OFF-DIAGONAL GENERALIZED VECTOR DOMINANCE IN DIS AND QCD* **

G. Cvetič[a,b], D. Schildknecht[a] and A. Shoshi[a]

[a]Dept. of Physics, University of Bielefeld, 33501 Bielefeld, Germany
[b]Dept. of Physics, University of Dortmund, 44221 Dortmund, Germany

e-mail: cvetic@physik.uni-bielefeld.de
e-mail: Dieter.Schildknecht@physik.uni-bielefeld.de
e-mail: shoshi@physik.uni-bielefeld.de

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We review the generalized vector dominance (GVD) approach to DIS at small values of the scaling variable, x. In particular, we concentrate on a recent formulation of GVD that explicitly incorporates the configuration of the $\gamma^* \to q\bar{q}$ transition and a QCD–inspired ansatz for the $(q\bar{q})p$ scattering amplitude. The destructive interference, originally introduced in off-diagonal GVD is traced back to the generic structure of two-gluon exchange. Asymptotically, the transverse photoabsorption cross section behaves as $(\ln Q^2)/Q^2$, implying a logarithmic violation of scaling for F_2, while the longitudinal-to-transverse ratio decreases as $1/\ln Q^2$. We also briefly comment on vector–meson production.

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1. Introduction

As a starting point for the present talk, I will briefly return to the experimental results on photoproduction at high energies obtained in the late sixties and their interpretation in terms of vector meson $(\rho^0, \omega, \phi, J/\psi)$ dominance (*e.g.* Refs. [1, 2]). I will subsequently introduce the concept of generalized vector dominance (GVD) [3], particularly relevant as soon as the photon four momentum enters the spacelike region, $q^2 = -Q^2 < 0$. I will briefly sketch the basic points of the most recent paper on off-diagonal GVD and its connection with QCD [4]. Before entering the detailed discussions in

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Section 2, I will also present the motivation, essentially based on the results obtained at HERA [5,6] on low–x deep-inelastic scattering, for coming back to concepts that existed already in the pre-QCD era. Essentially, it will be shown [4] that off-diagonal GVD, relevant in deep-inelastic scattering at low values of the scaling variable $x \approx Q^2/W^2$, is very well compatible with and in fact contained in QCD.

Experiments in the late sixties revealed that photoproduction [1] from nucleons above the resonance region has simple features characterized by "hadronlike" behaviour. Indeed, the energy dependence of the total cross section is much like the energy dependence of typical hadron interactions, such as pion-nucleon scattering. Likewise, as a function of the momentum transfer, the closely related reaction of Compton scattering, $\gamma p \to \gamma p$, shows a forward diffraction peak and has a predominantly imaginary forward scattering amplitude. Similar features are observed in vector–meson production, $\gamma p \to (\rho^0, \omega, \phi, J/\psi)p$. These empirical facts are quantitatively summarized by vector–meson dominance [2]. In intuitive terms, the photon virtually fluctuates into the low-lying vector–meson states, which are subsequently scattered from the nucleon. Upon applying the optical theorem, the total photoproduction cross section is thus represented as a sum of vector–meson forward production amplitudes [7],

$$\sigma_{\gamma p}(W^2) = \sum_{V=\rho^0,\omega,\phi,J/\psi} \sqrt{16\pi}\sqrt{\frac{\alpha\pi}{\gamma_V^2}} \left(\frac{d\sigma^0}{dt}|_{\gamma p \to Vp}(W^2) \right)^{\frac{1}{2}} , \tag{1}$$

and as a sum of total cross sections for the scattering of transversely polarized vector mesons on protons [8],

$$\sigma_{\gamma p}(W^2) = \sum_{V=\rho^0,\omega,\phi,J/\psi} \frac{\alpha\pi}{\gamma_V^2} \sigma_{Vp}(W^2) \,. \tag{2}$$

