GRAVITATIONAL WAVES — NEW PERSPECTIVES*

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Laser interferometric experiments planned for 2002 will open up a new window onto the Universe. The first part of the paper gives a brief intuitive introduction to gravity waves, detection techniques and enumeration of main astrophysical sources and frequency bands to which they contribute. Then two more specific issues are discussed concerning cosmological perspectives of gravity waves detection. First one is the problem of gravitational lensing of the signal from inspiralling NS–NS binaries. The magnitude of the so called magnification bias is estimated and found non-negligible for some quite realistic lens models, but strongly model-dependent. The second problem is connected with estimates of galactic and extragalactic parts of the stochastic background. The main conclusion from these two examples is that in so far as the cosmological payoff of gravitational wave detection would be high, we should substantially deepen our understanding of basic astrophysical properties of galaxies and their clusters (in terms of mass distribution) in order to draw clear cosmological conclusions.

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1. Introduction

As we are approaching the end of the XXth century and think of challenging problems with which we enter the next millennium, we inevitably encounter the issue of the gravity waves. The gravity waves are with us for more than 80 years, we have an indirect evidence of them (from the binary pulsars [1]) for about 25 years but we are still waiting for the direct detection which is expected to take place after ca. 2002 [2] when the laser interferometric experiments LIGO/VIRGO are scheduled to start.

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Almost all information about the universe comes from the electromagnetic waves — this is the standard window onto universe (slightly supplemented by cosmic rays and solar neutrinos). When classical visual electromagnetic window was enlarged in the sixties by the radio band a true revolution in modern astrophysics occurred (just to recall the discovery of the cosmic microwave background radiation (CMBR), quasars or active galaxies). Therefore we have good reasons to expect that similar revolution will take place when the new window onto the universe – the gravitational one opens up.

It would be instructive to compare main points of difference between electromagnetic and gravitational waves [2].

1. First of all electromagnetic waves propagate in spacetime whereas gravitational waves are the disturbances of the spacetime itself — the ripples on the spacetime appearing to us as waves as our history in time unfolds.

2. Almost all types of electromagnetic waves generated in astrophysical setting are incoherent superposition of emissions from individual atoms. Gravitational waves are supposed to be produced as a result of coherent bulk motions of huge amounts of mass-energy.

3. Typical wavelength of electromagnetic waves are (much) lower than typical size of the source — we can make pictures of the emitting objects (geometric optics). On the other side, gravitational waves have wavelengths typically greater than the size of the source. Hence we cannot make pictures out of them. In terms of everyday experience we can say that information carried by gravitational waves is similar to that carried by sound. When this new window is open we will be able to "hear" "the inside" of violent astrophysical phenomena.

4. Electromagnetic waves are easily scattered or absorbed and quickly thermalize in opaque environment. On the contrary gravitational waves travel to us almost undistorted from the place where they were generated.

5. Frequency of typical astrophysical electromagnetic radiation begins at about $f \sim 10^7$ Hz and extends 20 orders of magnitudes upwards. Gravitational waves begin at $f \sim 10^4$ Hz and extend about 20 orders of magnitude downward.

Already this short discussion makes it clear that when detected gravitational waves will give us information about astrophysical sources complementary to that obtainable from the electromagnetic waves.

The most obvious way to characterize gravitational waves is by means of dimensionless amplitude (the wave strain at the detector) [3] $h = \frac{\Delta L}{L}$, where L is the separation between two "masses" — the elements of the detector and ΔL is the change of L due to gravitational wave. Another useful characteristic of the gravitational waves is the spectral omega function: $\Omega(f) = \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d(\ln f)}$ where $\rho_{\rm crit} = \frac{3H_0^2}{8\pi G}$ is the critical density of the Universe, and the meaning of $\Omega(f)$ is the fraction of closure density contained in gravitons per unit logarithmic frequency interval. The first indicator is usually employed to characterize individual sources, whereas the second one is more useful to characterize the background radiation and the connection between these two characteristics is given by the following formula [3]:

$$h(f) = 1.3 \times 10^{-20} h_{100} \sqrt{\Omega(f)} \frac{100~\text{Hz}}{f} \,. \tag{1}$$

Hence if we have the background radiation in the band 50 Hz $< f <$ 150 Hz with $\Omega = 10^{-8}$, this means that the wave strain at the detector would be $h \approx 10^{-24}$.

Gravitational wave detectors currently being constructed fall into two categories: resonant and interferometric detectors [1]. The first group follows the idea that incoming gravitational wave should deform a very massive metal block driving its oscillatory normal modes which could then be detected by piezoelectric devices. In order to suppress thermal vibrations of the atoms the detector should be kept frozen in very low temperature (a few degrees Kelvin). Currently operating EXPLORER network comprise cylindric detectors at Rome (Al), Louisiana (Al) and Perth (Nb) the working temperature of these detectors is $T = 3$ K and the net sensitivity is $h \sim 6 \times 10^{-19}$. This network would have detected a gravitational signal from the SN1987a had it been working at that time. Two more cylindric detectors NAUTILUS in Rome and AURIGA in Legarno are under construction. They are designed to work in $T \sim 0.05$ K and have sensitivity of $h \sim 10^{-20}$. Another project TIGA (Truncated Icosahedron Gravitational Antenna) assumes the construction of nearly spherical 100 ton resonant detector operating at $T \sim 0.01$–0.05 K with the sensitivity of $h \sim 10^{-21}$.

