

ARE NEUTRINOS DIRAC OR MAJORANA PARTICLES?* **

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In spite of the general belief that neutrinos are Majorana particles, their character should be revealed experimentally. We begin by discussing why it is so difficult in terrestrial experiments. If neutrinos are Majorana particles, the first signal should come from neutrinoless double β decay. Still the search for such a decay of various nuclei is negative. We outline how the present knowledge of neutrino masses and mixing matrix elements combined with the bound from $(\beta\beta)_{0\nu}$ decay could help to determine their nature.

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1. Introduction

There are two main problems in neutrino physics. First is the problem of neutrino masses, which in the light of present data [1] seems to be solved. Neutrinos are massive. The second is that of the neutrino nature. As massive they can be Dirac (with particles and antiparticles being different objects: $\nu \neq \bar{\nu}$) or Majorana (with particles and antiparticles being the same, just as

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for photons: $\nu = \bar{\nu}$). An experimental distinction between these two seems to be much more complicated than the confirmation of non-vanishing mass. While experimentalists are trying to find some way of doing it, theorists have no doubts. They widely believe in the Majorana nature [2]. Almost all extensions of the Standard Model (SM) predict it. The only way to have Dirac neutrinos is to impose lepton number conservation. However, there is no particular reason for this, since it is not a fundamental quantity like the electric charge. If we do, we immediately run into trouble. Let us mention only a loss of the natural ‘see-saw’ mechanism to explain the smallness of the neutrino mass.

Theoretical reasons aside, the scientific method obliges us to perform experimental studies, that would falsify either option. So then, why is it difficult? Naively one might think it to be rather easy. Imagine for example, neutrinos from π^+ decay ($\pi^+ \rightarrow \mu^+ \nu_\mu$) scattering on a nuclear target. The result is a flux of μ^- (antineutrinos $\bar{\nu}_\mu$ coming from π^- decay ($\pi^- \rightarrow \mu^- \bar{\nu}_\mu$) always produce antiparticles μ^+). Unfortunately the lepton number L ($L(\nu_\mu) = +1$, $L(\bar{\nu}_\mu) = -1$) is not the only property characterizing neutrinos. We know also from experiment [3] that neutrinos and antineutrinos have opposite helicity ($\nu_\mu = \nu(-)$ and $\bar{\nu}_\mu = \nu(+)$). Therefore, we are not able to state which is responsible for μ^- (μ^+) production, lepton number conservation or helicity. In the first case the left-handed neutrino fields $\nu_L(x)$

$$\nu_L(x) = \int \frac{d^3k}{(2\pi)^3} \left(A(-) e^{-ikx} - B^+(+) e^{ikx} \right) \chi(-) \quad (1)$$

are composed of two different operators (see [4] for a detailed definition). $A(-)$ which annihilates particles has negative helicity and $B^+(+)$ which creates particles has positive helicity.

For a massless Majorana field $N_L(x)$ only one operator $A = B \equiv a$ appears

$$N_L(x) = \int \frac{d^3k}{(2\pi)^3} \left(a(-) e^{-ikx} - a^+(+) e^{ikx} \right) \chi(-). \quad (2)$$

In order to check whether lepton number conservation ($A \neq B$) or particle helicity ($a(-) \neq a(+)$) is responsible for μ^- (μ^+) production, we have to compare neutrino interactions in the same helicity states

$$A(-) \quad \text{with} \quad B(-), \quad (3)$$

or

$$A(+) \quad \text{with} \quad B(+). \quad (4)$$

Unfortunately, the visible neutrino interactions are such that only particles in the states $A(-)$ and $B(+)$ are produced. No neutrinos in the states $A(+)$ and $B(-)$ appear in known experiments.

In the next section we would like to show the connection between the presence (or absence) of the states given in Eqs. (3) and (4) with the symmetries of the theory.

Next in Chapter 3 examples which explain the origin of the experimental difficulties of discerning Dirac from Majorana neutrinos are given. The main background being the small mass of neutrinos which causes that they are produced as highly relativistic particles and their visible left-handed interaction.

