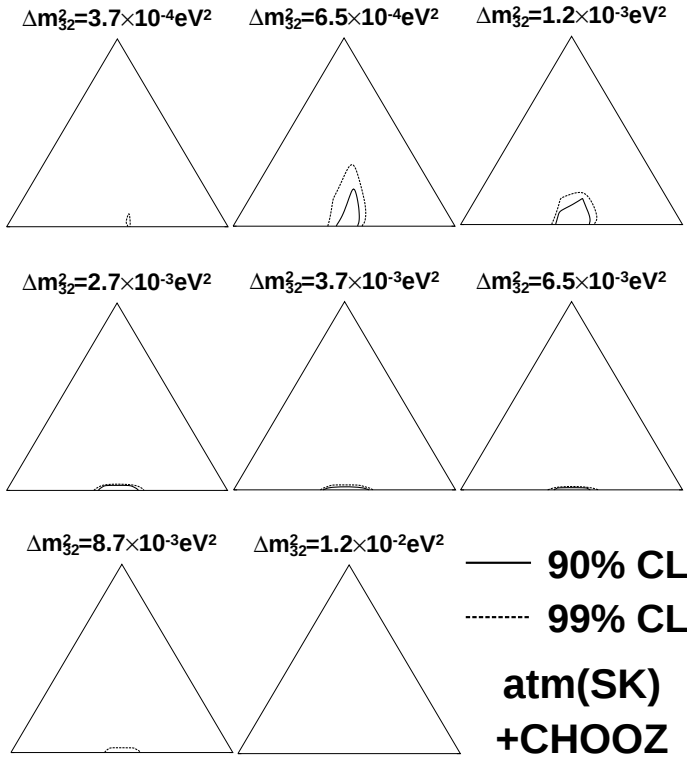


Fig. 2. The allowed regions for various Δm_{32}^2 by the constraints of atmospheric neutrino data of the Superkamiokande contained and upward going μ events, and the CHOOZ reactor data.

To obtain more stringent bound on $\sin^2 2\theta_{13}$, I include the three flavor analysis [16] of the contained events of the Kamiokande atmospheric neutrino data [6], for which the value of the mass squared difference in the allowed region tends to be higher than that of Superkamiokande. In fact $\Delta m_{32}^2 < 2 \times 10^{-3} \text{eV}^2$ is excluded at 90%CL by including the Kamiokande data. Combining the atmospheric neutrino data of Superkamiokande, Kamiokande and

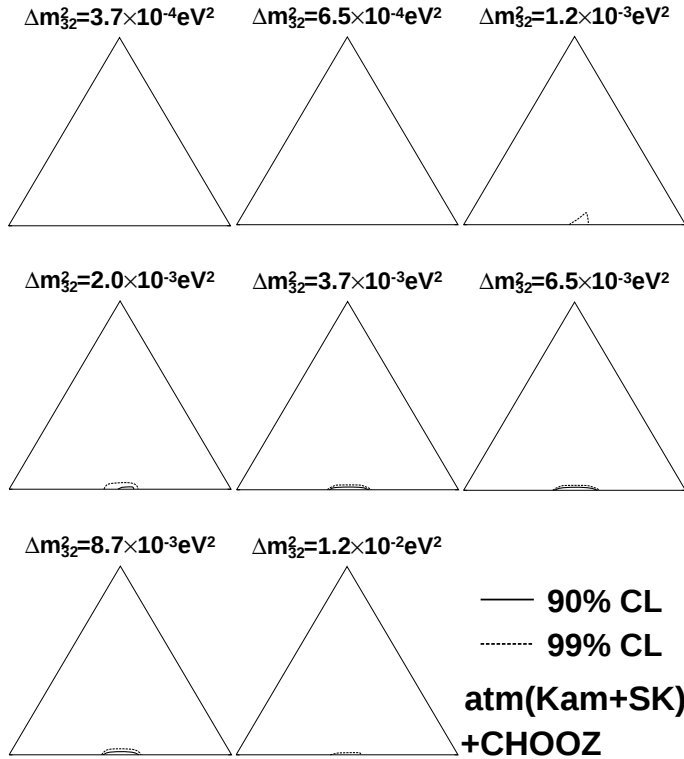


Fig. 3. The allowed regions for various Δm_{32}^2 by the constraints of atmospheric neutrino data of the Kamiokande contained events, the Superkamiokande contained and upward going μ events, and the CHOOZ reactor data. All the shadowed regions are located near the $\nu_\mu - \nu_\tau$ line.

the CHOOZ reactor data, I have obtained the allowed region which is depicted in Fig. 3. Fig. 3 shows that $\sin^2 2\theta_{13} < 0.1$ has to be satisfied, which is basically the consequence from the CHOOZ data.