The most promising class of detectors involves laser interferometry technique. The basic idea is to measure the mutual separation between two freely suspended masses. Basic design is to have an L-shaped detector where in the corner as well as at the ends of the arms four heavy reflecting blocks are suspended on vibration-isolated supports. Laser beam is injected through the beam splitter in two perpendicular directions forming a Fabry–Perrot

[1] For more detailed discussion of detection techniques see Thorne [2] or Finn [4].

interferometric cavity. It is a major advance of laser interferometry that it makes possible to measure displacements as minute as $\Delta L \sim 10^{-16}$ cm Consequently in order to detect a wave strain with $h \sim 10^{-21}$–10^{-22} one ought to have the arm's length of order of $L \sim \Delta L/h \sim 1$–10 km Indeed the forthcoming laser experiments American LIGO and French/Italian VIRGO have arm's length $L = 4$ km and $L = 3$ km respectively. Two other projects of this type: German GEO600 and Japanese TAMA300 have $L = 600$ m and $L = 300$ m respectively. LIGO/VIRGO type experiments will be sensitive for the gravitational waves at frequencies of 1–10^4 Hz. Lower frequency band: $f \sim 10^{-4}$–1 Hz will be covered by laser interferometric space experiment LISA planned for ca. 2035 where the role of test masses would be played by satellites located 5×10^6 km apart in the vertices of an equilateral triangle.

Astrophysically and cosmologically interesting sources of gravitational radiation split into four natural frequency bands:

1. High Frequency ($f \sim 1$–10^4 Hz) in the scope of LIGO/VIRGO where the primary sources are: coalescing NS–NS binaries, fast rotating pulsars, supernovae and moderately massive (1–10^3 $M_{\odot}$) black holes;

2. Low Frequency ($f \sim 10^{-4}$ Hz) — band expected to be probed by LISA, and the following sources contributing to this band: massive black holes, binary stars — ordinary, white dwarfs, NS and relic gravitons;

3. Very Low Frequency band ($f \sim 10^{-7}$–10^{-9} Hz) — sources: relic gravitons, early universe effects (cosmic strings, colliding topological defects), no detector achievable in the foreseeable future, constrained a little bit by milisecond pulsar timing;

4. Extremely Low Frequency band ($f \sim 10^{-15}$–10^{-18} Hz) — source: relic gravitons, no terrestrial detector available, but CMBR can serve as an indirect detector. Especially the measurements of CMBR polarization, which are a part of MAP and Planck missions create great hopes for separating scalar and tensorial components of observed CMBR anisotropies [5].

The significance of relic gravitons for our understanding of the universe can easily be appreciated. It is perhaps not that obvious that inspiralling NS–NS binaries would also offer a possibility to (independently) determine important cosmological parameters such like the Hubble constant or deceleration parameter. In the rest of the paper we shall discuss in more detailed way some astrophysical aspects of two above mentioned cosmological perspectives.

2. Cosmological parameters from NS–NS inspiral catalogs

It was in 1986 when B. Schutz first noticed that the (average) amplitude of gravitational waves from NS–NS binary system is given by the following formula [6]:

$$\langle h \rangle = 10^{-23} M_{\rm tot}^{2/3} \mu f_{100}^{2/3} r_{100}^{-1}\,, \tag{2}$$

where: $M_{\rm tot}$ and μ are the total and reduced masses of the system, respectively, f_{100} is the frequency in units of 100 Hz, r_{100} is the distance in 100 Mpc. On the other hand, the timescale of the frequency drift of the inspiralling system reads:

$$\tau = \frac{f}{\dot{f}} = 7.8 M_{\rm tot}^{-2/3} \mu^{-1} f_{100}^{-8/3} \quad \text{sec.} \tag{3}$$

This is a fortunate circumstance that (unknown a priori) masses of components can be eliminated if one combines two above formulae. In this way one acquires an opportunity to express the distance to the source through observable quantities:

$$r_{100} = 7.8 f_{100}^{-1} (\langle h_{23} \rangle \tau)^{-1}, \tag{4}$$

where by h_{23} we denoted an amplitude in the units of 10^{-23}. The formula (4) which means that distance to a merging binary is a direct observable quantity easy to obtain from the waveforms initiated the series of papers exploring an intriguing possibility of accurate measurements of cosmological parameters such as the Hubble constant, or deceleration parameter [7–9]. It is worth noticing that again we encounter a substantial difference between gravitational and electromagnetic windows. In the former case it is not possible to measure the distance to the source directly (excluding perhaps few very nearby objects having measurable parallaxes).

It is clear that if we knew the distance and measured the redshift z of the parent galaxy we could infer the Hubble constant in an independent way. However, one immediately encounters a severe problem with poor directionality of gravitational wave detectors, which for a single detector can be as large as a hemisphere and for a network of 3 or more detectors [9] (separated by long distances) can be reduced to several degrees. Again there can be thousands of potentially parent galaxies for the merging binary on one squared degree on the celestial sphere leaving little hope for reliable identification of just one correct. Fortunately, Markovic [8] noticed that observable (from gravitational wave strains) quantities having dimensions of $[\text{mass}]^p$ scale like $(1+z)^p$. The masses of neutron stars apparently have sharp distribution peaked at 1.4 $M_\odot$ [10] and because of stability reasons cannot exceed the maximal value (a bit higher than ca. 1.5 $M_{\odot}$). Hence whenever one reads "too large" a mass from the waveforms one can attribute this enhancement to the redshift effect. In this way the redshift becomes an observable extracted from the waveforms. However, this extraction can only be of a statistical nature [7].