It is common belief that the first place to search is the neutrinoless double β decay $((\beta\beta)_{0\nu})$ of nuclei. Unfortunately up to now such a decay has not been found and experimental data gives lower bounds on $(\beta\beta)_{0\nu}$ decay modes of various nuclei. These in turn lead to the limit [5] on the so-called effective neutrino mass $\langle m_\nu \rangle$

$$|\langle m_\nu \rangle| \equiv \left| \sum U_{ei}^2 m_i \right| < 0.2 \text{ eV}. \quad (5)$$

There are plans to increase the sensitivity of the bound(s) down to 0.01 or even 0.001 eV [6]. If $\langle m_\nu \rangle \neq 0 (= 0)$ the neutrinos are massive Majorana (Dirac) particles. Currently, however, the bound (5) alone is not conclusive. There are nevertheless different experiments from which independent information on the neutrino mixing matrix elements U_{ei} and masses m_i can be inferred. Then, we can check whether the bound (5) is satisfied or not. If not, neutrinos are Dirac particles. If it is satisfied, no conclusion can be drawn. Such an analysis is performed in Chapter (4). Finally, in Chapter (5) the conclusions are given.

2. Dirac or Majorana nature of particles, and symmetries

We would like to explain how the particle content of a theory is connected with its symmetries.

We believe up to now [7], that the fundamental symmetry of any theory which describes elementary particle interactions is Lorentz invariance. This statement means precisely that the theory must be invariant under the proper orthochronous group of Lorentz transformations L_+^\uparrow . For massive particles, they mix states with all helicities, for massless, helicity is Lorentz invariant. So, from L_+^\uparrow invariance it follows that:

- for massive particles ($m \neq 0$) with spin j all states

$$|\vec{p}, \lambda\rangle \quad \text{for } \lambda = -j, -j + 1, \dots, +j \quad (6)$$

must be present in the theory

- for massless spin j particles ($m = 0$) only one state

$$|\vec{p}, \lambda = j\rangle \text{ or } |\vec{p}, \lambda = -j\rangle \quad (7)$$

must be introduced.

For example, it is possible to build a theory which has L_+^\uparrow invariance with three helicity states of the W^+ $\lambda = -1, 0, +1$ with no W^- and a photon of one polarization e.g. $|photon, \lambda = +1\rangle$ or a neutrino in the state $|\text{neutrino}, \lambda = -1/2\rangle$.

The next symmetry is invariance under the CPT transformation [8] which changes particles into antiparticles and helicity $\lambda \rightarrow -\lambda$.

$$\text{CPT } |\vec{p}, \lambda\rangle_{\text{particle}} = |\vec{p}, -\lambda\rangle_{\text{antiparticle}}. \quad (8)$$

In any theory with CPT symmetry, particles and antiparticles with opposite helicities must exist. In our example this means that W^- particles with $\lambda = \pm 1, 0$, and an antiphoton with $\lambda = -1$, and antineutrinos with $\lambda = +1/2$ must be present.

There are theories like QED where also the separate symmetries C, P and T hold. The helicity states transform as

$$\text{P } |\vec{p}, \lambda\rangle = \eta_{\text{P}} e^{i\pi\lambda} |-\vec{p}, -\lambda\rangle, \quad (9)$$

$$\text{T } |\vec{p}, \lambda\rangle = \eta_{\text{T}} e^{i\pi\lambda} |-\vec{p}, \lambda\rangle, \quad (10)$$

and

$$\text{C } |\vec{p}, \lambda\rangle_{\text{particle}} = \eta_{\text{C}} e^{i\pi\lambda} |\vec{p}, \lambda\rangle_{\text{antiparticle}}. \quad (11)$$

For massive particles these symmetries do not introduce new necessary particle states above those already present because of Lorentz invariance and CPT symmetry. For massless particles, however, P leads to the existence of particle (antiparticle) states with opposite helicities. Once more in our example there has to be a photon and an antiphoton.

Now we can go back to our previous statement: in order to determine the nature of neutral objects we need to compare the interaction of particles and antiparticles in the same helicity states.

$$|\vec{p}, \lambda\rangle_{\text{particle}} \quad \text{with} \quad |\vec{p}, \lambda\rangle_{\text{antiparticle}}. \quad (12)$$

In a theory with C, P and T symmetry:

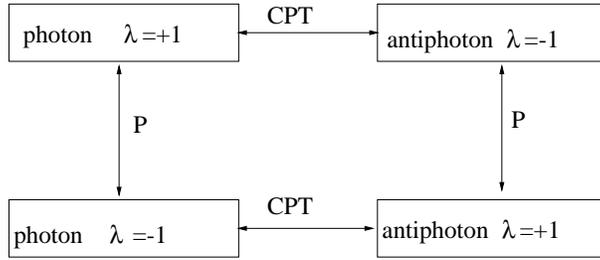


Fig. 1. Four ‘photon’ states connected by CPT and P transformations

- (i) such states exist for massive and massless particles
- (ii) from C symmetry particles and antiparticles interact in the same way so there is no way to distinguish them.

