

NEUTRINO MASSES AND MIXINGS WITH FLAVOR SYMMETRIES*

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Recent atmospheric neutrino data at Super-Kamiokande suggest the large flavor mixing of neutrinos. Models for the lepton mass matrix, which give the near-maximal flavor mixing, are discussed in the three family model. Especially, details of the models with the S_3 or $O(3)$ flavor symmetry are studied.

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1. Introduction

Recent experimental data of neutrinos make big impact on the neutrino masses and their mixings. Most exciting one is the results at Super-Kamiokande on the atmospheric neutrinos, which indicate the large neutrino flavor oscillation of $\nu_\mu \rightarrow \nu_\tau$ [1]. Solar neutrino data also provide the evidence of the neutrino oscillation, however, this problem is still uncertain [2].

Furthermore, a new stage is represented by the Long Baseline(LBL) neutrino oscillation experiments. The first LBL reactor experiment CHOOZ has provided a bound of the neutrino oscillation [3], which gives a strong constraint of the flavor mixing pattern. The LBL accelerator experiment K2K [4] begins taking data, whereas the MINOS [5] and ICARUS [6] experiments will start in the first year of the next century. Those LBL experiments are expected to clarify masses, flavor mixings and CP violation of neutrinos.

The short baseline experiments may be helpful to understand neutrino masses and flavor mixings. The tentative indication has been already given by the LSND experiment [7], which is an accelerator experiment for $\nu_\mu \rightarrow \nu_e$ ($\bar{\nu}_\mu \rightarrow \bar{\nu}_e$). The CHORUS and NOMAD experiments [8,9] have reported

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the new bound for $\nu_\mu \rightarrow \nu_\tau$ oscillation, which has already improved the E531 result [10]. The KARMEN experiment [11] is also searching for the $\nu_\mu \rightarrow \nu_e (\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ oscillation as well as LSND. However, they did not observe any evidence of the oscillation. The Bugey [12] and Krasnoyarsk [13] reactor experiments and CDHS [14] and CCFR [15] accelerator experiments have given bounds for the neutrino mixing parameters as well as E776 [16].

What can we learn from these experimental results? We want to get clues for the origin of neutrino masses and neutrino flavor mixings. In this paper, we concentrate our discussion on the flavor symmetry, which controls the flavor structure of quark-lepton masses and mixings.

2. Possible neutrino mass hierarchy and mass matrix texture

Our starting point as to the neutrino mixing is the large $\nu_\mu \rightarrow \nu_\tau$ oscillation of the atmospheric neutrino oscillation with $\Delta m_{\text{atm}}^2 = (2 \sim 6) \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta_{\text{atm}} \geq 0.84$, which are derived from the recent data of the atmospheric neutrino deficit at Super-Kamiokande [1]. In the solar neutrino problem [2], there are three solutions: the small or large mixing angle MSW [17] solution and the vacuum oscillation solution (just so solution) [18]. These mass difference scales are much smaller than the atmospheric one.

Once we put $\Delta m_{\text{atm}}^2 = \Delta m_{32}^2$ and $\Delta m_{\odot}^2 = \Delta m_{21}^2$, there are three typical mass patterns: $m_3 \gg m_2 \geq m_1$, $m_3 \simeq m_2 \simeq m_1$ and $m_1 \simeq m_2 \gg m_3$. In this case, the LSND data is disregarded because there are only two mass difference scales in the three family model.

If one goes beyond three neutrinos, the sterile neutrinos are introduced. These reconcile LSND result [7]. Then one can explain the difference of the mixing pattern between quarks and leptons because the sterile neutrino couples to active neutrinos. This case has been discussed by Grimus at this school [19].

The neutrino mixing is defined as $\nu_\alpha = U_{\alpha i} \nu_i$ [20], where α denotes the flavor e, μ, τ and i denotes mass eigenvalues 1, 2, 3. Now we have two typical mixing patterns:

$$U_{\text{MNS}} \simeq \begin{pmatrix} 1 & U_{e2} & U_{e3} \\ U_{\mu 1} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ U_{\tau 1} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & U_{e3} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

the first one is the single maximal mixing pattern, in which the solar neutrino deficit is explained by the small mixing angle MSW solution, and the other is the bi-maximal mixings pattern [21], in which the solar neutrino deficit is explained by the just so solution or the large mixing angle of MSW solution. In both cases U_{e3} is constrained by the CHOOZ data [3].