2.1. Gravitational lensing effect on gravitational waves from inspiralling binaries

Having in mind that inspiralling binaries observable by LIGO/VIRGO type experiments are distant extragalactic sources there exists a potential possibility that gravitational signal from them can be magnified by intervening clumps of matter acting as gravitational lenses. Because of this effect a part of such signal-to-noise limited sample would be drawn from a fainter source population, which could not otherwise be detected had not they be lensed. In the first estimate concerning this effect, Wang *et al.* [11] claimed that an advanced LIGO experiment should see a few strongly lensed events per year. The optimistic prediction of Wang *et al.* [11] derives from the assumption that considered population of lenses can be modelled as compact Schwarzschild lenses which is in conflict with assumed geometric optics approximation. In [12] it was shown that this estimate could be significantly lowered if one considered the mixture of spiral and elliptical galaxies modelled as simple singular isothermal spheres in the role of lenses. This discrepancy illustrates the sensitive dependence of predictions on the lens model adapted. Here, as an illustration we shall consider the population of lenses modelled as singular isothermal spheres (SIS) embedded into a sheet of matter [13]. Such choice of model is dictated by observation that galaxies do not occur in isolation but are members of clusters. Therefore, they are immersed in an X-ray emitting gas and in the intracluster dark matter. What follows is a sketchy outline of main points — the details can be found in a separate paper [14].

Assuming flat Einstein–deSitter cosmological models we can parametrize them by two quantities: Ω_0 and Ω_Λ, where Ω_0 denotes the current matter density as a fraction of critical density for closing the Universe, Ω_Λ is analogous fraction of critical density contained in the cosmological constant Λ and these two sum up to the value one. The gravity wave detector would register only those inspiral events for which the signal-to-noise ratio exceeded certain threshold value ρ_0 [7, 15] which is estimated as $\rho_0 = 8.$ for LIGO-type detectors.

Let us denote by $\dot{n}_0$ the local binary coalescing rate per unit comoving volume. One can use "the best guess" for local rate density $\dot{n}_0 \approx 9.9$ h 10^{-8} $\mathrm{Mpc}^{-3}\mathrm{yr}^{-1}$ as inferred from the three observed binary pulsar systems that will coalesce in less than a Hubble time [16].

The relative orientation of the binary with respect to the detector is described by the factor Θ. This complex quantity cannot be measured nor assumed a priori. However, its probability density averaged over binaries and orientations $P_\Theta(\Theta)$ can be calculated [15].

The rate $\frac{dN}{dz_s}$ at which we observe the inspiral events that originate in the redshift interval $[z_s,\ z_s + dz_s]$ is given by [17]:

$$\frac{dN}{dz_s} = \frac{\dot{n}_0}{1+z_s}\ 4\pi d_M^2 \frac{d}{dz_s} d_M(z_s)\ C_\Theta(x)\,, \tag{5}$$

where $C_\Theta(x) = \int_x^\infty P_\Theta(\Theta) d\Theta$ denotes the probability that given detector registers inspiral event at redshift z_s with $\rho > \rho_0$, $d_M(z_s)$ denotes the proper distance of the source. Figure 1 shows the expected detection rate of inspiralling events for the cosmological models considered. It has been obtained by numerical integration of the formula (5).

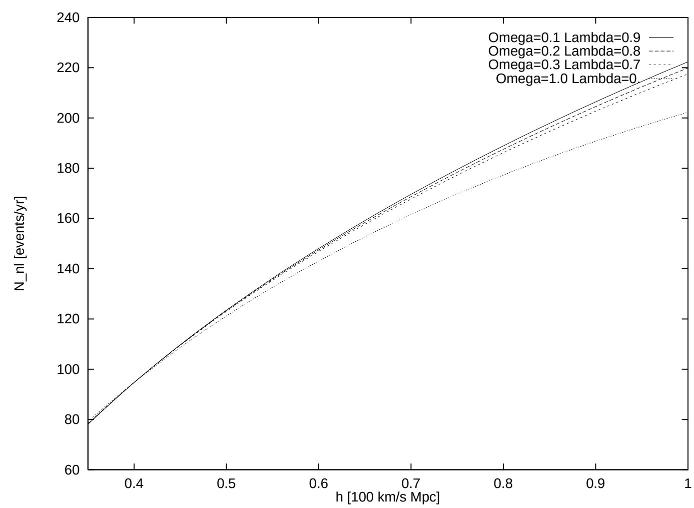

Fig. 1. The detection rate prediction for the advanced gravity wave detectors *i.e.* with signal–to–noise threshold $\rho_0 = 5.$ and probing distance $r_0 = 355$ Mpc.

Differential lensing rate $\frac{dN_L}{dz_s}$ for distant gravity wave sources is given by

$$\frac{dN_L}{dz_s} = \frac{dN}{dz_s}\,\frac{\sigma_{\rm tot}(\mu_0, z_s)}{4\pi}\,, \tag{6}$$

where $\frac{dN}{dz_s}$ is the inspiral events rate like in the formula (5) (but without $C_\Theta(x)$ factor), $\sigma_{\rm tot}(\mu_0, z_s)$ is the total cross-section of all lenses affecting the source located in the redshift interval $[z_s, z_s + dz_s]$ by magnifying it more than μ_0.

Since by assumption the galaxies are lensing masses, one can write:

$$\sigma_{\rm tot}(\mu_0, z_s) = \int_0^{z_s} dz_l \frac{dV}{dz_l} \int_0^{\infty} \sigma(\mu_0, z_l, z_s, L)\, \Phi(L) dL\,, \tag{7}$$

where $\sigma(\mu_0, z_l, z_s, L)$ is the cross-section for a single galaxy, z_l is the redshift of the lens, V is the comoving volume, L denotes the luminosity and $\Phi(L)$ is galaxy luminosity function assumed to be Schechter function [17, 18]:

The elementary cross-section $\sigma(\mu_0, z_l, z_s, L)$ can be estimated as

$$\sigma(\mu_0, z_l, z_s, L) = \pi r_{cr}^2\, \sigma_{\Theta}(\mu_0) \tag{8}$$

$\sigma_{\Theta}(\mu_0)$ is the dimensionless part of the cross-section for magnification larger than μ_0, which for the Singular Isothermal Sphere plus smooth sheet of matter reads [13]: $\sigma_{\Theta}(\mu_0) = \frac{4\pi}{(1-\kappa_0)^4 \mu_0^2}$ where $\kappa_0 = \Sigma / \Sigma_{\rm crit}$ is the fraction of projected lens surface mass density to the critical one (for details see [13]); r_{cr} denotes the outermost angular distance from the source to the lens axis that produces multiple images. For our model r_{cr} can be expressed by the line of sight velocity dispersion related to L via Tully–Fisher or Faber–Jackson relation (see [14]).