3. A possible way to measure θ_{13}

As we have seen in Sec. 2, the data of atmospheric neutrinos and the CHOOZ experiment gives $\sin^2 2\theta_{13} \lesssim 0.1$. The two sets of parameters $(\Delta m_{21}^2, \sin^2 2\theta_{12})$ and $(\Delta m_{32}^2, \sin^2 2\theta_{23})$ will be determined with more and more accuracy in the future by various experiments of solar and atmospheric neutrinos, respectively, so the next thing we would like to pursue is to determine θ_{13} . There have been discussions on the future intense muon beam [17] which could be hundreds times as high as the present one, and it would enable us to have very long baseline experiments, where the neutrino path

length is comparable to the radius of the Earth. In this talk I would like to point out that the oscillation probability $P(\nu_\mu \rightarrow \nu_e)$ (in the case of $\Delta m_{32}^2 > 0$) or $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ (in the case of $\Delta m_{32}^2 < 0$) is enhanced for a certain region of the neutrino energy due to the matter effect of the Earth when $\sin^2 2\theta_{13} \gtrsim 0.01$ and therefore it is possible to deduce the magnitude of θ_{13} by measuring experimentally $P(\nu_\mu \rightarrow \nu_e)$ or $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ as a function of the neutrino energy in very long baseline experiments which may be possible with the intense muon beam technology in the future.

Let us now consider the situation where ΔE_{21} is completely negligible. In that case the positive energy part of the Dirac equation for three flavors of neutrinos in matter is given by

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = M \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (3)$$

with

$$\begin{aligned} M &\equiv U \text{diag}(0, 0, \Delta E_{32}) U^{-1} + \text{diag}(A, 0, 0) \\ &= D e^{i\theta_{23}\lambda_7} \left[e^{i\theta_{13}\lambda_5} \text{diag}(0, 0, \Delta E_{32}) e^{-i\theta_{13}\lambda_5} + \text{diag}(A, 0, 0) \right] e^{-i\theta_{23}\lambda_7} D^{-1} \\ &= D e^{i\theta_{23}\lambda_7} e^{i\theta_{13}^{M(-)}\lambda_5} \left[\frac{\Delta E_{32} + A}{2} \text{diag}(1, 0, 1) - \frac{B^{(-)}}{2} \text{diag}(1, 0, -1) \right] \\ &\quad \times e^{-i\theta_{13}^{M(-)}\lambda_5} e^{-i\theta_{23}\lambda_7} D^{-1}, \end{aligned} \quad (4)$$

where the unit matrix $\text{diag}(E_2, E_2, E_2)$ which contributes only to the overall phase has been subtracted from M , $\Delta E_{ij} \equiv \Delta m_{ij}^2/2E$, E is the neutrino energy, $A \equiv \sqrt{2}G_F N_e(x)$ stands for the matter effect [18] of the Earth, $D \equiv \text{diag}(e^{i\delta}, 1, 1)$ and the standard parametrization [19] has been used for the MNS matrix (1)

$$\begin{aligned} U &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ &= D e^{i\theta_{23}\lambda_7} e^{i\theta_{13}\lambda_5} D^{-1} e^{i\theta_{12}\lambda_2}, \end{aligned} \quad (5)$$

with the Gell-Mann matrices

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \quad (6)$$

$\theta_{13}^{M(\pm)}$ is the mixing angle in matter given by

$$\tan 2\theta_{13}^{M(\pm)} \equiv \frac{\Delta E_{32} \sin 2\theta_{13}}{\Delta E_{32} \cos 2\theta_{13} \pm A} \quad (7)$$

as in the two flavor case [18] , and

$$B^{(\pm)} \equiv \sqrt{(\Delta E_{32} \cos 2\theta_{13} \pm A)^2 + (\Delta E_{32} \sin 2\theta_{13})^2}, \quad (8)$$

where (+) sign is for antineutrinos, as the sign of A is reversed for antineutrinos. Assuming the constant density of the matter, the appearance probability $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in the three flavor framework can be written as