The factor $\alpha\pi/\gamma_V^2$ in (1) and (2) denotes the strength of the coupling of the (virtual) photon to the vector meson V, as measured in e^+e^- annihilation by the integral over the vector–meson peak,

$$\frac{\alpha\pi}{\gamma_V^2} = \frac{1}{4\pi^2\alpha} \sum_F \int \sigma_{e^+e^- \to V \to F}(s) ds \,, \tag{3}$$

or by the partial width of the vector meson,

$$\Gamma_{V \to e^+e^-} = \frac{\alpha^2 m_V}{12(\gamma_V^2/4\pi)} \,. \tag{4}$$

The sum rule (1) is an approximate one. The fractional contributions of the ρ^0, ω, and ϕ add up to approximately 72% [1, 3], and an additional contribution of 1% to 2% is to be added for the J/ψ.

The lack of complete saturation of the sum rule (1) in conjunction with the coupling of the photon to a continuum of more massive states observed in e^+e^- annihilation provides the starting point of GVD. In obvious generalisation of $\rho^0, \omega, \phi, J/\psi, \ldots$ dominance, one assumes a double dispersion relation for the transverse photoabsorption cross section $\sigma_{\gamma^*_{\rm T}p}(W^2, Q^2)$ [3],

$$\sigma_{\gamma^*_{\rm T}p}(W^2, Q^2) = \int dM^2 \int dM'^2 \frac{\rho_{\rm T}(W^2, M^2, M'^2) M^2 M'^2}{(Q^2 + M^2)(Q^2 + M'^2)} \qquad (5)$$

and a generalisation of (5) to the longitudinal cross section, $\sigma_{\gamma^*_L p}(W^2, Q^2)$. The spectral weight function, $\rho_{\rm T}(W^2, M^2, M'^2)$, contains the coupling strengths of the (timelike virtual) photon to the hadronic vector states of masses M and M', as well as the (imaginary parts of the) forward-scattering amplitudes of these states on the nucleon. The representation (5) was expected [3] to be valid in particular in the small-x diffraction region of deep inelastic scattering (DIS), *i.e.* at small x and all Q^2.[1]

The frequently employed diagonal approximation

$$\rho_{\rm T} \sim \delta(M^2 - M'^2) \qquad (6)$$

together with the photon coupling, *i.e.* $\sigma_{e^+e^- \to {\rm hadrons}}(M^2) \sim 1/M^2$, requires that vector state–nucleon scattering, σ_{Vp}, is to decrease as $\sigma_{Vp} \sim 1/M^2$. Otherwise, the representation (5) becomes divergent, and scaling of the nucleon structure function becomes violated by a power of Q^2. One arrives at what sometimes [10] has been called the "Gribov paradox" [11]. Due to the existence of diffraction dissociation in hadron-induced reactions, corresponding to off-diagonal transitions, the validity of the diagonal approximation was doubtful right from the outset. The $1/M^2$ law for σ_{Vp} has nevertheless been frequently applied as an effective one. In the HERA energy range, it led to acceptable fits to the experimental data [12]. Compare Fig. 1.

Taking into account the known empirical behaviour of diffraction dissociation (*i.e.* $M \neq M'$) in hadron physics, it was noted [13] a long time ago that indeed the justification for the diagonal approximation (6) stands on extremely weak grounds. By invoking destructive interference between diagonal and off-diagonal transitions, motivated by quark-model arguments for photon–vector–meson couplings, it was shown [13][2] that indeed scaling in

[1] Compare Ref. [9] for related lifetime arguments.

[2] Compare also [14] for a comparison of off-diagonal GVD with the experimental data then available.