Before discussing possible mass matrices of neutrinos, we show how to get U_{MNS} from the mass matrix as follows:

$$U_{\text{MNS}} = L_E^\dagger L_\nu, \quad m_E^{\text{diagonal}} = L_E^\dagger m_E R_E, \quad m_\nu^{\text{diagonal}} = L_E^T m_\nu L_E, \quad (2)$$

where neutrinos are assumed to be Majorana particles. So the large mixing in U_{MNS} could come from $L_E^\dagger$ or/and L_ν . The pattern of the 2×2 sub-matrix with $\sin^2 2\theta = 1$ is given in terms of the small parameter ε as

$$\begin{aligned} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} &\Rightarrow (0, 2), & \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} &\Rightarrow (1 - \varepsilon, 1 + \varepsilon), \\ \begin{pmatrix} \varepsilon & 1 \\ 1 & \varepsilon \end{pmatrix} &\Rightarrow (-1 - \varepsilon, 1 - \varepsilon), & \begin{pmatrix} \varepsilon & 1 \\ \varepsilon & 1 \end{pmatrix} &\Rightarrow (0, \sqrt{2(1 + \varepsilon^2)}). \end{aligned} \quad (3)$$

The first matrix gives the hierarchical eigenvalues, so it is useful for the neutrino and charged lepton mass matrices. The second and third ones give almost degenerate eigenvalues, which are useful only for neutrino masses. The last one is the asymmetric mass matrix with the hierarchical eigenvalues. So it is useful only for the charged lepton. If the 3×3 mass matrix includes these sub-matrices, the maximal mixing is derived. Moreover, there are some additional patterns in the 3×3 matrix [22].

The left handed neutrino masses are supposed to be at most $\mathcal{O}(1)\text{eV}$. We need some physical reasons for the smallness of the neutrino mass. In the case of Majorana neutrino, we know two classes of models which lead naturally to a small neutrino mass: *(i)* models in which the see-saw mechanism works [23] and *(ii)* those in which the neutrino mass is induced by a radiative correction. The central idea of models *(i)* supposes some higher symmetry which is broken at the high energy scale. If this symmetry breaking takes place so that it allows the right-handed neutrino to have a mass, and a small mass induced for the left handed neutrino by the see-saw mechanism. In the classes of model *(ii)* one introduces a scalar particle with a mass of the order of the electroweak (EW) energy scale which breaks the lepton number in the scalar sector. A left-handed neutrino mass is then induced by a radiative correction from a scalar loop without the right-handed neutrinos. This model requires some new physics at the EW scale.

Anyway, models of *(i)* and *(ii)* reduce to the effective dimension-five operator

$$\frac{\kappa_{ij}}{\Lambda} \phi^0 \phi^0 \nu_i \nu_j, \quad (4)$$

where ϕ^0 is the SU(2) doublet Higgs in the SM, which generates Majorana neutrino masses and mixings. The structure of the κ_{ij}/Λ depends on details of models [24]. In the following, we present typical mass matrix models which lead to the large flavor mixing of the atmospheric(solar) neutrinos.

See-saw enhancement:

We begin with discussing the see-saw enhancement. The see-saw mechanism of neutrino mass generation gives a very natural and elegant understanding for the smallness of neutrino masses. This mechanism may play another important role, which is to reproduce the large flavor mixing. In the standpoint of the quark-lepton unification, the Dirac mass matrix of neutrinos is similar to the quark mass matrices. Therefore, the neutrino mixings are expected to be typically of the same order of magnitude as the quark mixings. However, the large flavor mixings of neutrinos could be obtained in the see-saw mechanism as a consequence of a certain structure of the right-handed Majorana mass matrix [25,26]. That is the so called see-saw enhancement of the neutrino mixing due to the cooperation between the Dirac and Majorana mass matrices.