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) &= s_{23}^2 \sin^2 2\theta_{13}^{M(-)} \sin^2 \left(\frac{B^{(-)}L}{2} \right) \\ P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) &= s_{23}^2 \sin^2 2\theta_{13}^{M(+)} \sin^2 \left(\frac{B^{(+)}L}{2} \right), \end{aligned} \quad (9)$$

respectively. Note that the only difference between the formulae (9) and that in vacuum is $\sin^2 2\theta_{13}^{M(\pm)}$ which is replaced by $\sin^2 2\theta_{13}$ in vacuum. Thus the effective mixing angle in matter is enhanced if $\Delta m_{32}^2 > 0$ and $\Delta E_{32} \cos 2\theta_{13} - A$ becomes small for some E . On the other hand, if $\Delta m_{32}^2 < 0$, then there is no enhancement in the probability $P(\nu_\mu \rightarrow \nu_e)$, but the probability $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is enhanced instead. The probability $P(\nu_e \rightarrow \nu_x)$ with matter effect of the Earth has been discussed by many people in the framework of two flavors [20] and three flavors [21]. The equation (9) is almost the same as that for two flavor case [20], the only difference being that the counterpart to ν_e is the linear combination $s_{23}\nu_\mu + c_{23}\nu_\tau \simeq (\nu_\mu + \nu_\tau)/\sqrt{2}$.

I have computed numerically the probability $P(\nu_\mu \rightarrow \nu_e)$ for $\theta_{23} = \pi/4$, $\theta_{13} = 1^\circ, 3^\circ, 5^\circ, 7^\circ, 9^\circ$ (or $\sin^2 2\theta_{13} = 1.2 \times 10^{-3}, 1.1 \times 10^{-2}, 3.0 \times 10^{-2}, 5.9 \times 10^{-2}, 9.5 \times 10^{-2}$) and for $\cos \Theta = -1, -0.75, -0.5, -0.25$ (or $L = 12,800$ km, 9,600 km, 6,400 km, 3,200 km) where the zenith angle Θ , the neutrino path length L and the radius R of the Earth are related by $L = -2R \cos \Theta$. The results are shown in Fig. 4. The shape of the probability $P(\nu_\mu \rightarrow \nu_e)$ is almost the same as that for the two flavor oscillation [20], but it is scaled by the normalization s_{23}^2 (*cf.* (9)). As can be seen in Fig. 4, the case of $\cos \Theta = -1$ (or $L = 12,800$ km) is most advantageous to get the enhancement in the probability for $\theta_{13} \gtrsim 2^\circ$, while in the case of $\cos \Theta = -0.25$ it is difficult to see the enhancement for smaller values of θ_{13} .

To measure the probability in practical experiments, one has to measure the momentum of the recoiled nucleon as well as that of the outgoing charged