This means that in those fully symmetric theories, there are only MAJORANA neutral particles. That is why photons must be Majorana particles in QED.

All looks different in theories where C, P, T symmetries do not hold (like in the weak interactions).

For massive particles two states (Eq. (12)) exist and we can compare their interactions. Particles and antiparticles in the same helicity states can interact (i) in different ways or (ii) identically.

In case (i):

- some additive quantum number exists, which differentiate particles from antiparticles,
- particles and antiparticles are not the same,
- it is the case of massive Dirac neutrinos (described by bispinors) with lepton number conservation.

$$\begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix} \equiv \Psi_D \neq \Psi_D^C. \tag{13}$$

In the case (ii):

- additive quantum numbers cannot exist,
- particles and antiparticles are not indistinguishable, they are Majorana objects,

- there are two important examples of such particles: the Z_0 gauge boson, and massive Majorana neutrinos described by Majorana bispinors

$$\begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix} \equiv \Psi_M = \Psi_M^C. \quad (14)$$

For massless particles the symmetries do not require the existence of both states in Eq. (12). It is possible to build theories where particles and antiparticles in the same helicity state (*i*) do not exist or (*ii*) are introduced.

In the same case (*i*):

- the discussion about Dirac or Majorana nature of such particles is meaningless, there is nothing to compare,
- in the case of spin 1/2 objects there is a kinematical theorem [9], which proves that Weyl neutrinos are identical with massless Majorana neutrinos.

In the case (*ii*):

- two spinors ν_L and ν_R are introduced. As in the L-R symmetric model, four states described by $A(\pm)$ and $B(\pm)$ annihilation operators exist,
- objects $A(\pm)$ and $B(\pm)$ can interact in different ways so we have massless Dirac neutrinos (or if CP is conserved, two Majorana neutrinos with opposite η_{CP} parities), or
- objects $A(\pm)$ and $B(\pm)$ can interact in the same way and we have two identical massless Majorana neutrinos (these Majorana neutrinos have the same η_{CP} parity).

3. Why is it difficult to distinguish experimentally Dirac and Majorana neutrinos?

There are two main reasons, which cause that practically it is impossible, at least with the present experimental precision to determine nature of neutrinos [10]. Firstly, the created neutrinos are usually relativistic ($E \gg m$). On the other hand, cross sections for neutrino interaction are proportional to the energy E , so that nonrelativistic neutrinos interact with matter very weakly. Secondly, visible neutrino interactions are either left-handed $\frac{1}{2}\gamma^\mu(1 - \gamma_5)$ for gauge bosons or proportional to neutrino mass for scalar particles (m_ν/m_W for Higgs particles).

The forthcoming examples will demonstrate these problems.

Let us assume that a beam of muon neutrinos with helicity h_ν , scatters on a nuclear target. To be more general we consider the neutrino charged current interaction to be of the form

$$L_{CC} = \frac{g}{\sqrt{2}} [A_L (\bar{N}\gamma^\mu P_L l) + A_R (\bar{N}\gamma^\mu P_R l)] W_\mu^+ + \text{h.c.} \quad (15)$$

with a left-handed (A_L) and a hypothetical right-handed (A_R) part. Dirac neutrinos generate only μ^- 's (with helicity h_μ)

$$\nu^D + N \rightarrow \mu^- + X, \quad (16)$$

with the amplitude proportional to

$$\begin{aligned} A_{\mu^-}^D(h_\nu, h_\mu) &\sim A_L^* [(E_\mu - 2h_\mu p_\mu)(E_\nu - 2h_\nu p_\nu)]^{1/2} \\ &+ A_R^* [(E_\mu + 2h_\mu p_\mu)(E_\nu + 2h_\nu p_\nu)]^{1/2}, \end{aligned} \quad (17)$$

where E_μ, p_μ (E_ν, p_ν) is the energy and momentum of the muons (neutrinos).

Majorana neutrinos generate μ^- 's with exactly the same amplitude Eq. (17) and μ^+ 's. The amplitude for μ^+ production is now proportional to

$$\begin{aligned} A_{\mu^+}^M(h_\nu, h_\mu) &\sim A_L [(E_\mu + 2h_\mu p_\mu)(E_\nu + 2h_\nu p_\nu)]^{1/2} \\ &+ A_R [(E_\mu - 2h_\mu p_\mu)(E_\nu - 2h_\nu p_\nu)]^{1/2}. \end{aligned} \quad (18)$$