Mass matrix of light Majorana neutrinos m_ν has the following form

$$m_\nu \simeq -m_D M_R^{-1} m_D^T, \quad (5)$$

where m_D is the neutrino Dirac mass matrix and M_R is the Majorana mass matrix of the right-handed neutrino components. Then, the lepton mixing matrix is [25] $U_{\text{MNS}} = S_\ell^\dagger \cdot S_\nu \cdot U_{ss}$, where S_ℓ , S_ν are transformations which diagonalize the Dirac mass matrices of charged leptons and neutrinos, respectively. The U_{ss} specifies the effect of the see-saw mechanism, *i.e.* the effects of the right-handed Majorana mass matrix. It is determined by

$$U_{ss}^T m_{ss} U_{ss} = \text{diag}(m_1, m_2, m_3) \quad \text{with} \quad m_{ss} = -m_D^{\text{diag}} M_R^{-1} m_D^{\text{diag}}. \quad (6)$$

In the case of two generations, the mixing matrix U_{ss} is easily investigated in terms of one angle θ_s . This angle could be maximal with the same set of parameters in the Dirac mass matrix and the right handed Majorana mass matrix. That is the enhancement due to the see-saw mechanism. The rich structure of right-handed Majorana mass matrix can lead to the maximal flavor mixing of neutrinos. The detailed studies have been given recently in Refs. [27,28].

Asymmetric mass matrix:

The large mixing angle could be derived from the asymmetric mass matrix of charged leptons. In the standpoint of the quark-lepton unification, the charged lepton mass matrix is connected with the down quark one. The mixing following from the charged lepton mass matrix may be considered to be small like quarks in the hierarchical base. However, this expectation is not true if the mass matrix is non-Hermitian (asymmetric mass matrix). In the $SU(5)$, fermions belong to **10** and **5***:

$$\mathbf{10} : \chi_{ab} = u^c + Q + e^c, \quad \mathbf{5}^* : \psi^a = d^{c1} + L, \quad (7)$$

where Q and L are $SU(2)$ doublets of quarks and leptons, respectively. The Yukawa couplings are given by $10_i 10_j 5_H$ (up-quarks) and $5_i^* 10_j 5_H^*$ (down-quarks and charge leptons) ($i, j=1, 2, 3$). Therefore, we get $m_E = m_D$ at the GUT scale.

It should be noticed that observed quark mass spectra and the CKM matrix constrains only the down quark mass matrix typically as follows:

$$m_{\text{down}} \sim K_D \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^4 \\ x & \lambda^2 & \lambda^2 \\ y & z & 1 \end{pmatrix} \quad \text{with } \lambda = 0.22. \quad (8)$$

Three parameters x , y , z are not determined by observed quark mass spectra and the CKM matrix. Those are related to the left-handed charged lepton mixing due to $m_E = m_D^T$. The left(right)-handed down quark mixings are related to the right(left)-handed charged lepton mixings in the $SU(5)$. Therefore, there is a source of the large flavor mixing in the charged lepton sector if $z \simeq 1$ or/and $y \simeq 1$ is derived from some models as follows:

$$m_\ell m_\ell^\dagger = m_{\text{down}}^\dagger m_{\text{down}} \sim K_D^2 \begin{pmatrix} x^2 + y^2 & yz + \lambda^2 x & y + \lambda^2 x \\ yz + \lambda^2 x & z^2 & z \\ y + \lambda^2 x & z & 1 \end{pmatrix}. \quad (9)$$

This mechanism was used by some authors [29–31].

Radiative neutrino mass:

In the class of models in (ii), neutrino masses are induced from the radiative corrections even if the right-handed neutrino is absent. The typical one is the Zee model, in which charged gauge singlet scalar induces the neutrino mass [32]. The diagonal terms of the Zee mass matrix are exactly zero due to the symmetry as follows:

$$m_\nu \sim \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix}. \quad (10)$$

In the case of $m_{e\mu} \simeq m_{e\tau} \gg m_{\mu\tau}$, both solar neutrino problem and atmospheric neutrino deficit can be explained. Then, the inverse hierarchy $m_1 \simeq m_2 \gg m_3$ and the bi-maximal mixing matrix are obtained [33].