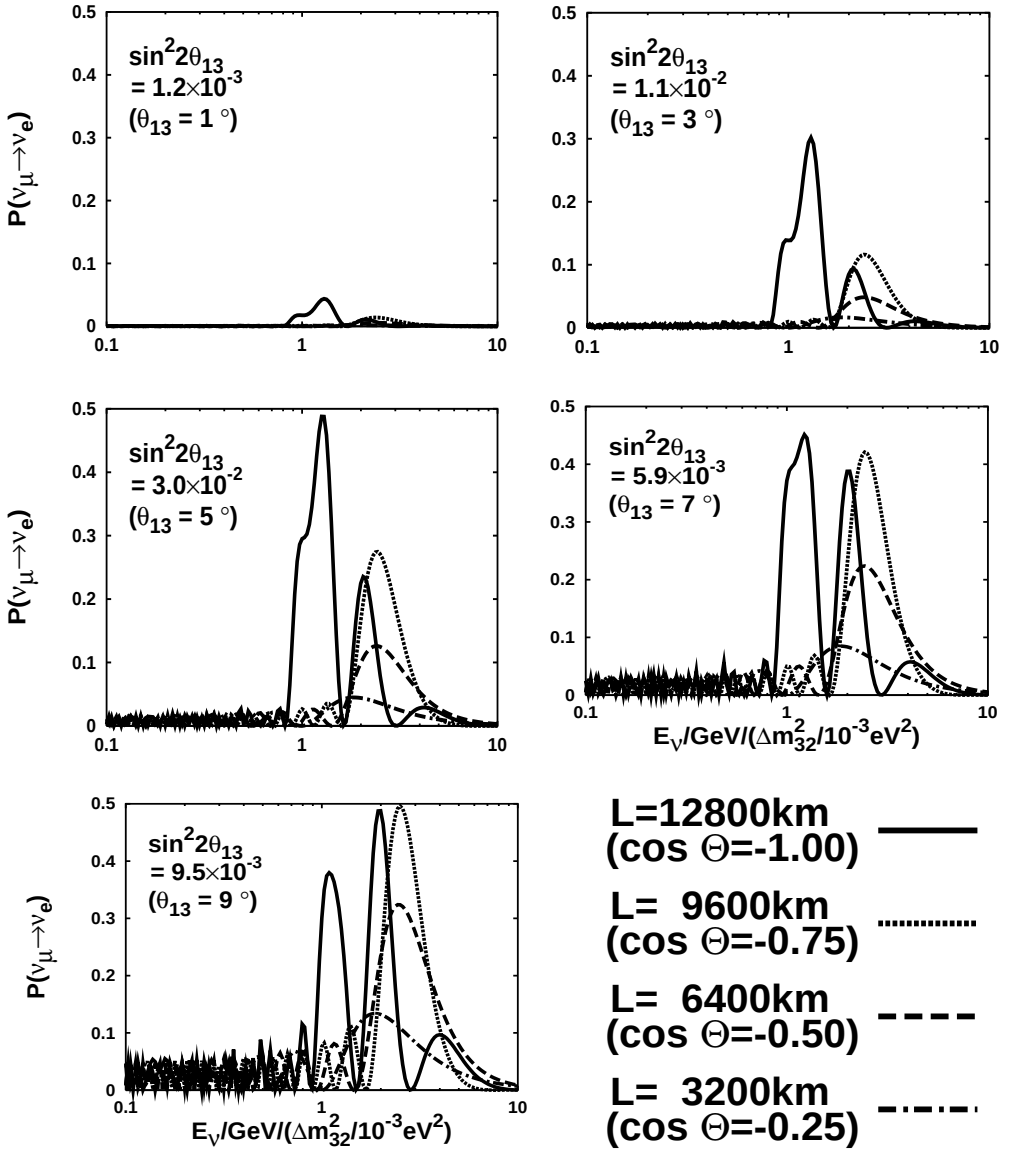


Fig. 4. The appearance probability $P(\nu_\mu \rightarrow \nu_e)$ as a function of $E_\nu/\Delta m_{32}^2$ for $\Delta m_{32}^2 > 0$. $\Delta m_{21}^2 = 0$, $\theta_{23} = \pi/4$ have been assumed. In the case of $\Delta m_{32}^2 < 0$ one has to look at $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ instead.

lepton in a quasi elastic scattering $\nu_\alpha + N \rightarrow \ell + N'$. From Fig. 4 we see that the maximum probability is obtained for $E_\nu/\text{GeV} \simeq 1.2 \times \Delta m_{32}^2/(10^{-3} \text{eV}^2)$ with $\cos \Theta = -1$ for each value of θ_{13} . If we assume $\Delta m_{32}^2 \simeq 3.5 \times 10^{-3} \text{eV}^2$

which is the best fit value in the Superkamiokande atmospheric neutrino data [8], then the maximum probability is obtained for $E_\nu \simeq 4$ GeV. The smaller $|\Delta m_{32}^2|$ becomes, the better it works, since the cross section of quasi elastic scatterings decreases as the neutrino energy increases [22].

4. A possible way to measure δ

There have been a lot of works which discussed CP violation in neutrino oscillations [23, 24]. From the oscillation probability in vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta; L) = \delta_{\alpha\beta} - 4 \sum_{j < k} \text{Re} (U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k}) \sin^2 \left(\frac{\Delta E_{jk} L}{2} \right) + 2 \sum_{j < k} \text{Im} (U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k}) \sin (\Delta E_{jk} L), \quad (10)$$

the CP violation in vacuum is given by

$$\begin{aligned} & P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \\ &= 4 \sum_{j < k} \text{Im} (U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k}) \sin (\Delta E_{jk} L) \\ &= 4 J [\sin (\Delta E_{12} L) + \sin (\Delta E_{23} L) + \sin (\Delta E_{31} L)], \end{aligned} \quad (11)$$

where

$$J \equiv \text{Im} (U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}) \quad (12)$$

is the Jarlskog factor, and

$$\text{Im} (U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}) = \text{Im} (U_{\alpha 2} U_{\beta 2}^* U_{\alpha 3}^* U_{\beta 3}) = \text{Im} (U_{\alpha 3} U_{\beta 3}^* U_{\alpha 1}^* U_{\beta 1}) \quad (13)$$

has been used. This Jarlskog factor, which is written as

$$J = c_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin \delta \quad (14)$$

in the standard parametrization, contains a small factor $\sin 2\theta_{13}$ which is constrained by the CHOOZ data ($\lesssim \sqrt{0.1}$), and a factor $\sin 2\theta_{12}$ which is small in the case of the small mixing angle MSW solution ($\sin^2 2\theta_{12} \simeq 6 \times 10^{-3}$). So in general the Jarlskog factor is expected to be small. In vacuum the CP violation happens to be the same as the T violation.