The signal–to–noise ratio for a lensed source reads: $\rho_l(z_s) = \sqrt{\mu_0}\, \rho(z_s)$, where μ_0 is magnification due to lensing, $\rho(z_s)$ is the signal–to–noise ratio for unlensed source at redshift z_s ([11]). This formula shows that there may exist sources for which $\rho(z_s) < \rho_0$ but with $\rho_l(z_s) > \rho_0$. It means that some (usually small) part of a flux (signal–to–noise) limited survey is constituted by an intrinsically fainter population, which would not be detected without lensing.

Figure 2 shows the total lensing cross-sections as a function of z for different values of the Hubble constant in the cosmological model $\Omega = 1$. $\Lambda = 0$.

It is evident that the lensing cross-section $\sigma_{\rm tot}(z)$ for inspiral events has noticeable dependence on h and it takes maximal values at moderate redshifts of the source $z_s \sim 0.5$–1.5. This is totally consistent with our model assumption that galaxies are lenses. Fig. 3 displays the lensing rate for $\kappa_0 = 0.5$. Lensed events as originating at redshifts larger than average events in the catalog can be identified by their observed chirp masses (higher than the average). It should be noticed that lensing rate strongly depends on the cosmological model adopted. The effect of the assumed lens model is even more pronounced and is summarized on Fig. 4. The range of κ_0 is suggested by empirical lens parameter estimation from lensed quasars [19].

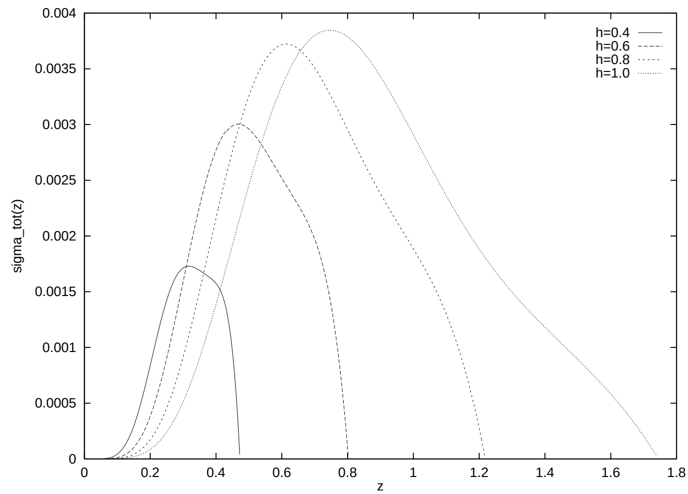

Fig. 2. The total lensing cross-section $\sigma_{\rm tot}(z_s)/4\pi$ as a function of the source redshift for four values of dimensionless Hubble constant h in the cosmological model with $\Omega = 1, \ \ \Lambda = 0$.

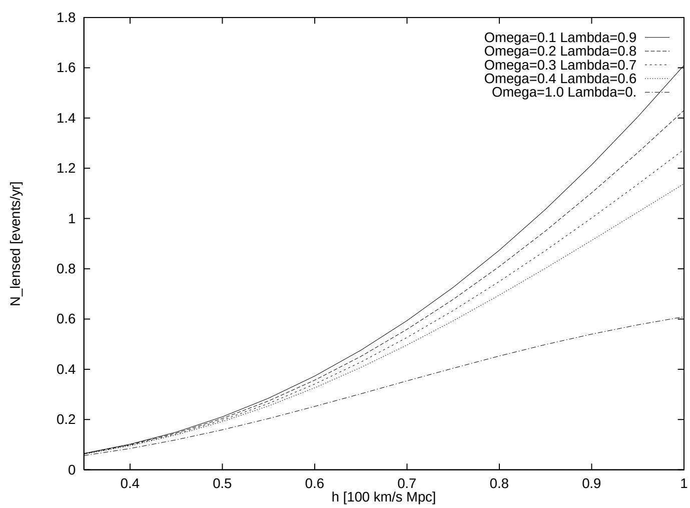

Fig. 3. The yearly lensing rate prediction for the gravitational inspiral events in the model of SIS lens embedded in a sheet of matter with $\kappa_0 = 0.5$.

Our results show that magnification bias becomes worth further consideration since it could produce a systematic distortion of the catalogs created in order to make cosmological inferences [7,8]. Moreover, yearly lensing rate as high as a few per year will add to gravitational wave astrophysics a new

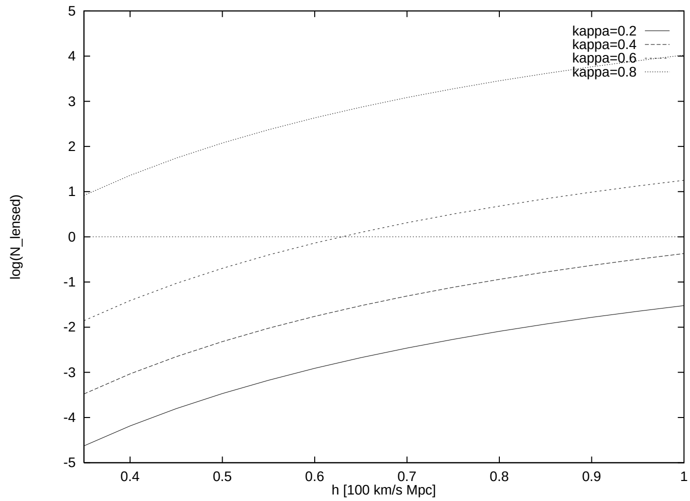

Fig. 4. The dependence of the logarithm of the yearly lensing rate prediction for the gravitational inspiral events as a function of κ_0. Note the strong dependence of lensing rate on the lens model.

dimension of becoming a tool for investigating the dark matter in the universe. However, in order to fully appreciate this cosmological perspective we should gain more confidence in our lens models which is another challenge for astrophysics.