In the laboratory frame we are able to obtain a beam of muon neutrinos with helicity $h_\nu = -1/2$ (*e.g.* from $\pi^+ \rightarrow \mu^+ \nu_\mu$). The cross section for μ^+ production is unfortunately proportional to

$$\begin{aligned} \sigma_{\mu^+}^M(h_\nu = -1/2) &\sim \left| A_L \sqrt{E_\nu - p_\nu} + \zeta(h_\mu) A_R \sqrt{E_\nu + p_\nu} \right|^2 \\ &\approx \left| A_L \frac{m_\nu}{\sqrt{2E_\nu}} + \zeta(h_\mu) A_R \sqrt{2E_\nu} \right|^2, \end{aligned} \quad (19)$$

where

$$\zeta(h_\mu) = \left[\frac{E_\mu - 2h_\mu p_\mu}{E_\mu + 2h_\mu p_\mu} \right]^{\frac{1}{2}}. \quad (20)$$

Both terms in (19) are small in the high β limit.

For neutral current interactions the situation seems at first sight to be even more promising. There are two characteristic features, which are completely different for Dirac and Majorana neutrinos.

(i) the vector current $\bar{\nu}_M \gamma^\mu \nu_M = 0$, for Majorana neutrinos and

(ii) Majorana neutrinos, as identical particles, need symmetrization.

Let us consider shortly both of them. The respective neutral current interactions are of the form

$$L_{NC}(D) = \bar{\nu}_D \gamma^\mu (g_V^D - g_A^D \gamma_5) \nu_D Z_\mu, \quad (21)$$

and

$$L_{NC}(M) = \bar{\nu}_M \gamma^\mu (-g_A^M \gamma_5) \nu_M Z_\mu. \quad (22)$$

Despite this striking difference, both cases are again indistinguishable [10]. Let us consider the measurement of the total cross section for inclusive production (Fig.2)

$$\nu + N \rightarrow \nu + X. \quad (23)$$

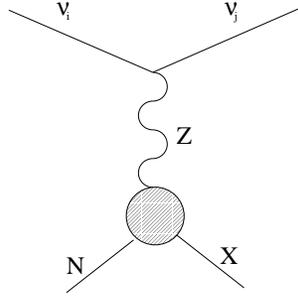


Fig.2. The Z exchange describes the process $\nu + N \rightarrow \nu + X$. The amplitude responsible for $N - Z - X$ interaction B_ν is schematically described in the text.

The amplitudes are given by

$$A_{i \rightarrow f}^D = \bar{u}_f \gamma^\mu (g_V^D - g_A^D \gamma_5) u_i D_{\mu\nu} B^\mu, \quad (24)$$

and

$$A_{i \rightarrow f}^M = (-g_A^M) [\bar{u}_f \gamma^\mu \gamma_5 u_i - \bar{v}_i \gamma^\mu \gamma_5 v_f] D_{\mu\nu} B^\nu, \quad (25)$$

where $D_{\mu\nu}$ is the Z_0 propagator and B^ν describes the Z_0 interaction with nuclei $Z_0 N \rightarrow X$.

Both amplitudes look different, but if we approximate them for relativistic neutrinos ($E_\nu \gg m_\nu$), with the relation

$$\bar{v}_i \gamma^\mu v_f = -\bar{u}_f \gamma^\mu u_i, \quad (26)$$

and

$$\gamma_5 u_i = -u_i + 0 \left(\frac{m_\nu}{E_\nu} \right), \quad (27)$$

we find in both cases

$$A_{i \rightarrow f} = \Omega \bar{u}_f \gamma^\mu u_i D_{\mu\nu} B^\nu, \tag{28}$$

where $\Omega = g_V^D + g_A^D$ for Dirac and $\Omega = 2g_A^M$ for Majorana neutrinos.

The measurement of the total cross section $\sigma(\nu N \rightarrow \nu X)$ gives one number Ω and we are not able to say whether $\Omega = g_V^D + g_A^D$ or $\Omega = 2g_A^M$. Therefore, even if the neutral current interaction is so different for Dirac and Majorana neutrinos, it cannot be used to distinguish them.

To see possible differences in the behavior of Dirac and Majorana neutrinos, which could follow from the symmetrization procedure let us consider the process

$$e^- e^+ \rightarrow \nu_M \nu_M \text{ or } \rightarrow \nu_D \bar{\nu}_D. \tag{29}$$

We suppose that the measurement of the angular distribution of final neutrinos (of course in the case that such a distribution is measured which is not the case up to now) is the simplest way to find their character: if the angular distribution has forward-backward symmetry the neutrinos are Majorana particles if not, Dirac neutrinos were produced.