On the other hand, in the presence of matter, the expression (10) for the probability is modified. The eigen matrix in matter can be formally diagonalized by a unitary matrix V :

$$U \text{diag} (-\Delta E_{21}, 0, \Delta E_{32}) U^{-1} + \text{diag} (A, 0, 0) = V \text{diag} (\zeta_1, \zeta_2, \zeta_3) V^{-1}. \quad (15)$$

Assuming constant density, the oscillation probability can be written as

$$\begin{aligned}
 P(\nu_\alpha \rightarrow \nu_\beta; L) = & \delta_{\alpha\beta} - 4 \sum_{j < k} \text{Re} (V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}) \sin^2 \left(\frac{\Delta \zeta_{jk} L}{2} \right) \\
 & + 2 \sum_{j < k} \text{Im} (V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}) \sin (\Delta \zeta_{jk} L), \quad (16)
 \end{aligned}$$

as in the case of the probability in vacuum. The sign for the matter term A is reversed for antineutrinos $\bar{\nu}_\alpha$ and the unitary matrix $\bar{V}$ and the eigenvalues $\bar{\zeta}_j$ for $\bar{\nu}_\alpha$ are different from V and ζ_j for neutrinos ν_α . Therefore it is not illuminating to see the CP violation in matter [24], since it contains terms which vanish in the limit $\delta \rightarrow 0$ and those which do not:

$$\begin{aligned}
 & P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \\
 = & -4 \sum_{j < k} \text{Re} \left[V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k} \sin^2 \left(\frac{\Delta \zeta_{jk} L}{2} \right) \right. \\
 & \left. - \bar{V}_{\alpha j} \bar{V}_{\beta j}^* \bar{V}_{\alpha k}^* \bar{V}_{\beta k} \sin^2 \left(\frac{\Delta \bar{\zeta}_{jk} L}{2} \right) \right] \\
 & + 2 \sum_{j < k} \text{Im} [V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k} \sin (\Delta \zeta_{jk} L) \\
 & - \bar{V}_{\alpha j} \bar{V}_{\beta j}^* \bar{V}_{\alpha k}^* \bar{V}_{\beta k} \sin (\Delta \bar{\zeta}_{jk} L)] , \quad (17)
 \end{aligned}$$

where $\Delta \bar{\zeta}_{jk} \equiv \bar{\zeta}_j - \bar{\zeta}_k$. It has been pointed out [25] that T violation is useful to probe the CP violating phase in the presence of matter. In fact from (16) we have T violation in matter:

$$\begin{aligned}
 \Delta P & \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \\
 = & 4 \sum_{j < k} \text{Im} (V_{\alpha j} V_{\beta j}^* V_{\alpha k}^* V_{\beta k}) \sin (\Delta \zeta_{jk} L) \\
 = & 4 \mathcal{J} [\sin (\Delta \zeta_{12} L) + \sin (\Delta \zeta_{23} L) + \sin (\Delta \zeta_{31} L)] , \quad (18)
 \end{aligned}$$

where

$$\mathcal{J} \equiv \text{Im} (V_{\alpha 1} V_{\beta 1}^* V_{\alpha 2}^* V_{\beta 2}) \quad (19)$$

is the modified Jarlskog factor in matter and $\Delta \zeta_{jk} \equiv \zeta_j - \zeta_k$. Here I will consider T violation under two situations where one of the mixing angles in (19) is enhanced due to the matter effect of the Earth in our scheme with mass hierarchy.

4.1. (a) $|\Delta E_{21}| \ll |A| \simeq |\Delta E_{32}|$

First let us consider the case where $|\Delta E_{21}| \ll |A| \simeq |\Delta E_{32}|$. This case has been discussed by Arafune and Sato [24]. The unitary matrix which diagonalizes the eigen matrix (15) to first order in $|\Delta E_{21}/\Delta E_{32}|$ is given by

$$V \equiv e^{i\theta_{23}\lambda_7} e^{i\theta_{13}^{M(-)}\lambda_5} \exp \left\{ i\Delta E_{21} \left[\frac{1}{u_-} \tilde{c}_{13}s_{12}c_{12}(\lambda_1 \sin \delta - \lambda_2 \cos \delta) - \frac{1}{u_+ - u_-} \tilde{s}_{13}\tilde{c}_{13}s_{12}^2\lambda_5 - \frac{1}{u_+} \tilde{s}_{13}s_{12}c_{12}(\lambda_6 \sin \delta + \lambda_7 \cos \delta) \right] \right\}, \quad (20)$$

where I have assumed $\Delta E_{32} > 0$, $\theta_{13}^{M(-)}$ and $B^{(-)}$ are given in (7) and (8), respectively, and

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ u_{\pm} &\equiv \frac{1}{2}(\Delta E_{32} + A \pm B^{(-)}), \\ \tilde{\theta}_{13} &\equiv \theta_{13} - \theta_{13}^{M(-)}, \quad \tilde{s}_{13} \equiv \sin \tilde{\theta}_{13}, \quad \tilde{c}_{13} \equiv \cos \tilde{\theta}_{13}. \end{aligned} \quad (21)$$