3. Astrophysical stochastic background

Another important theoretical payoff is expected from successful detection of gravitational background radiation [20, 22]. Its cosmological part would carry almost undistorted information about very early stages of evolution of the Universe [21]. By building up a picture of the universe at about 10^{-22} seconds after big-bang it would lead to a progress in high energy physics. Also a negative result *i.e.* a nondetection in frequency ranges where cosmological generation processes are predicted to be dominant would be important ingredient falsifying the underlying model assumptions, such as specific inflationary scenarios. However, the whole problem is complicated by the competition from astrophysical sources of gravitational background. This part of the background arises from an extremely large number of individual, independent, uncorrelated astrophysical sources (like spinning neutron stars, close binary systems). It is therefore important to build up an intuition of how to disentangle these substantially different parts. An important observation on road to disentangle these two competing contributions has been made by Postnov and Prokhorov [23]. In particular they found that at frequencies higher than 0.07 Hz we do not expect unresolved binaries contributing above rms LISA sensitivity. Hence the successful detection of gravitational background by LISA experiment in this frequency range would reveal its cosmological origin.

In this Section we address the question of how actual (anisotropic) distribution of galactic binary sources (located in the disk or halo) influences the properties of the stochastic signal. Complementary discussion of multipole expansion coefficients of anisotropic astrophysical gravitational background can be found in [24].

In the theory of gravity wave signal detection [4] one distinguishes two components in the detector output $s(t)$: $s(t) = h(t) + n(t)$, where: $h(t)$ represents the response to gravitational radiation and $n(t)$ is the detector's noise. Suppose that we have two detectors (labeled by superscript 1 and 2) and we cross-correlate the signals:

$$\langle s_1, s_2 \rangle := \int\limits_{-T/2}^{T/2} s_1(t) s_2(t) dt \approx \langle h_1, h_2 \rangle + \langle n_1, n_2 \rangle$$

cross-correlation between gravitational background signals grows linearly with observation time T [15, 20]:

$$\langle h_1, h_2 \rangle \sim |\tilde{h}(f)|^2 \, \Delta f \; T \sim \Omega(f) \; \Delta f \; T$$

$\tilde{h}$ and $\tilde{n}$ denote Fourier transformed h and n respectively, Δf is the bandwidth. Noise behaves in slightly different way (as a one dimensional random walk), *i.e.*:

$$\langle n_1, n_2 \rangle \sim |\tilde{n}(f)|^2 \, \sqrt{\Delta f \; T} \, .$$

This means that minimal detectable stochastic background has spectral density

$$\Omega_{\min} \sim \frac{|\tilde{n}(f)|^2}{\sqrt{\Delta f \; T}}$$

and longer observation time improves sensitivity of background detection.

The response to gravitational waves from an ensemble of unresolved binaries can be decomposed into two polarization modes (plus and cross — $h_+(t)$ and $h_\times(t)$ respectively): $h(t) = \sum_{i=1}^{N} F_+^i(t) h_+^i(t) + F_\times^i h_\times^i(t)$, where the expansion coefficients $F_+^i(t)$ and $F_\times^i(t)$ are beam-pattern functions accounting for actual orientation of the i^{th} source with respect to the detector's arms. If we approximate the actual distribution of discrete unresolved sources by continuous distribution $N(r, \theta, \phi)$ (assuming the galactic reference frame) then after performing an average one ends up with the following formula

$$\langle h^2(t)\rangle \sim \int\limits_0^{\infty}\int\limits_{-\pi/2}^{\pi/2}\int\limits_0^{2\pi}[\hat{F}_{+,\times}(\theta,\phi)]^2 N(r,\theta,\phi)\cos(\theta)\;dr\;d\theta\;d\phi\,. \tag{9}$$

As discussed by Postnov and Prokhorov [23] the most significant fraction of binary stochastic signal in the LISA frequency band is formed by the merging ordinary binary stars (in contrast to NS–NS binaries — main sources for LIGO). The population of inspiraling close binary systems composed of compact objects splits up in a natural way into Galactic and extragalactic part. The first is connected with the sources located in our Galaxy which belongs either to disk or to the halo. The second portion of sources is constituted by the binaries in other galaxies.

Let us consider the galactic sources first. One can estimate the spectral function $\Omega_{\rm GW}(f)$ for the binary stochastic background as follows:

$$\Omega_{\rm GW}\,\rho_{cr}c^2 = \sum_i \frac{L_i(f)}{4\pi c r_i^2}\,, \tag{10}$$

where $L(f)$ is gravitational wave luminosity per unit logarithmic frequency interval and the sum is performed over the ensemble of sources.