The MSSM with R -parity violation can also give the neutrino masses and mixings [34,35]. The MSSM allows renormalizable B and L violation. The R -parity conservation forbids the B and L violation in the superpotential in order to avoid the proton decay. However, the proton decay is avoided in the tree level if either of B or L violating term vanishes. The simplest model is the bi-linear R -parity violating model with $\varepsilon_i H_u L_i$ for the lepton-Higgs coupling [34]. This model provides the large mixing which is consistent with atmospheric and solar neutrinos.

3. Search for flavor symmetry

Masses and mixings of the quark-lepton may suggest some flavor symmetry. The simple flavor symmetry is U(1), which was discussed intensively by Ramond *et al.* [36]. In their model, they assumed (1) Fermions carry U(1) charge, (2) U(1) is spontaneously broken by $\langle\theta\rangle$, in which θ is the EW singlet with U(1) charge -1 , and (3) Yukawa couplings appear as effective operators a la Froggatt–Nielsen mechanism [37],

$$h_{ij}^D Q_i \bar{d}_j H_d \left(\frac{\theta}{\Lambda}\right)^{m_{ij}} + h_{ij}^U Q_i \bar{u}_j H_u \left(\frac{\theta}{\Lambda}\right)^{n_{ij}} + \dots, \quad (11)$$

where $\langle\theta\rangle/\Lambda = \lambda \simeq 0.22$. The powers m_{ij} and n_{ij} are determined from the U(1) charges of fermions in order that the effective operators are U(1) invariants as,

$$m_{ij} = Y_{Q_i} + Y_{\bar{d}_j} + Y_{H_d}, \quad n_{ij} = Y_{Q_i} + Y_{\bar{u}_j} + Y_{H_u}, \quad (12)$$

where Y denotes the U(1) charge. The U(1) charges of the fermions are fixed by the experimental data of the fermion masses and mixings. Their naive U(1) symmetric mass matrices could be modified by taking account of new fields or new symmetries.

Another approach is based on the non-Abelian flavor symmetry S_3 . The $S_{3L} \times S_{3R}$ symmetric mass matrix is the so called democratic mass matrix [38],

$$M_q = c_q \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (13)$$

which needs the large rotation in order to move to the diagonal base as $A^T M_q A$, where

$$A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}. \quad (14)$$

In the CKM mixing matrix, this large rotation matrix A is completely canceled out between down quarks and up quarks. This democratic mass matrix is not a realistic one because two quarks are massless. There are many works in which realistic quark mass matrices are discussed including symmetry breaking terms in the quark sector [39]. However, the situation of the lepton sector is very different from the quark sector since the effective neutrino mass matrix m_{LL}^ν could be far from the democratic one and the charged lepton one is still the democratic one.

The neutrino mass matrix is different from the democratic one if they are Majorana particles. The S_{3L} symmetric mass term is given as follows:

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + c_\nu r \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (15)$$

where r is an arbitrary parameter. The eigenvalues of this matrix are given as $c_\nu(1+2r, 1-r, 1-r)$, which means that there are at least two degenerate masses in the S_{3L} symmetric Majorana mass matrix [40–42].

In order to explain both solar and atmospheric neutrinos, three neutrinos should be almost degenerate in this model. If three degenerate light neutrinos are required, the parameter r should be taken as $r = 0$ or $r = -2$. The first case was discussed in Ref. [40] and the second one in Ref. [41]. The difference of r leads to the difference in the CP property of neutrinos.

In order to reproduce the atmospheric neutrino deficit by the large neutrino oscillation, the symmetry breaking terms are required. Since the results are almost the same, we show the numerical analyses in Ref. [40].

Let us start with discussing the following charged lepton mass matrix:

$$M_\ell = \frac{c_\ell}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + m_{\text{break}}. \quad (16)$$

The first term is a unique representation of the $S_{3L} \times S_{3R}$ symmetric matrix and the second one m_{break} is a symmetry breaking one. The unitary matrix that diagonalises the charged lepton mass matrix is $U_\ell = AB_\ell$, where the matrix A is defined in Eq. (14) and B_ℓ depends on the symmetry breaking term m_{break} .