The modified Jarlskog factor $\mathcal{J}_1$ in this case is given by

$$\mathcal{J}_1 = \frac{2\Delta E_{21}}{A \cos 2\theta_{13}} \frac{c_{13}}{8} \sin 2\theta_{12} \sin 2\theta_{13}^{M(-)} \sin 2\theta_{23} \sin \delta. \quad (22)$$

If $\Delta E_{32} < 0$ then we have to look at $\Delta \bar{P} \equiv P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$ instead, and $\theta_{13}^{M(-)}$ and $B^{(-)}$ have to be replaced by $\theta_{13}^{M(+)}$ and $B^{(+)}$, respectively. $\mathcal{J}_1$ is similar to the Jarlskog factor J in vacuum, but one important difference between J and $\mathcal{J}_1$ is that $\mathcal{J}_1$ has $\sin 2\theta_{13}^{M(-)}$ which could be enhanced by the matter effect of the Earth. Another difference is that $\mathcal{J}_1$ has a factor $\Delta E_{21}/A$ whose absolute value is small by assumption and therefore the T violating effect under this condition (a) is supposed to be small.

4.2. (b) $|\Delta E_{21}| \simeq |A| \ll |\Delta E_{32}|$

Next let us consider the case where $|\Delta E_{21}| \simeq |A| \ll |\Delta E_{32}|$. In this case it is more convenient to adopt the original parametrization for U proposed by Kobayashi and Maskawa [26]:

$$\begin{aligned} U &= \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \\ &= e^{-i\theta_2\lambda_7} e^{-i\theta_1\lambda_2} \mathcal{D} e^{i\theta_3\lambda_7}, \end{aligned} \quad (23)$$

where $\mathcal{D} \equiv \text{diag}(1, 1, -e^{i\delta})$. The eigen matrix (15) is decomposed as the zero-th order term M_0 and the first order contribution M_1 with respect to $|\Delta E_{21}/\Delta E_{32}|$:

$$U \text{diag}(-\Delta E_{21}, 0, \Delta E_{32}) U^{-1} + \text{diag}(A, 0, 0) = U(M_0 + M_1)U^{-1} \quad (24)$$

with

$$\begin{aligned} M_0 &\equiv (0, 0, \Delta E_{32}), \\ M_1 &\equiv \text{diag}(-\Delta E_{21}, 0, 0) + U^{-1} \text{diag}(A, 0, 0) U \\ &= U^{-1} e^{-i\theta_2 \lambda_7} e^{-i\theta_1^M \lambda_2} \text{diag}(t_-, t_+, 0) e^{i\theta_1^M \lambda_2} e^{i\theta_2 \lambda_7} U \\ &= e^{-i\theta_3 \lambda_7} \mathcal{D}^{-1} e^{i\tilde{\theta}_1 \lambda_2} \text{diag}(t_-, t_+, 0) e^{-i\tilde{\theta}_1 \lambda_2} \mathcal{D} e^{i\theta_3 \lambda_7} \\ &\equiv \text{diag}(\xi_1, \xi_2, \xi_3) + \eta_1 \lambda_1 + \eta_4 \lambda_4 + \eta_6 \lambda_6, \end{aligned} \quad (25)$$

where

$$\begin{aligned} t_{\pm} &\equiv \frac{1}{2} \left(A - \Delta E_{21} \pm \sqrt{(\Delta E_{21} \cos 2\theta_1 - A)^2 + (\Delta E_{21} \sin 2\theta_1)^2} \right), \\ \tan 2\theta_1^M &\equiv \frac{\Delta E_{21} \sin 2\theta_1}{\Delta E_{21} \cos 2\theta_1 - A}, \\ \eta_1 &\equiv (t_+ - t_-) \tilde{c}_1 \tilde{s}_1 c_3, \quad \eta_4 \equiv (t_+ - t_-) \tilde{c}_1 \tilde{s}_1 s_3, \quad \eta_6 \equiv (t_- \tilde{s}_1^2 + t_+ \tilde{c}_1^2) c_3 s_3, \\ \xi_1 &\equiv t_- \tilde{c}_1^2 + t_+ \tilde{s}_1^2, \quad \xi_2 \equiv (t_- \tilde{s}_1^2 + t_+ \tilde{c}_1^2) c_3^2, \quad \xi_3 \equiv (t_- \tilde{s}_1^2 + t_+ \tilde{c}_1^2) s_3^2, \\ \tilde{\theta}_1 &\equiv \theta_1 - \theta_1^M, \quad \tilde{s}_1 \equiv \sin \tilde{\theta}_1, \quad \tilde{c}_1 \equiv \cos \tilde{\theta}_1. \end{aligned} \quad (26)$$