We assume that the dominant energy losses are due to gravity waves radiated away at frequency f (which manifests itself as inspiraling of the components). This assumption may not be true since other evolutionary processes like the mass exchange may in reality be leading ones. However, it is often (if not exclusively) encountered in the literature. The luminosity of a single source reads [23]

$$\left(\frac{dE}{dt}\right)_{\rm GW} = \left(\frac{dE}{dt}\right)_{\rm orb} = \frac{2}{3}E_{\rm orb}\left(\frac{\dot{f}}{f}\right)_{\rm GW}\,, \tag{11}$$

where $E_{\rm orb} = \frac{1}{2}\frac{GM_1M_2}{a}$ is the absolute value of the orbital energy of a binary system with the semimajor axis a composed of stars with masses M_1 and M_2, $(\dot{f}/f)_{\rm orb}$ denotes the orbital frequency change due to gravity wave radiation. Using the Kepler's 3rd law one can express $E_{\rm orb}$ in terms of the frequency f and the so called chirp mass $\mathcal{M} = M^{2/5}\mu^{3/5}$ where M is the total and μ is the reduced mass of the system:

$$E_{\rm orb} = \frac{(\pi G)^{2/3}}{2}\mathcal{M}^{5/3}f^{2/3}\,. \tag{12}$$

Consequently, the luminosity per unit logarithmic frequency interval reads:

$$L(f) = \frac{2}{3} E_{\rm orb}(\mathcal{M}, f)\mathcal{R}\,, \tag{13}$$

where $\mathcal{R}$ is the rate at which the binary system enters the considered frequency interval. By analogy with [23] we assume that the systems are sweeping through the accessible frequency band in such a manner that a steady state is maintained. Hence $\mathcal{R}$ equals the rate of binary mergers. For the white dwarf binaries (WD–WD) we assume $\mathcal{R}$ of order of 0.001 yr^{-1} per galaxy and for the neutron-star binaries (NS–NS) $\mathcal{R}$ is of order of 10^{-4}–10^{-6} yr^{-1} per galaxy [23]. In our further estimates we shall use some "typical" values of the gravity wave luminosity for given types of sources. The comparison of rates $\mathcal{R}$ indicates that WD–WD sources will dominate in Galactic sources whereas NS–NS contribution to the background may become important in extragalactic part.

Let us consider the disk population of WD–WD binaries. Following [25] one can use a double exponential model for galactic disk

$$N(r,z) = \frac{N_0}{2z_0} \exp\left(-\frac{R-R_0}{R_c}\right) \exp\left(-\frac{|z|}{z_0}\right), \tag{14}$$

where z is the distance from the Galactic plane, *i.e.* $z = r\, sin b$ and r is the distance to the source, b is Galactic latitude, R denotes the distance from the Galactic center to the source, $R_0 \approx 7.5$ kpc is our distance to the Galactic center. Another parameters in the formula (14) are following: $N_0 = 0.05\ \mathrm{M}_\odot/\mathrm{pc}^3$ — the density of stars in the vicinity of the solar system, $z_0 = 0.3$ kpc. is the disk height scale and $R_c \approx 0.5\ R_0$ is the halo core radius [25].

Introducing an angle θ between the source and the Galactic center one has $R = (R_0^2 + r^2 - 2rR_0\cos\theta)^{1/2}$. We are now able to write down an expression for the spectral density distribution $\Omega_{\rm GW}(f)$ contributing from the disk sources:

$$\begin{aligned}\Omega_{\rm GW}^{\rm disk}(f) &= (\rho_{cr}c^3)^{-1}\, L(f) \int_0^{D_{\max}} dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{\pi/2} N(R(\theta), z)\cos b\, db \\ &= (\rho_{cr}c^3)^{-1}\, L(f)\frac{N_0}{z_0} R_0 \mathcal{F}^{\rm disk}. \end{aligned} \tag{15}$$

The behavior of $\mathcal{F}^{\rm disk}$ as a function of $D_{\max}/R_0$ for double exponential disk model is shown in Fig. 5.

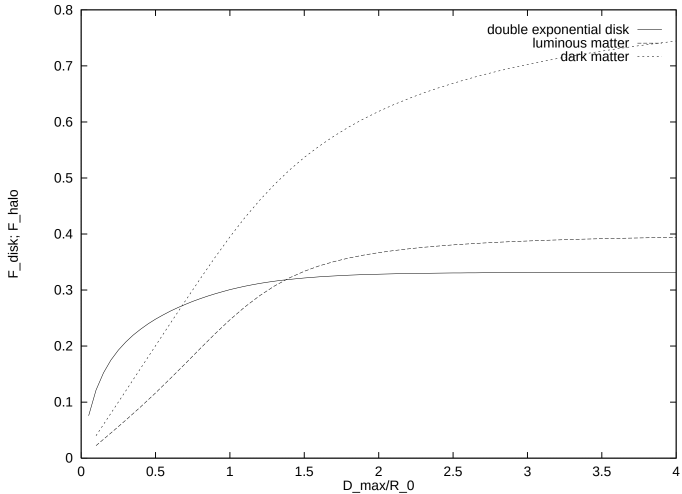

Fig. 5. Geometric factors in spectral omega function for Galactic stochastic gravitational wave background — double exponential disk, luminous and dark matter halo models.

Assuming that the detectors are probing the distances larger than saturation values we arrive at the following estimate for the spectral density of the stochastic background:

$$\Omega_{\rm GW}^{\rm disk}(f) = 9.1\ 10^{-9}\ R_{100}\ \left(\frac{\mathcal{M}}{\mathrm{M}_\odot}\right)^{5/3}\ \left(\frac{f}{10^{-3}\,\mathrm{Hz}}\right)^{2/3}. \tag{16}$$

The density of luminous halo objects *i.e.* the ordinary stars is falling off as R^{-3} where R is the distance from the Galactic center [26]. In the case of dark matter the density profile falls off as R^{-2} [27]. One can therefore model the density distribution in the following way:

$$N(R) = \frac{N_0}{1+\left(\frac{R}{R_c}\right)^{\alpha}}, \tag{17}$$

where $R_0 = 7.5\ kpc$ is the distance to the Galactic center, $R_c \approx 0.5\,R_0$, $\alpha = 2$ for luminous objects and $\alpha = 3$ for the dark matter. We assume $N_0 = 0.01\ \mathrm{M}_\odot/\mathrm{pc}^3$ for luminous and $N_0 = 0.1\ \mathrm{M}_\odot/\mathrm{pc}^3$ for dark matter respectively. Consequently introducing an angle θ between the source and the Galactic center one has $R = (R_0^2 + r^2 - 2rR_0\cos\theta)^{1/2}$ and the halo contribution to the $\Omega(f)$ reads:

$$\Omega_{\rm GW}^{\rm halo}(f)\ =\ (2\rho_{cr}c^3)^{-1}\ L(f)\ \int_0^{D_{\max}} dr\ \int_0^{\pi} N(R(\theta))\sin\theta\,d\theta$$

$$= (2\rho_{cr}c^3)^{-1}\, L(f) N_0 R_0 \mathcal{F}^{\rm halo}\,. \tag{18}$$

In Fig. 5 one can see the behaviour of $\mathcal{F}^{\rm halo}$ as a function of $D_{\rm max}/R_0$ for two halo models: with $\alpha = 2.$ and $\alpha = 3.$ We see that the values of $\mathcal{F}^{\rm halo}$ saturate at distances $D_{\rm max}^{\rm lum} \approx 2R_0$ and $D_{\rm max}^{\rm dark} \approx 4R_0$ respectively. Assuming that the detectors are probing the distances larger than saturation values we arrive at the following estimate for the spectral density of the stochastic background (sum of luminous and dark matter contributions):

$$\Omega_{\rm GW}^{\rm halo}(f) = 3.4\; 10^{-8}\; R_{100} \left(\frac{\mathcal{M}}{\rm M_\odot}\right)^{5/3} \left(\frac{f}{10^{-3}\,{\rm Hz}}\right)^{2/3}\,. \tag{19}$$

The above calculations substantiate this intuitively obvious conclusion that the finite extent of our Galaxy limits the spectral density of Galactic stochastic background.

The contribution of extragalactic binaries can be estimated in an analogous manner if we properly account for non-Euclidean character of the space-time in the cosmic scale. First modification comes from the redshift effect - the frequency f at the detector corresponds to $f' = f(1+z)$ at the source. The second one is the distinction between three types of distances [28]: the proper motion distance $d_M(z)$, the angular diameter distance $d_A(z)$ and the luminosity distance $d_L(z)$. So the sources inside the comoving volume $dV(z)$ at redshift z contribute

$$\rho_{cr}c^2\; d\Omega_{\rm GW}(f) = \frac{L(f')}{4\pi c d_L^2(z)}\, N(z) dV(z)\,, \tag{20}$$

where $N(z)$ is the density of galaxies, $d_L(z)$ is the luminosity distance corresponding to the redshift z. We neglect the source evolutionary effects $N(z) = N_0(1+z)^3$ where $N_0 = 0.025 h^3 {\rm Mpc}^{-3}$ is the present local value of galaxy density [29]. Now, let us note that

$$dV(z) = 4\pi d_M^2(z) d(d_M(z)) = 4\pi d_M^2(z) \frac{d(d_M(z))}{dz} dz = 4\pi d_M^2(z) \frac{c}{H_0}\frac{dz}{\mathcal{D}}\,, \tag{21}$$

where by $\mathcal{D}(z)$ we have denoted the following quantity:

$$\mathcal{D}(z) = \sqrt{\Omega_0(1+z)^3 + \Omega_\Lambda} = \sqrt{(1+z)^2(1+\Omega_0 z) - z(2+z)\Omega_\Lambda}\,.$$

Then by virtue of (20) and (13) we arrive at

$$\rho_{cr}c^2\; d\Omega_{\rm GW}(f) = \frac{(\pi G)^{2/3}}{3} \frac{d_H}{c} N_0 h_{100}^{-1} \mathcal{R} \mathcal{M}^{5/3} f^{2/3}\; (1+z)^{5/3} \frac{dz}{\mathcal{D}(z)}\,. \tag{22}$$

Assuming that the first sources capable of contributing to the binary stochastic signal have formed at redshifts z_* we get

$$\Omega_{\rm GW}(f) = \frac{1}{\rho_{cr} c^2} \frac{(\pi G)^{2/3}}{3} \frac{N_0}{H_0} \mathcal{R} \mathcal{M}^{5/3} f^{2/3} \int_0^{z_*} \frac{(1+z)^{5/3}}{\mathcal{D}(z)}\, dz\,. \qquad (23)$$

Again the cosmological model assumed intervenes through the integral

$$\mathcal{F}^{\Omega}(z_*) = \int_0^{z_*} \frac{(1+z)^{5/3}}{\mathcal{D}(z)}\, dz\,. \qquad (24)$$

The values of this integral as a function of limiting redshift z_* are shown on Fig. 6. One can see a noticeable dependence of this geometric factor on the cosmological constant.

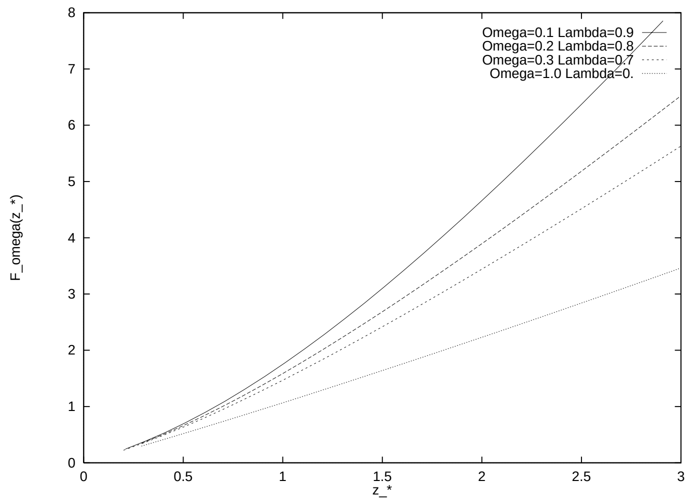

Fig. 6. Geometric factors in spectral omega function for extragalactic stochastic gravitational wave background. Noticeable dependence on the cosmological constant can be seen.