Let us turn to the neutrino mass matrix, in which $r = 0$ is taken as:

$$M_\nu = c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon_\nu & 0 \\ \varepsilon_\nu & 0 & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}, \quad (17)$$

where the symmetry breaking is given by a small term with two adjustable parameters. It is seen that S_{3L} is broken by δ_ν but S_{2L} is still preserved in Eq. (17). The mass eigenvalues are $c_\nu \pm \varepsilon_\nu$, and $c_\nu + \delta_\nu$, and the matrix that diagonalises M_ν ($U_\nu^T M_\nu U_\nu = \text{diagonal}$) is

$$U_\nu = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (18)$$

That is, our M_ν represents three degenerate neutrinos, with the degeneracy lifted by a small parameter.

The lepton mixing angle as defined by $U_{\text{MNS}} = (U_\ell)^\dagger U_\nu = (AB_\ell)^\dagger U_\nu$ is thus given by

$$U_{\text{MNS}} \simeq \begin{pmatrix} 1 & \frac{1}{\sqrt{3}}B_{\ell 21} & -\frac{2}{\sqrt{6}}B_{\ell 21} \\ B_{\ell 12} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad (19)$$

where $B_{\ell 21}$ and $B_{\ell 12}$ are correction terms in the charged lepton sector, typically, $B_{\ell 21} \sim \sqrt{m_e/m_\mu}$. We have predictions

$$\sin^2 2\theta_{\text{atm}} \simeq \frac{8}{9}, \quad U_{e2} \simeq -\frac{1}{\sqrt{2}}U_{e3}, \quad (20)$$

where $U_{\alpha i}$ denotes the MNS mixing. If $B_{\ell 21} \simeq \sqrt{m_e/m_\mu}$, we get $U_{e2} \simeq 0.04$ and $U_{e3} \simeq 0.057$, which leads to $\sin^2 2\theta_\odot \simeq 6.5 \times 10^{-3}$. This prediction also agrees with the neutrino mixing corresponding to the small mixing angle MSW solution $(4 - 13) \times 10^{-3}$ for the solar neutrino problem [2]. In the future, this prediction will be tested in the following long baseline experiments $\nu_\nu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$.

Let us briefly discuss the consequence of the other symmetry breaking of neutrino masses. If we adopt the symmetry breaking term alternative to Eq. (17),

$$\begin{pmatrix} \rho_\nu & 0 & 0 \\ 0 & \varepsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}, \quad (21)$$

in which $S_{3\text{L}}$ is completely broken, we obtain the lepton mixing matrix to be

$$U_{\text{MNS}} \simeq A^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}. \quad (22)$$

This is identical to the matrix presented by Fritzsch and Xing [43]. For this case one gets

$$\sin^2 2\theta_\odot \simeq 1, \quad \sin^2 2\theta_{\text{atm}} \simeq 8/9. \quad (23)$$

This case can accommodate the “just-so” solution for the solar neutrino problem due to neutrino oscillation in vacuum and may be also consistent with the large mixing angle MSW solution including correction terms. This matrix has been investigated in detail [44] focusing on recent data at Super-Kamiokande.

In the model, the symmetry breaking terms are not unique, and moreover, the neutrino mass degeneracy is put by hand, $r = 0$. In order to avoid these ambiguities, we should go to higher symmetry of flavors.

4. O(3) flavor symmetry and phenomenology

We assume that neutrinos are almost degenerate. Since the quark-lepton masses are hierarchical, one may raise a question: How can one gets the consistent picture in these mass generation? The O(3) flavor symmetry [45, 46] has a unique prediction, that is almost degenerate neutrino masses. Masses of quarks and charged leptons vanish in the O(3) symmetric limit. Therefore, mass matrices of quarks and leptons are determined by details of breaking pattern of the flavor symmetry. Although there are some symmetry breaking mechanisms [45, 46] we discuss a possible flavor O(3) breaking mechanism [47] that leads to “successful” phenomenological mass matrices with S_3 symmetry in the previous section.

We consider the supersymmetric standard model and impose $O(3)_L \times O(3)_R$ flavor symmetry. Three lepton doublets $\ell_i (i = 1 - 3)$ transform as an $O(3)_L$ triplet and three charged leptons $\bar{e}_i (i = 1 - 3)$ as an $O(3)_R$ triplet, while Higgs doublets H and $\bar{H}$ are $O(3)_L \times O(3)_R$ singlets. We will discuss the quark sector later.