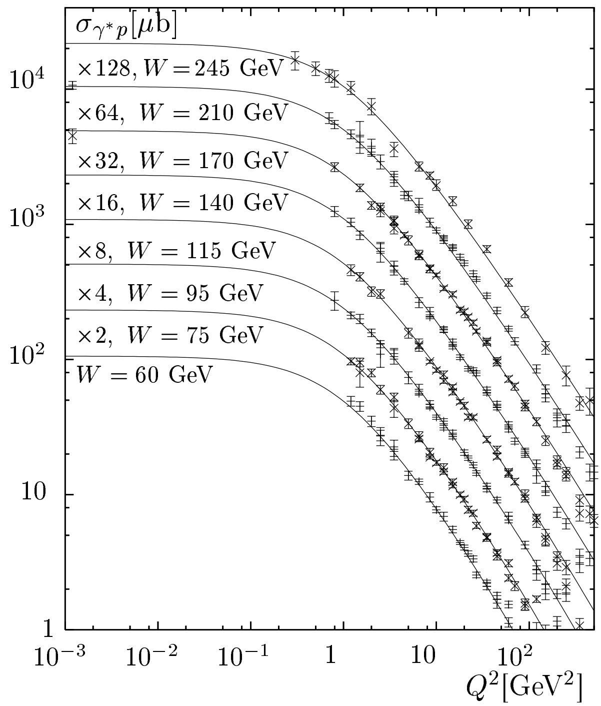

Fig. 1. Generalized Vector Dominance prediction for $\sigma_{\gamma^* p}$ [12] compared with the experimental data from the H1 and ZEUS collaborations at HERA [5,6].

e^+e^- annihilation ($\sigma_{e^+e^-} \sim 1/M^2$) and a constant total vector state–nucleon cross section ($\sigma_{VN} \sim$ const, independent of M) are completely compatible with scaling ($\sigma_{\gamma^*_{\rm T} p} \sim 1/Q^2$) in DIS.

I finally turn to the most recent work [4] by Cvetic, Shoshi and myself. In contrast to the pre-QCD formulation of GVD, we explicitly take into account the configuration of the $\gamma^* \to q\bar{q}$ transitions (seen in quark-jets in e^+e^- annihilation) and a QCD-inspired structure for the $(q\bar{q})p$-forward-scattering amplitude. We will see that the destructive interference introduced in off-diagonal GVD [13] is recovered in this formulation and traced back to the generic structure of two-gluon exchange. While the general structure of the expression for the transverse and longitudinal cross sections is the same as in the original formulation of off-diagonal GVD, there is nevertheless an important modification in terms of a $\log Q^2$ factor that originates precisely from the explicit incorporation of the $q\bar{q}$ configuration in the $\gamma^* \to q\bar{q}$ coupling. In what follows, as in the original paper [4], theoretical issues will be stressed,

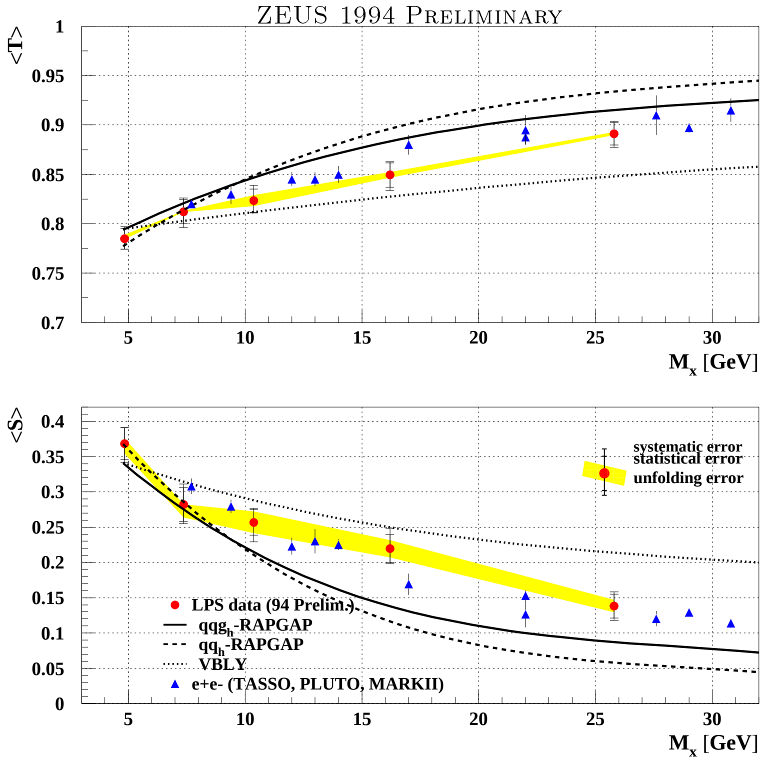

Fig. 2. Thrust $\langle T\rangle$ and sphericity $\langle S\rangle$ in diffractive production and e^+e^- annihilation.

while the comparison with the data will at most be qualitative. A quantitative comparison with the data, which needs a more careful treatment of the energy dependence, will hopefully be provided in the near future.