Final estimate of the spectral density of extragalactic stochastic background reads:

$$\Omega_{\rm GW}(f) = 1.9 \times 10^{-11}\, R_{100} \left(\frac{\mathcal{M}}{\rm M_\odot}\right)^{5/3} \left(\frac{f}{10^{-3}\,\rm Hz}\right)^{2/3} \mathcal{F}^{\Omega}(z_*) h_{100}^{-3}\,. \qquad (25)$$

Again from the formulae (16), (19) and (25) we see that geometric factors (denoted by $\mathcal{F}$ with respective superscripts) substantially influence the final estimate. Therefore our estimates depend crucially on our knowledge of galactic mass distributions and on the cosmological model adapted.

4. Conclusions

If the source estimates obtained within two last decades are approximately correct then the planned for the near future (year 2002 or so) experiments will open up a new window onto the Universe. The perspectives of gravitational wave observations are indeed fascinating and go beyond studying the sources themselves but they offer possibilities to gain information about the Universe as a whole. However, before we can fully appreciate this offer concerning distant (in space and time) universe we should deepen our understanding of the local universe, especially the distribution of mass in galaxies and in their clusters. We demonstrated this claim in two examples: the effect of gravitational lensing on the inspiral NS–NS catalogs and on the astrophysical component of stochastic background. This conclusion can (philosophically) be perceived as another manifestation of the intimate relation between the local and the global having its deepest roots in self-consistent formulation of the theory of gravity (the interaction that shapes the world in the largest scales).

REFERENCES

[1] J.H. Taylor, J.M. Weisberg, *Astrophys. J.* **253**, 908 (1982); *Astrophys. J.*, **345**, 434 (1989).

[2] K.S. Thorne, Gravitational Waves in *Proc. of the Snowmass 95 Summer Study on Particle and Nuclear Astrophysics and Cosmology*, eds. E.W.Kolb and R.Peccei, World Scientific Publ., Singapore 1996a; K.S. Thorne, Gravitational Waves From Compact Bodies in *Proc. of IAU Symposium 165, Compact Stars in Binaries*, eds. J.van Paradijs, E.van den Heuvel and E.Kuulkers, Kluwer Acad. Publ., 1996b.

[3] B.F. Schutz, *A first course in general relativity*, Cambridge University Press, 1990.

[4] L.S. Finn, Gravitational Radiation Sources and Signatures, gr-qc/9903107.

[5] R.R. Caldwell, M. Kamionkowski, L. Wadley, The first space-based gravitational wave detectors, astro-ph/9807319; M. Kamionkowski, A. Kosowsky, Detectability of Inflationary Gravitational Waves with Microwave Background Polarization, astro-ph/9705219; M. Kamionkowski, A. Kosowsky, A. Stebbins, *Phys. Rev. Lett.* **78**, 2058 (1997).

[6] B. Schutz, *Nature* **323**, 310 (1986).

[7] L.S. Finn, D.F. Chernoff, *Phys. Rev.* **D47**, 2198 (1993); D.F. Chernoff, L.S. Finn, *Astrophys. J.* **411**, L5 (1993).

[8] D. Marković, *Phys. Rev.* **D48**, 4738 (1993).

[9] B.F. Schutz, *Nature* **323**, 310 (1986); A. Krolak, B.F. Schutz, *Gen. Relativ. Gravitation* **19**, 1163 (1987); B.F. Schutz, *Classical Quantum Gravity* **6**, 1761 (1989).

[10] L.S. Finn, *Phys. Rev. Lett.* **73**, 1878 (1994).

[11] Wang Yun, E. Turner, A. Stebbins, *Phys. Rev. Lett.* **77**, 2875 (1996).

[12] M. Biesiada, *Acta Cosmol.* **28**, (1998).

[13] P. Schneider, J. Ehlers, E.E. Falco, *Gravitational Lenses*, Springer Verlag, Berlin 1992.

[14] M. Biesiada, submitted to *Astrophys. J.*.

[15] L.S. Finn, *Phys. Rev.* **D53**, 2878 (1996).

[16] R. Narayan, T. Piran, A. Shemi, *Astrophys. J.* **379**, L17 (1991); E.S. Phinney, *Astrophys. J.* **380**, L17 (1991).

[17] M.A. Nowak, S.A. Grossman, *Astrophys. J.* **435**, 557 (1994).

[18] S. Mao, *Astrophys. J.* **241**, 389 (1992).

[19] L.L.R. Williams, R.A.M.J. Wijers, *Mon. Not. R. Astron. Soc.* **286**, L11 (1997).

[20] B. Allen, The Stochastic Gravity-Wave Background: Sources and Detections, in *Les Houches School on Astrophysical Sources of Gravitational Waves*, eds. J.A. Marck and J.P. Lasota, Cambridge University Press, 1996.

[21] L.P. Grishchuk, Yu. Sidorov, *Classical Quantum Gravity* **6**, L161 (1989).

[22] L.P. Grishchuk, The detectability of relic (squeezed) gravitational waves by laser interferometers, gr-qc/9810055.

[23] K. Postnov, Astrophysical Sources of Stochastic Gravitational Radiation in the Universe, astro-ph/9706053.

[24] G. Giampieri, A.G. Polnarev, *Mon. Not. R. Astron. Soc.* **291**, 149 (1997); G. Giampieri, *Mon. Not. R. Astron. Soc.* **292**, 218 (1997).

[25] J.N. Bahcall, R.M. Soneira, *Astrophys. J. Suppl.* **44**, 73 (1980).

[26] B. Paczynski, *Acta Astron.* **41**, 157 (1991).

[27] J. Binney, S. Tremaine, *Galactic Dynamics*, Princeton University Press, 1987.

[28] P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton University Press, 1993.

[29] M.J. Geller *et al.*, *Astron. J.***114**, 2398 (1997).