We introduce, to break the flavor symmetry, a pair of fields $\Sigma_L^{(i)} (i = 1, 2)$ and $\Sigma_R^{(i)} (i = 1, 2)$ which transform as symmetric traceless tensors **5**'s of $O(3)_L$ and $O(3)_R$, respectively. We assume that the $\Sigma_L^{(i)}(\mathbf{5}, \mathbf{1})$ and $\Sigma_R^{(i)}(\mathbf{1}, \mathbf{5})$ take values

$$\Sigma_{L,R}^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} w_{L,R}^{(1)}, \quad (24)$$

and

$$\Sigma_{L,R}^{(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} w_{L,R}^{(2)}. \quad (25)$$

We consider that these are explicit breakings of $O(3)_L \times O(3)_R$ rather than vacuum-expectation values of $\Sigma_{L,R}^{(i)}$ (spontaneous breaking), otherwise we have unwanted massless Nambu–Goldstone multiplets. In the following discussion we use dimensionless breaking parameters $\sigma_L^{(i)}$ and $\sigma_R^{(i)}$, which are defined as

$$\sigma_{L,R}^{(1)} \equiv \frac{\Sigma_{L,R}^{(1)}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_{L,R}, \quad (26)$$

and

$$\sigma_{L,R}^{(2)} \equiv \frac{\Sigma_{L,R}^{(2)}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \varepsilon_{L,R} . \quad (27)$$

Here, M_f is the large flavor mass scale, $\delta_{L,R} = w_{L,R}^{(1)}/M_f$ and $\varepsilon_{L,R} = w_{L,R}^{(2)}/M_f$. We assume $\delta_{L,R}, \varepsilon_{L,R} \leq 1$.

The neutrinos acquire small Majorana masses from the superpotential,

$$W = \frac{H^2}{M} \ell (1 + \alpha_{(i)} \sigma_L^{(i)}) \ell , \quad (28)$$

which yields a neutrino mass matrix as

$$\begin{aligned} \hat{m}_\nu = \frac{\langle H \rangle^2}{M} \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \alpha_{(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_L \right. \\ \left. + \alpha_{(2)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \varepsilon_L \right\} . \quad (29) \end{aligned}$$

Here, $\alpha_{(i)}$ are $\mathcal{O}(1)$ parameters and the mass M denotes a cut-off scale of the present model which may be different from the flavor scale M_f . We take $M \simeq 10^{14-15} \text{ GeV}$ to obtain $m_{\nu_i} \simeq 0.1 - 1 \text{ eV}$ indicated from the atmospheric neutrino oscillation [1] for degenerate neutrinos.

The above breaking is, however, incomplete, since the charged leptons remain massless. We introduce an $O(3)_L$ -triplet and an $O(3)_R$ -triplet fields $\phi_L(\mathbf{3}, \mathbf{1})$ and $\phi_R(\mathbf{1}, \mathbf{3})$ to produce masses of the charged leptons. The vacuum expectation values of ϕ_L and ϕ_R are determined by the following superpotential:

$$\begin{aligned} W = & Z_L(\phi_L^2 - 3v_L^2) + Z_R(\phi_R^2 - 3v_R^2) \\ & + X_L(a_{(i)} \phi_L \sigma_L^{(i)} \phi_L) + X_R(a'_{(i)} \phi_R \sigma_R^{(i)} \phi_R) \\ & + Y_L(b_{(i)} \phi_L \sigma_L^{(i)} \phi_L) + Y_R(b'_{(i)} \phi_R \sigma_R^{(i)} \phi_R) . \quad (30) \end{aligned}$$

Here, the fields $Z_{L,R}$, $X_{L,R}$ and $Y_{L,R}$ are all singlets of $O(3)_L \times O(3)_R$.