Before coming to the main subject, a brief remark on the motivation for taking up the subject of off-diagonal GVD again may be appropriate. The motivation is essentially provided by the fact that HERA is able to explore low–x DIS in detail. Moreover, qualitatively, the experimental HERA results are as expected from GVD and strongly support the GVD ansatz,

(i) the existence of diffractive production of high-mass states, the "large-rapidity-gap events" first announced at the '93 Marseille International Conference [15],

(ii) the similarity in shape to e^+e^- final states, compare Fig. 2 from [16],

(iii) the persistence of shadowing on complex nuclei in DIS, observed since about the year 1988 [17], after many years of confusion.

2. Off-diagonal GVD from QCD

2.1. The classical GVD approach in momentum space

The classical GVD approach is summarized in Fig. 3. A virtual timelike photon undergoes a transition to a quark-antiquark, $q\bar{q}$, state of mass $M_{q\bar{q}}$. The $q\bar{q}$ state, upon being boosted to high energy in the proton rest frame,

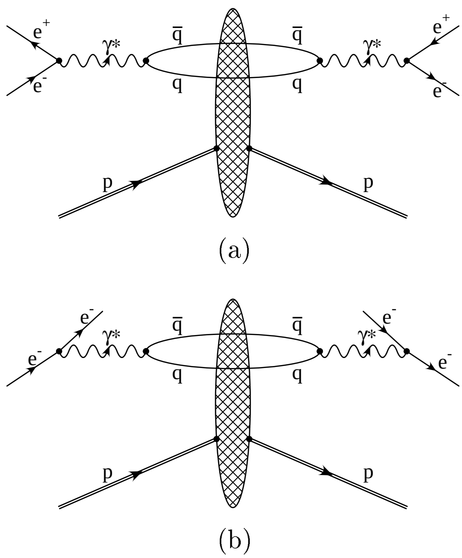

Fig. 3. The Compton forward amplitude in the proton rest frame, a) in the Gedankenexperiment where a timelike photon of mass $q^2 = M^2_{q\bar{q}}$ interacts with the nucleon, b) upon continuation from $q^2 = M^2_{q\bar{q}}$ to $q^2 = -Q^2 < 0$, with $x(\approx Q^2/W^2) \ll 1$.

is being scattered in the forward direction. The appropriate introduction of the propagators for the $q\bar{q}$ system of mass $M_{q\bar{q}}$ (and of an additional Q^2–dependent factor [3, 18] from current conservation for longitudinal photon polarisation) takes us to spacelike four-momentum transfer $q^2 \equiv -Q^2 < 0$ with $x \approx Q^2/W^2 \ll 1$ relevant for DIS in the diffraction region.

The configuration of the $q\bar{q}$ vector state depends on the transverse momentum of the quark, $\vec{k}_\perp$, with respect to the three-momentum of the photon, and on the (lightcone) variable z ($0 \leq 1 \leq z$) that is related to the angle, ϑ, of the three momentum of the quark in the $q\bar{q}$-rest-system via

$$\sin^2\vartheta \equiv 4z(1-z)\,. \tag{7}$$

The mass of the $q\bar{q}$ system is thus given by[3]

$$M_{q\bar{q}}^2 = \frac{k_\perp^2}{z(1-z)}\,, \tag{8}$$

such that either the pair of variables $(k_\perp^2, z)$ or the pair $(M_{q\bar{q}}, z)$ may be used to characterize the $q\bar{q}$ vector state.