Using perturbation theory in $|\Delta E_{21}/\Delta E_{32}|$, it can be shown that the unitary matrix

$$V \equiv U e^{i(\eta_4 \lambda_5 + \eta_6 \lambda_7)/\Delta E_{32}} e^{-i\psi_1^M \lambda_2} \quad (27)$$

diagonalizes $U(M_0 + M_1)U^{-1}$ to first order in $|\Delta E_{21}/\Delta E_{32}|$:

$$U(M_0 + M_1)U^{-1} = V \text{diag} \left(\frac{\xi_1 + \xi_2}{2} + b, \frac{\xi_1 + \xi_2}{2} - b, \xi_3 + \Delta E_{32} \right) V^{-1}, \quad (28)$$

where

$$\begin{aligned} b &\equiv \sqrt{\left(\frac{\xi_1 - \xi_2}{2} \right)^2 + \eta_1^2} \\ \tan 2\psi_1^M &\equiv \frac{2\eta_1}{\xi_1 - \xi_2} = \frac{2(t_+ - t_-) \tilde{c}_1 \tilde{s}_1 c_3}{t_- \tilde{c}_1^2 + t_+ \tilde{s}_1^2 - (t_- \tilde{s}_1^2 + t_+ \tilde{c}_1^2) c_3^2}. \end{aligned} \quad (29)$$

In the limit $|\Delta E_{21}/\Delta E_{32}| \rightarrow 0$, (27) becomes

$$V \simeq U e^{-i\psi_1^M \lambda_2} = \begin{pmatrix} c_\psi U_{e1} + s_\psi U_{e2} & -s_\psi U_{e1} + c_\psi U_{e2} & U_{e3} \\ c_\psi U_{\mu 1} + s_\psi U_{\mu 2} & -s_\psi U_{\mu 1} + c_\psi U_{\mu 2} & U_{\mu 3} \\ c_\psi U_{\tau 1} + s_\psi U_{\tau 2} & -s_\psi U_{\tau 1} + c_\psi U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \quad (30)$$

where $s_\psi \equiv \sin \psi_1^M$, $c_\psi \equiv \cos \psi_1^M$. Notice that ψ_1^M survives even in the limit $|\Delta E_{21}/\Delta E_{32}| \rightarrow 0$. Therefore the modified Jarlskog factor $\mathcal{J}_2$ is given by

$$\begin{aligned} \mathcal{J}_2 &\simeq \text{Im} [(c_\psi U_{e1} + s_\psi U_{e2})(-s_\psi U_{e1} + c_\psi U_{e2}) \\ &\quad \times (c_\psi U_{\mu 1} + s_\psi U_{\mu 2})^*(-s_\psi U_{\mu 1} + c_\psi U_{\mu 2})] \\ &= \frac{1}{2} \left[\frac{1}{2} \sin 2\psi_1^M (1 - U_{e3}^2) - U_{e1} U_{e2} \cos 2\psi_1^M \right] U_{e3} \sin 2\theta_2 \sin \delta. \end{aligned} \quad (31)$$

In the case of the small mixing angle MSW solution for which we may neglect small factors U_{e3}^2 and U_{e2} , we have

$$\mathcal{J}_2 = \frac{1}{4} \sin 2\psi_1^M \sin 2\theta_2 U_{e3} \sin \delta \simeq \frac{1}{4} \sin 2\theta_1^M \sin 2\theta_2 U_{e3} \sin \delta, \quad (32)$$

where we have used the fact $\tan 2\psi_1^M \simeq \tan 2\theta_1^M$ which follows from (29) when $|\theta_3| \ll 1$. (32) could be large since $\mathcal{J}_2$ does not contain the suppression factor $|\Delta E_{21}/\Delta E_{32}|$ like in $\mathcal{J}_1$, $\sin 2\theta_1^M$ could be enhanced by the matter effect of the Earth (*cf.* (26)), and the only factor which is always small is U_{e3} ($|U_{e3}| \lesssim \sqrt{0.1}/2$). Note that we know $\Delta E_{21} > 0$ from the solar neutrino deficit.