We obtain vacuum-expectation values from the superpotential Eq. (30) by solving $|F_X| = 0$, $|F_Y| = 0$ and $|F_Z| = 0$:

$$\langle \phi_L \rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_L , \quad \langle \phi_R \rangle \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_R . \quad (31)$$

Notice that only with the first two terms in Eq. (30) we have $O(3)_L \times O(3)_R$ global symmetry and hence unwanted Nambu-Goldstone multiplets appear in broken vacua. The couplings to the explicit breakings $\sigma_{L,R}^{(i)}$ are necessary to eliminate the Nambu-Goldstone multiplets in the low energy spectrum, which determine vacuum-expectation values of ϕ_L and ϕ_R as in Eq. (31).

With the non-vanishing $\langle \phi_L \rangle$ and $\langle \phi_R \rangle$ in Eq. (31), the Dirac masses of charged leptons arise from the superpotential,

$$W = \frac{\kappa_E}{M_f^2} (\bar{e} \phi_R) (\phi_L \ell) \bar{H}. \quad (32)$$

This produces so-called “democratic” mass matrix of the charged leptons,

$$\hat{m}_E = \kappa_E \left(\frac{v_L v_R}{M_f^2} \right) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \langle \bar{H} \rangle. \quad (33)$$

Diagonalization of this mass matrix yields large lepton mixings [40, 43] and one non-vanishing eigenvalue, m_τ . The masses of e and μ are derived from distortion of the “democratic” form of mass matrix in Eq. (33), which is given by a superpotential containing the explicit $O(3)_L \times O(3)_R$ breaking parameters $\sigma_{L,R}^{(i)}$,

$$\delta W = \frac{\kappa_E}{M_f^2} \left\{ A_i^\ell (\bar{e} \sigma_R^{(i)} \phi_R) (\phi_L \ell) + B_i^\ell (\bar{e} \phi_R) (\phi_L \sigma_L^{(i)} \ell) + C_{ij}^\ell (\bar{e} \sigma_R^{(i)} \phi_R) (\phi_L \sigma_L^{(j)} \ell) \right\} \bar{H}. \quad (34)$$

Then, the charged lepton mass matrix is given in the hierarchical base by

$$A^T \hat{m}_E A = \frac{\kappa_E v_L v_R}{M_f^2} \langle \bar{H} \rangle \begin{pmatrix} 2C_{22}^\ell \varepsilon_L \varepsilon_R & 2\sqrt{3}C_{21}^\ell \varepsilon_R \delta_L & \sqrt{6}A_2^\ell \varepsilon_R \\ 2\sqrt{3}C_{12}^\ell \varepsilon_L \delta_R & 6C_{11}^\ell \delta_L \delta_R & 3\sqrt{2}A_1^\ell \delta_R \\ \sqrt{6}B_2^\ell \varepsilon_L & 3\sqrt{2}B_1^\ell \delta_L & 3 \end{pmatrix}_{RL}, \quad (35)$$

where the matrix A is defined in Eq. (14). The mass eigenvalues of this lepton mass matrix are

$$m_\tau \simeq 3\kappa_E \frac{v_L}{M_f} \frac{v_R}{M_f} \langle \bar{H} \rangle, \quad \frac{m_\mu}{m_\tau} \simeq \mathcal{O}(\delta_L \delta_R), \quad \frac{m_e}{m_\tau} \simeq \mathcal{O}(\varepsilon_L \varepsilon_R), \quad (36)$$

where we assume that all coupling parameters A_i^ℓ , B_i^ℓ and C_{ij}^ℓ ($i, j = 1, 2$) are of $\mathcal{O}(1)$.

We now turn to the quark sector, in which three doublet quarks q_i transform as an $O(3)_L$ triplet while three down quarks $\bar{d}_i$ and the three up quarks $\bar{u}_i$ as $O(3)_R$ triplets. Quark mass matrices are the same ones as in Eq. (35) apart from $O(1)$ coefficients A_i^ℓ , B_i^ℓ and C_{ij}^ℓ . The CKM mixing angles are given by

$$|V_{us}| \simeq \frac{\varepsilon_L}{\delta_L}, \quad |V_{cb}| \simeq \delta_L, \quad |V_{ub}| \simeq \varepsilon_L. \quad (37)$$

Putting the experimental quark mass ratios and CKM matrix elements:

$$\frac{m_d}{m_b} \simeq \lambda^4, \quad \frac{m_s}{m_b} \simeq \lambda^2, \quad |V_{us}| \simeq \lambda, \quad |V_{cb}| \simeq \lambda^2, \quad (38)$$

we obtain the order of parameters as follows:

$$\delta_L \simeq \lambda^2, \quad \delta_R \simeq 1, \quad \varepsilon_L \simeq \lambda^3, \quad \varepsilon_R \simeq \lambda, \quad (39)$$

with $\lambda \simeq 0.2$. Then, we predict $|V_{ub}| \simeq \varepsilon_L \simeq \lambda^3$, which is consistent with the experimental value [48]. Thus our model successfully explains both lepton and quark mass matrices.