Upon carrying out the above-mentioned steps, the transverse and longitudinal photoabsorption cross sections, $\sigma_{\gamma^*_{\mathrm{T,L}}p}(W^2, Q^2)$, become [4]

$$\begin{aligned}\sigma_{\gamma^*_{\mathrm{T,L}}p}(W^2,Q^2) &= \left[\frac{1}{2(2\pi)^3}\right]^2 \sum_{\lambda,\lambda'=\pm 1} \int dz \int dz' \int_{|\vec{k}_\perp|\geq k_{\perp 0}} d^2k_\perp \int_{|\vec{k}'_\perp|\geq k_{\perp 0}} d^2k'_\perp \\ &\times \mathcal{M}_{\mathrm{T,L}}^{(\lambda,\lambda')}(\vec{k}'_\perp, z'; Q^2)^* \mathcal{T}_{(q\bar{q})p\to(q\bar{q})p}(\vec{k}'_\perp, z'; \vec{k}_\perp, z; W^2) \mathcal{M}_{\mathrm{T,L}}^{(\lambda,\lambda')}(\vec{k}_\perp, z; Q^2)\,. \end{aligned} \tag{9}$$

The (imaginary part of the) $(q\bar{q})p$ forward-scattering amplitude (including a factor $1/W^2$ from the use of the optical theorem) has been denoted by $\mathcal{T}_{(q\bar{q})p\to(q\bar{q})p}(\vec{k}'_\perp, z'; \vec{k}_\perp, z; W^2)$. The factors $\mathcal{M}^*$ and $\mathcal{M}$ in (9) contain the $\gamma^*(q\bar{q})$ coupling as well as the propagators and read

$$\mathcal{M}_{\mathrm{T}}^{(\lambda,\lambda')}(M_{q\bar{q}}, z, Q^2) = -\frac{e_q}{Q^2 + M_{q\bar{q}}^2} \frac{j_{\mathrm{T}}^{(\lambda,\lambda')}}{\sqrt{z(1-z)}}\,, \tag{10}$$

and

$$\mathcal{M}_{\mathrm{L}}^{(\lambda,\lambda')}(M_{q\bar{q}}, z, Q^2) = -\frac{e_q}{Q^2 + M_{q\bar{q}}^2} \sqrt{\frac{Q^2}{M_{q\bar{q}}^2}} \frac{j_{\mathrm{L}}^{(\lambda,\lambda')}}{\sqrt{z(1-z)}}\,. \tag{11}$$

We refer to the original paper [4] for the explicit expressions for the transverse and longitudinal currents, $j_{\mathrm{L}}^{(\lambda,\lambda')}$ and $j_{\mathrm{T}}^{(\lambda,\lambda')}$. The lower limits of the integrations over the transverse momenta in (9) correspond to a finite transverse extension of the $q\bar{q}$ state in position space (confinement). The thresholds, $k_{\perp 0}$, allow (9) to be used as an effective description at low values of Q^2, where the low-lying vector mesons actually dominate the forward Compton amplitude.

So far, the $(q\bar{q})p$ scattering amplitude has been left unspecified. Quite generally, transitions diagonal (final mass $M'_{q\bar{q}} = M_{q\bar{q}}$) and off-diagonal ($M'_{q\bar{q}} \neq M_{q\bar{q}}$) in the mass of the $q\bar{q}$ state are possible. Assuming z to be "frozen" during the scattering process, diagonal transitions correspond to zero transverse-momentum transfer to the quark as well as to the antiquark,

[3] Here, we work in the approximation of massless quarks.

$\vec{k}'_\perp = \vec{k}_\perp$. For off-diagonal transitions to occur, the same amount of transverse momentum $|\vec{l}_\perp|$, opposite in sign for quark and antiquark, has to be transfered to the system, $\vec{k}'_\perp = \vec{k}_\perp + \vec{l}_\perp$.

It is a general feature of quantum field theory, in particular of QED, or more appropriately for the present case, of pQCD, that fermion and antifermion couple with opposite sign. This implies that for every perturbative diagonal transition amplitude there exists an off-diagonal one of the same strength, but opposite in sign. Compare the two generic diagrams for the case of two-gluon exchange in Fig. 4.