To check whether the above statement is true, let us calculate the helicity amplitudes (for simplicity we neglect the electron and neutrino masses)(see for details Ref. [4, 11]. For Majorana neutrinos, four helicity amplitudes do not vanish,

$$M_M(\Delta\sigma = \pm 1, \Delta\lambda = \pm 1) \neq 0, \tag{30}$$

where $\Delta\sigma = \sigma - \bar{\sigma}$, $\Delta\lambda = \lambda - \bar{\lambda}$ and σ ($\bar{\sigma}$) and λ ($\bar{\lambda}$) are helicities of the electron (positron) and the final neutrino (antineutrino).

Whereas, there are only two amplitudes for Dirac neutrinos

$$M_D(\Delta\sigma = \pm 1, \Delta\lambda = -1) = \sqrt{2}M_M(\Delta\sigma = \pm 1, \Delta\lambda = -1). \tag{31}$$

If we calculate the unpolarized cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4} \sum_{\Delta\sigma, \Delta\lambda} \frac{d\sigma(\Delta\sigma, \Delta\lambda)}{d\cos\theta}, \tag{32}$$

we find out that there is difference between the Dirac and the Majorana cases. The important feature of a detector, that does not measure helicity is that it also is not able to distinguish a neutrino from an antineutrino (Fig. 3). Therefore we have to add the cross section for Dirac neutrinos and antineutrinos. Due to the formulae:

$$\frac{d\sigma^D}{d\cos\theta}(\sigma) = \frac{d\sigma^M}{d\cos\theta}(\Delta\lambda = -1), \tag{33}$$

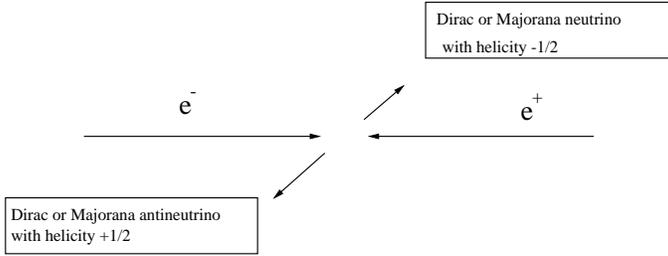


Fig. 3. Detectors do not distinguish lepton number from helicity

and

$$\frac{d\sigma^D}{d\cos\theta}(\pi - \theta) = \frac{d\sigma^M}{d\cos\theta}(\Delta\lambda = +1),$$

the final result will be now symmetric. For the total cross section we recover once more the equivalence between both types of neutrinos. In order not to take into account the same spin configuration two times, we have to integrate the Majorana cross section only over half of the full solid angle and we have

$$\sigma_{tot}(M) = \int_0^1 d\cos\theta \frac{d\sigma}{d\cos\theta} = \int_{-1}^1 d\cos\theta \frac{d\sigma^D}{d\cos\theta} = \sigma_{tot}(D). \quad (34)$$

There is only one terrestrial experiment, which currently promises to state whether neutrinos are Majorana or Dirac particles. It is the neutrinoless double β decay of nuclei $(\beta\beta)_{0\nu}$ [12].

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-. \quad (35)$$

There are many different mechanisms which could be responsible for $(\beta\beta)_{0\nu}$ decay [13]. The most important one is massive Majorana neutrino exchange [13] (see Fig. (4)).

It has been proved that independently of the mechanism which governs the $(\beta\beta)_{0\nu}$, there is a generic relation between the amplitude of $(\beta\beta)_{0\nu}$ decay and the Majorana mass term for neutrinos [14]. If any of these two quantities vanishes, the other one vanishes, too, and vice versa if one of them is not zero, the other also differs from zero.

Taking into account the most obvious mechanism from Fig.4 the $(\beta\beta)_{0\nu}$ amplitude is given by

$$A_{(\beta\beta)_{0\nu}} = A_{\text{nucl}} \langle m_\nu \rangle, \quad (36)$$

where A_{nucl} describes the nuclear transition and $\langle m_\nu \rangle$ is given by Eq.(5). Many experiments on the search for $(\beta\beta)_{0\nu}$ decay of different nuclei are going on at present. Unfortunately, up to now such a decay has not been found