Let us discuss neutrino masses and the mixings. Basing on the analysis on the quark mass matrices we take $\delta_L \simeq 0.1$ and $\varepsilon_L \simeq 10^{-3} - 10^{-2}$. We should remark that there is an additional contribution to the neutrino mass matrix in Eq. (29) as

$$\delta W = \frac{H^2}{M} \ell \left(\beta \frac{\phi_L}{M_f} \frac{\phi_L}{M_f} \right) \ell. \quad (40)$$

The neutrino mass matrix is now given by

$$\begin{aligned} \hat{m}_\nu = & \frac{\langle H \rangle^2}{M} \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \alpha_{(1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_L \right. \\ & \left. + \alpha_{(2)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \varepsilon_L + \beta \left(\frac{v_L}{M_f} \right)^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\}. \quad (41) \end{aligned}$$

The large MNS mixing angle between ν_μ and ν_τ is obtained if

$$\beta \left(\frac{v_L}{M_f} \right)^2 \ll \alpha_{(1)} \delta_L. \quad (42)$$

We also see large neutrino mixings between ν_e and $\nu_{\mu,\tau}$ for $\beta(v_L/M_f)^2 \leq \alpha_{(2)} \varepsilon_L$. By using $\Delta m_{23}^2 (\equiv m_{\nu_3}^2 - m_{\nu_2}^2) \simeq 10^{-3} \text{eV}^2$ for the $\nu_\mu - \nu_\tau$ oscillation [1]

(which corresponds to $m_{\nu_i} = \mathcal{O}(0.1)\text{eV}$), $\delta_L \simeq 0.1$ and $\varepsilon_L \simeq 10^{-3} - 10^{-2}$, we obtain

$$\Delta m_{12}^2 \simeq \frac{\varepsilon_L}{\delta_L} \Delta m_{23}^2 \simeq 10^{-5} - 10^{-4} \text{eV}^2, \quad (43)$$

for the $\nu_e - \nu_{\mu,\tau}$ oscillation. This is consistent with the large angle MSW solution [17] to the solar neutrino problem. The current analyses [49] of Super-Kamiokande experiments give $\Delta m_{12}^2 \simeq 2 \times 10^{-5} - 2 \times 10^{-4} \text{eV}^2$ and $\sin^2 2\theta_{12} = 0.60 - 0.97$ at the 99% confidence level, for the large MSW solution. It is remarked that we obtained the numerical prediction $\sin^2 2\theta_{12} = 0.60 - 0.97$ under the condition $\beta(v_L/M_f)^2 \leq \alpha_{(2)}\varepsilon_L$.

We considered a model where ℓ_i and q_i belong to triplets of one $O(3)$ and $\bar{e}_i$, $\bar{d}_i$ and $\bar{u}_i$ belong to triplets of the other $O(3)$. We note here that there is another interesting assignment that ℓ_i and $\bar{d}_i$ are triplets of the $O(3)$ while $\bar{e}_i$, q_i and $\bar{u}_i$ transform as triplets of the other $O(3)$ by imposing a discrete symmetry such as Z_6 [47].

5. Summary

We have presented some typical mechanisms which lead to models for the lepton mass matrix, which gives the near-maximal flavor mixing. Especially, details of the models with the S_3 or $O(3)$ flavor symmetry are presented. Since these models predict almost degenerate neutrino masses, double- β decay experiments will test the model in the future [50]. Moreover, as our $O(3)_L \times O(3)_R$ model predicts the large mixing angle MSW solution, we wait for results in KamLAND experiment [51]. More theoretical works on the flavor symmetry as well as experimental data are expected.

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