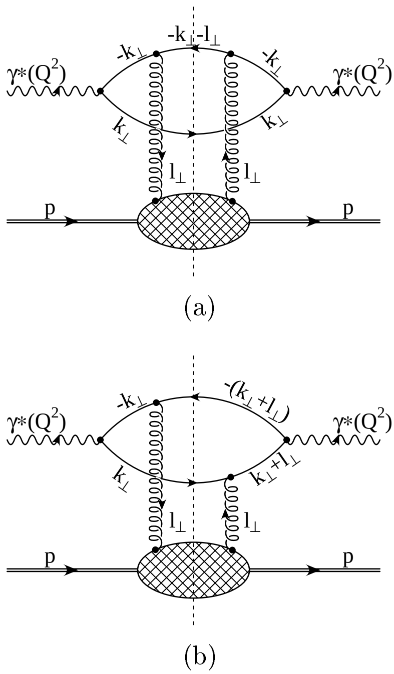

Fig. 4. The two-gluon exchange realisation of the destructive interference structure. The diagrams (a) and (b) correspond to transitions diagonal and off-diagonal in the masses of the $q\bar{q}$ pairs, respectively.

The $(q\bar{q})p$ forward-scattering amplitude is accordingly given by

$$\mathcal{T}_{(q\bar{q})p\to(q\bar{q})p}(\vec{k}'_\perp, z'\vec{k}_\perp, z; W^2) = 2(2\pi)^3 \int d^2 l_\perp \tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2) \times \left[\delta(\vec{k}'_\perp - \vec{k}_\perp) - \delta(\vec{k}'_\perp - \vec{k}_\perp - \vec{l}_\perp)\right] \delta(z - z') \ . \tag{12}$$

Substituting (12) into (9), and carrying out the integrations over $d^2k'_\perp$ and dz', yields

$$\sigma_{\gamma^*_{\mathrm{T,L}}p}(W^2,Q^2) = \frac{1}{16\pi^3}\int dz \int d^2l_\perp \tilde{\sigma}_{(q\bar{q})p}(l^2_\perp;W^2)$$
$$\times\Bigg\{\int_{|\vec{k}_\perp|\geq k_{\perp 0}} d^2k_\perp \sum_{\lambda,\lambda'=\pm 1}\left|\mathcal{M}^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{k}_\perp;Q^2)\right|^2$$
$$-\int_{|\vec{k}_\perp|\geq k_{\perp 0},|\vec{k}_\perp+\vec{l}_\perp|\geq k_{\perp 0}} d^2k_\perp \sum_{\lambda,\lambda'=\pm 1}\mathcal{M}^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{k}_\perp;Q^2)\mathcal{M}^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{k}_\perp+\vec{l}_\perp;Q^2)^*\Bigg\}. \tag{13}$$

The structure of this result, in particular the opposite signs of the diagonal and the off-diagonal term, coincide[4] with the structure of destructive interference embodied in off-diagonal GVD [13]. Arguments based on diffraction dissociation and $q\bar{q}$-bound-state wave functions were originally employed to arrive at off-diagonal GVD. Here the original ansatz finds an aposteriori justification from the general structure of the interaction of a fermion-antifermion ($q\bar{q}$) state with a hadron target.

2.2. The transverse-position-space approach

The arguments of Section 2.1 were based on the classical GVD approach, characterized by introducing the Q^2 dependence via the propagation of $q\bar{q}$ vector states. In the present section, I show that identical results are obtained in a treatment in transverse position space that introduces QCD in the $(q\bar{q})p$ interaction by invoking the notion of "colour transparency" [19,22].