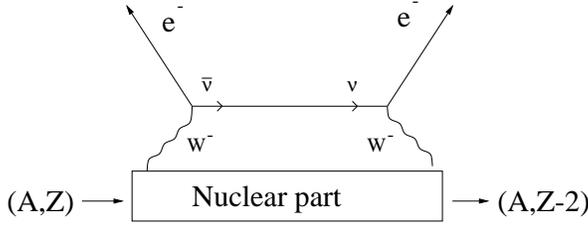


Fig. 4. Massive Majorana neutrino exchange mechanism describing the neutrinoless double β decay. The antineutrino $\bar{\nu}$ emitted in one vertex must be absorbed as a neutrino ν in other. Such a scenario is possible only if the neutrino is massive (then there is a chance that the emitted antineutrino has negative helicity $\bar{\nu}$ and must be a Majorana particle (then $\bar{\nu} = \nu$).

and experimentalists can only give a lower bound on the $(\beta\beta)_{0\nu}$ decay modes of various nuclei. The most stringent limit was found in the germanium Heidelberg–Moscow experiment. Their latest result on the half-life time $T_{1/2}^{0\nu} \sim |A_{(\beta\beta)_{0\nu}}|^{-2}$ is [5]

$$T_{1/2}^{0\nu} (Ge) > 5.7 \cdot 10^{25} \text{ year (90\% C.L.)}, \tag{37}$$

from which the bound on $|\langle m_\nu \rangle|$ (Eq. (5)) has been found. Such results alone give no chance to conclude about the nature of neutrinos. There are however other experimental data on mixing matrix elements U_{ei} and masses which are independent of the neutrino character. This information comes from flavor oscillation experiments (see Appendix) tritium β decay and cosmology. We can use this data and check whether the bound (Eq. (5)) is satisfied. If it is, the results are still not conclusive. If however the U_{ei} and m_i are such that the value of $\langle m_\nu \rangle$ is greater than the present bound, neutrinos must have Dirac character.

4. Checking the agreement of $(\beta\beta)_{0\nu}$ decay bounds with other experimental results

The discussion which follows depends on the number of light neutrinos. Three such neutrinos are necessary to explain solar [15] and atmospheric [1] anomalies. Four light neutrinos must be introduced if, in addition, the LSND results [16] is not disregarded. Here we will present results for three light neutrinos [17]. So that we have a relation between 3 flavor states $(\nu_e, \nu_\mu, \nu_\tau)$ and 3 eigenmass states (ν_1, ν_2, ν_3)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{38}$$

The three elements in the first row of the mixing matrix (U_{e1} , U_{e2} , U_{e3}) are the scenario of our discussion.

Besides the $(\beta\beta)_{0\nu}$ decay there are three main sources where information about mixing matrix elements U_{ei} and m_i masses of neutrinos are given:

- (i) tritium β decay;
- (ii) cosmology (dark matter and number of neutrino species induced by nucleosynthesis) and most importantly;
- (iii) solar and atmospheric neutrino oscillation.

Without going into details, we present only the required results (see [17] for detail).

Tritium β decay

The latest result from the Curie plot endpoint of tritium β decay gives the bound on

$$m(\nu_e) = \left[|U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2 \right]^{1/2} \leq m_\beta, \quad (39)$$

where

$$m_\beta = 2.7 \text{ eV [18]} \quad 3.4 \text{ eV [19]}.$$

Similar limits on $m(\nu_\mu)$ and $m(\nu_\tau)$ are much larger and less precise, so they are not interesting for our next analysis.

Cosmology

In order not to exceed the critical density of the Universe the sum of masses of light, stable neutrinos [20]

$$\sum_{\nu} m_{\nu} < 30 \text{ eV}. \quad (40)$$

Then there is no place for cold matter. If only 20% of all dark matter is formed by neutrinos then [20]

$$\sum_{\nu} m_{\nu} \sim 6 \text{ eV}. \quad (41)$$

Presently the best fit to cosmological observations is obtained if only 30% of the critical density is formed by dark matter. The rest ($\sim 70\%$) is explained by the cosmological constant. Then, if all hot dark matter (20% of all dark matter) is formed by neutrinos [20]

$$\sum_{\nu} m_{\nu} \sim 2 \text{ eV}. \quad (42)$$

There is also a bound on the equivalent number of neutrino species N_ν which follows from the present abundance of ${}^4\text{He}$. It was found [21] that $N_\nu \sim (2 \div 4)$ with 95% C.L.

Reactor, atmospheric and solar neutrino oscillation

From CHOOZ [22] and the atmospheric neutrino anomaly [1] we can find

$$|U_{e3}|^2 \leq 0.05. \tag{43}$$

There are three still accepted solutions of solar neutrino deficit [15] (i) vacuum oscillation VO , (ii) small mixing angle MSW transition (SMA), and (iii) large mixing angle MSW transition (LMA).