4.3. Numerical analysis

The results by numerical calculations are given in Fig. 5 where rapid oscillations coming from the larger eigenvalues are averaged over, maximal T violation is assumed ($\delta = \pi/2$) and the best fit values [27] of the large and small angle MSW solutions have been chosen for the set of parameters $(\Delta m_{21}^2, \sin^2 2\theta_{12})$, respectively. In the case of the large mixing angle MSW solution there is some resonance for $E_\nu \sim$ several GeV and $E_\nu \sim \mathcal{O}(0.1)$ GeV which satisfy the conditions (a) and (b), respectively. In the case of the small mixing angle MSW solution, the enhancement is hardly visible for the energy $E_\nu \sim$ several GeV which satisfies the condition (a) but it is significant for $E_\nu \sim \mathcal{O}(10)$ MeV which satisfies the condition (b). We see from Fig. 5 that T violating effects are significant for relatively low neutrino energy, *i.e.*, $E_\nu \sim \mathcal{O}(0.1)$ GeV and $E_\nu \sim \mathcal{O}(10)$ MeV in the case of the

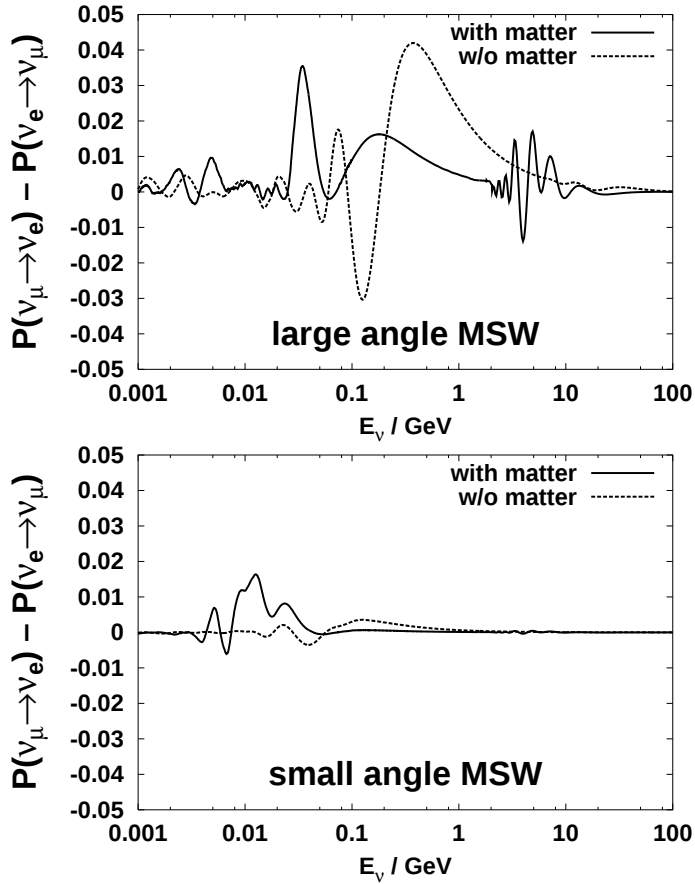


Fig. 5. T violation $P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu)$ in matter (solid lines) and in vacuum (dashed lines) for the large ($\Delta m_{21}^2 = 1.8 \times 10^{-5} \text{eV}^2$, $\sin^2 2\theta_{12} = 0.76$) and small ($\Delta m_{21}^2 = 5.4 \times 10^{-6} \text{eV}^2$, $\sin^2 2\theta_{12} = 0.006$) angle MSW solutions. $\sin^2 2\theta_{13} = 0.036$ and the set of the best fit parameters $(\Delta m_{32}^2, \sin^2 2\theta_{23}) = (3.5 \times 10^{-3} \text{eV}^2, 1.0)$ for the atmospheric neutrino data have been taken as reference values, maximum T violation $\delta = \pi/2$ has been assumed and rapid oscillations due to larger eigenvalues have been averaged over.

large and small angle MSW solutions, respectively. For the former case, the enhancement around $E_\nu \sim$ several GeV might be observable at neutrino factories [28], where intense beams of ν_e and ν_μ (or $\bar{\nu}_e$ and $\bar{\nu}_\mu$) are produced. For the latter case, it would be very difficult to see these effects unless we can produce intense beams of neutrinos with energy $E_\nu \sim \mathcal{O}(10)$ MeV. It should be also mentioned that it is hopeless to see T violation effects in the case of the vacuum oscillation solutions, since Δm_{21}^2 is extremely small.

5. Conclusions

To summarize, I have shown that it is possible to probe the small value of θ_{13} down to $\sin^2 2\theta_{13} \gtrsim 0.01$ in very long baseline experiments by looking for ν_e (in the case of $\Delta m_{32}^2 > 0$) or $\bar{\nu}_e$ (in the case of $\Delta m_{32}^2 < 0$) appearance which is enhanced due to the matter effect of the Earth. If we can produce very intense beams of ν_μ and ν_e with low energy then we may be able to determine the CP violating phase δ by looking at T violating effects in the case of the MSW solutions.

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