(i) for VO the constraints on $|U_{e2}|^2$ are not unique and two ranges of values are possible (which we denote as small = S or large = L)

$$0.24 \leq |U_{e2}|_S^2 \leq 0.48, \tag{44}$$

or

$$0.48 \leq |U_{e2}|_L^2 \leq 0.76. \tag{45}$$

For the MSW solution it is necessary that $|U_{e2}|^2 < |U_{e1}|^2$ in order to fulfill the resonance condition so we have only one range of values.

(ii) For SMA MSW transition we get:

$$0.0005 \leq |U_{e2}|^{(SMA)} \leq 0.0026. \tag{46}$$

(iii) Finally for LMA MSW resolution of solar neutrino anomaly there is:

$$0.204 \leq |U_{e2}|^{(LMA)} \leq 0.48. \tag{47}$$

There are two possible mass schemes, which can describe oscillation data. They are presented in Fig. 5.

In addition the total scale for neutrino masses is not fixed and different scenarios are possible (Fig. 6, Eq. (37), (39), (40))

Now we can combine all the information and check whether the bound on $\langle m_\nu \rangle$ (Eq. (5)) is satisfied. In the $\langle m_\nu \rangle$ there are squares of U_{ei} 's and large cancellations are possible. From other experiments we have only information about the modulus, not about phases. If we also take into account, that the scale of masses is not known the method is not powerful enough. Despite of that, in some cases the results can be conclusive. For example, for almost

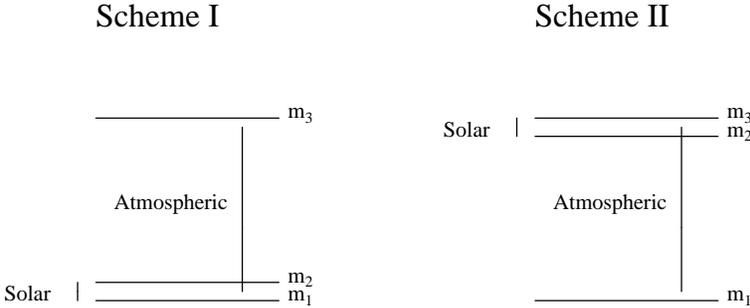


Fig. 5. Two possible neutrino mass spectra which can describe the oscillation data

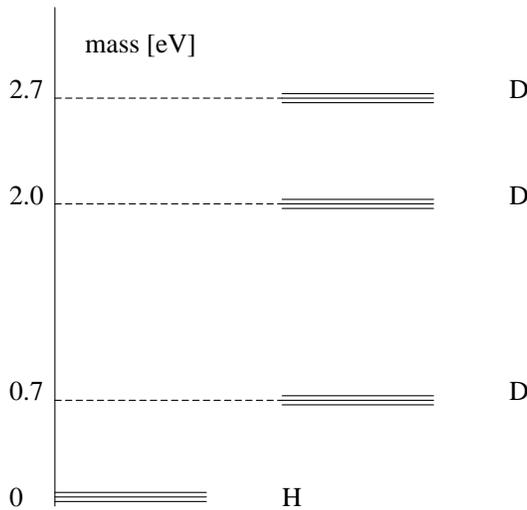


Fig. 6. Different scales for neutrino masses. In the first hierarchical scheme (H) $m_1 \simeq 0$ and $m_1 \ll m_2 \ll m_3$ neutrino masses are too small to be responsible for hot dark matter. In all other schemes — almost degenerate (D) — neutrinos can explain the existence of the hot dark matter without or with a non-vanishing cosmological constant.

degenerate neutrinos if we know that two elements of mixing matrix are small, then the third must be large, close to 1. In this case, independently of the possible cancellations, $\langle m_\nu \rangle$ is large ($|\langle m_\nu \rangle| \sim m_1$ and can be greater than the present bound on $(\beta\beta)_{0\nu}$ decay. Then we conclude that neutrinos must be Dirac particles. For details and discussions of different scenarios we refer to [17].

5. Conclusions

As (i) the SM works very well and no signal about non standard neutrino interaction is seen, and (ii) in any of the astrophysical sources and terrestrial experiments neutrinos are produced with an energy much larger than their mass, it is extremely difficult to find an experimental signal which would inform us about the nature of neutrinos.

There is only one terrestrial experimental test that can reveal the Majorana character of neutrinos - the neutrinoless double β decay. Unfortunately, experimental groups placed only the upper limit on the $(\beta\beta)_{0\nu}$ decay half life time. If neutrinos are Majorana particles, probably the next experiments which measure $\langle m_\nu \rangle$ up to 0.01 eV or 0.001 eV have a chance to measure it.

If neutrinos are Dirac particles we should get a signal about it by confronting the $(\beta\beta)_{0\nu}$ bound with independent information about masses and mixing matrix elements.

The present experimental precision is not good enough to find the answer. However, we are able to get some partial information *e.g.* if *SMA* MSW mechanism describes the solar neutrino deficit, and almost degenerate neutrinos have $m_\nu > 0.22$ eV then they must be Dirac particles. If the future GEMINI experiment still gives only a bound on $\langle m_\nu \rangle$ the next solar neutrino measurements (SNO and BOREXINO) have a chance to state that neutrinos are Dirac particles.

Appendix

We would like to clarify what the formulae for flavor oscillation $P_{\alpha\rightarrow\beta}(x)$ and effective neutrino mass $\langle m_\nu \rangle$ look like for Dirac neutrinos.

The mass term of n Dirac neutrinos is

$$L_{\text{mass}} = -\bar{\nu}_L M_D \nu_R + \text{h.c.} = -\frac{1}{2} (\bar{\nu}_L, \bar{\nu}_L^c) M_\nu (\nu_R, \nu_R^c)^T + \text{h.c.}, \quad (48)$$

where M_D is an arbitrary $n \times n$ matrix,

$$\nu_{R(L)}^c = c\bar{\nu}_{L(R)}^T, \quad \text{and} \quad M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & 0 \end{pmatrix}. \quad (49)$$

The M_D matrix can be diagonalized by the biunitary transformation

$$M_D \rightarrow V^T M_D V' = (M_D)_{\text{diag}}, \quad (50)$$

where V and V' are the $n \times n$ unitary matrices.

Then the M_ν matrix is diagonalized by the transformation

$$M_\nu \rightarrow U^T M_\nu U = \begin{pmatrix} (M_D)_{\text{diag}} & 0 \\ 0 & (M_D)_{\text{diag}} \end{pmatrix}, \quad (51)$$

where the $2n \times 2n$ matrix U is

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -iV, & V \\ iV', & V' \end{pmatrix}. \quad (52)$$

In the mass eigenstate basis for charged leptons the left-handed charged current interaction

$$L_{CC} \equiv -\frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu l_L W_\mu^+ + \text{h.c.} \quad (53)$$

can be written in the form

$$L_{CC} \equiv -\frac{g}{\sqrt{2}} \bar{\Psi}_L V^T \gamma^\mu l_L W_\mu^+ + \text{h.c.}, \quad (54)$$

where

$$\Psi_L = \frac{1}{\sqrt{2}} (iN_{1L} + N_{2L}),$$

and two Majorana bispinors N_{1i} and N_{2i} correspond to the same mass eigenvalue m_i of the matrix (49)

$$\nu_L = \frac{1}{\sqrt{2}} (iV^*, V^*) \begin{pmatrix} N_{1L} \\ N_{2L} \end{pmatrix} = V^* \Psi_L. \quad (55)$$

Now the effective neutrino mass is

$$\langle m_\nu \rangle = \sum_{i=1}^{2n} U_{ei}^2 m_i = \sum_{l=1}^n \frac{1}{2} \left((-iV_{ei})^2 + (V_{ei})^2 \right) m_i = 0. \quad (56)$$

For the probability that a neutrino born with flavor α will have flavor β after traveling distance x we get

$$\begin{aligned} P_{\alpha \rightarrow \beta}(x) &= \left| \sum_{i=1}^{2n} U_{\beta i}^* U_{\alpha i} e^{-i \frac{m_i^2}{2p} x} \right|^2 \\ &= \left| \sum_{i=1}^n e^{-i \frac{m_i^2}{2p} x} \frac{1}{2} \{ (iV_{\beta i}^*) (-iV_{\alpha i}) + (V_{\beta i}^*) (V_{\alpha i}) \} \right|^2 \\ &= \left| \sum_{i=1}^n e^{-i \frac{m_i^2}{2p} x} V_{\beta i}^* V_{\alpha i} \right|^2. \end{aligned} \quad (57)$$

We see that $P_{\alpha \rightarrow \beta}(x)$ looks exactly the same for n Dirac and n Majorana neutrino oscillation. The only difference is the number of CP violating

phases in mixing matrices $V_{\alpha i}$ in both cases. They are $(n-1)(n-2)/2$ for Dirac neutrinos and $n(n-1)/2$ for Majorana neutrino mixing. However, the physical phases by which the mixing matrices differ do not enter into transition probabilities [23]. Consequently by studying neutrino oscillation in vacuum or in matter it is impossible to distinguish the nature of neutrinos [24].

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