The Fourier transform of the amplitude for the propagating $q\bar{q}$ state, $\mathcal{M}^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{k}_\perp;Q^2)$, from (10), (11) yields what is sometimes called the "photon–$q\bar{q}$ wave function" [19],

$$\psi^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{r}_\perp;Q^2)\equiv\frac{\sqrt{4\pi}}{16\pi^3}\int_{k_{\perp 0}} d^2k_\perp \exp(\mathrm{i}\vec{k}_\perp\cdot\vec{r}_\perp)\mathcal{M}^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{k}_\perp;Q^2)\,. \tag{14}$$

The form of (12) and (13) suggests the transverse-position-space ansatz,

$$\sigma_{\gamma^*_{\mathrm{T,L}}p}(W^2,Q^2) = \sum_{\lambda,\lambda'=\pm 1}\int dz\int d^2r_\perp \left|\psi^{(\lambda,\lambda')}_{\mathrm{T,L}}(z,\vec{r}_\perp;Q^2)\right|^2 \sigma_{(q\bar{q})p}(r^2_\perp,W^2)\,. \tag{15}$$

[4] Compare Ref. [4] for a more detailed elaboration of this point. Similar conclusions, based on somewhat different reasoning but related arguments, were arrived at in refs. [19–21].

According to (15), the interaction is diagonal in transverse position space. The notion of colour transparency, no interaction for the $q\bar{q}$ state in the $r_\perp^2 \to 0$ limit of colour neutrality, together with constancy for $r_\perp^2$ sufficiently large transverse distance

$$\sigma_{(q\bar{q})p}(r_\perp^2, W^2) \to \begin{cases} 0, & \text{for} \quad r_\perp^2 \to 0, \\ \sigma^{(\infty)}_{(q\bar{q})p}(W^2), & \text{for} \quad r_\perp^2 \to \infty\ , \end{cases} \tag{16}$$

implies the following relation between “dipole cross section” $\sigma_{(q\bar{q})p}(r_\perp^2, W^2)$ and the momentum-space function $\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2)$ of the last section:

$$\sigma_{(q\bar{q})p}(r_\perp^2, W^2) = \int d^2 l_\perp \tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2) \left(1 - \mathrm{e}^{-\mathrm{i}\vec{l}_\perp \cdot \vec{r}_\perp}\right). \tag{17}$$

Substitution of (17) and (14) into (15) and integration over $d^2 r_\perp$ and $d^2 k'_\perp$ immediately leads us back to the result (13) of the last section.

The momentum-space GVD approach based on propagators for the $q\bar{q}$ states of mass $M_{q\bar{q}}$ and a QCD-motivated relative strength of diagonal and off-diagonal transitions is accordingly entirely equivalent to the transverse position-space ansatz based on the diagonal representation (15) and on the notion of colour transparency.

In Fig. 5, as an instructive example, we show the dipole cross section for the simple ansatz of a δ-function and a Gaussian for $\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2)$,

$$\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2) = \frac{\sigma^{(\infty)}_{(q\bar{q})p}}{\pi} \delta(l_\perp^2 - \Lambda^2) \Rightarrow \sigma_{(q\bar{q})p}(r_\perp^2) = \sigma^{(\infty)}_{(q\bar{q})p} \left(1 - J_0(\Lambda |\vec{r}_\perp|)\right)\ ; \tag{18}$$

$$\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2) = \frac{\sigma^{(\infty)}_{(q\bar{q})p}}{\pi} R_0^2 \mathrm{e}^{-l_\perp^2 R_0^2} \Rightarrow \sigma_{(q\bar{q})p}(r_\perp^2) = \sigma^{(\infty)}_{(q\bar{q})p} \left(1 - \mathrm{e}^{-\frac{r_\perp^2}{4R_0^2}}\right)\ . \tag{19}$$

Not surprisingly, one finds [4] similar results for $\sigma_{\gamma^*_{\mathrm{T,L}}p}$ in case of the δ-function and the Gaussian, provided the parameters Λ and R_0 are related via $\Lambda = 1/R_0$, where R_0 is of the order of the proton radius, $R_0 \approx 1$ fm ≈ 0.2 GeV^{-1}. A Gaussian ansatz was employed in a recent analysis [23] of the experimental data, while a polynomial representation was used in Ref. [21].

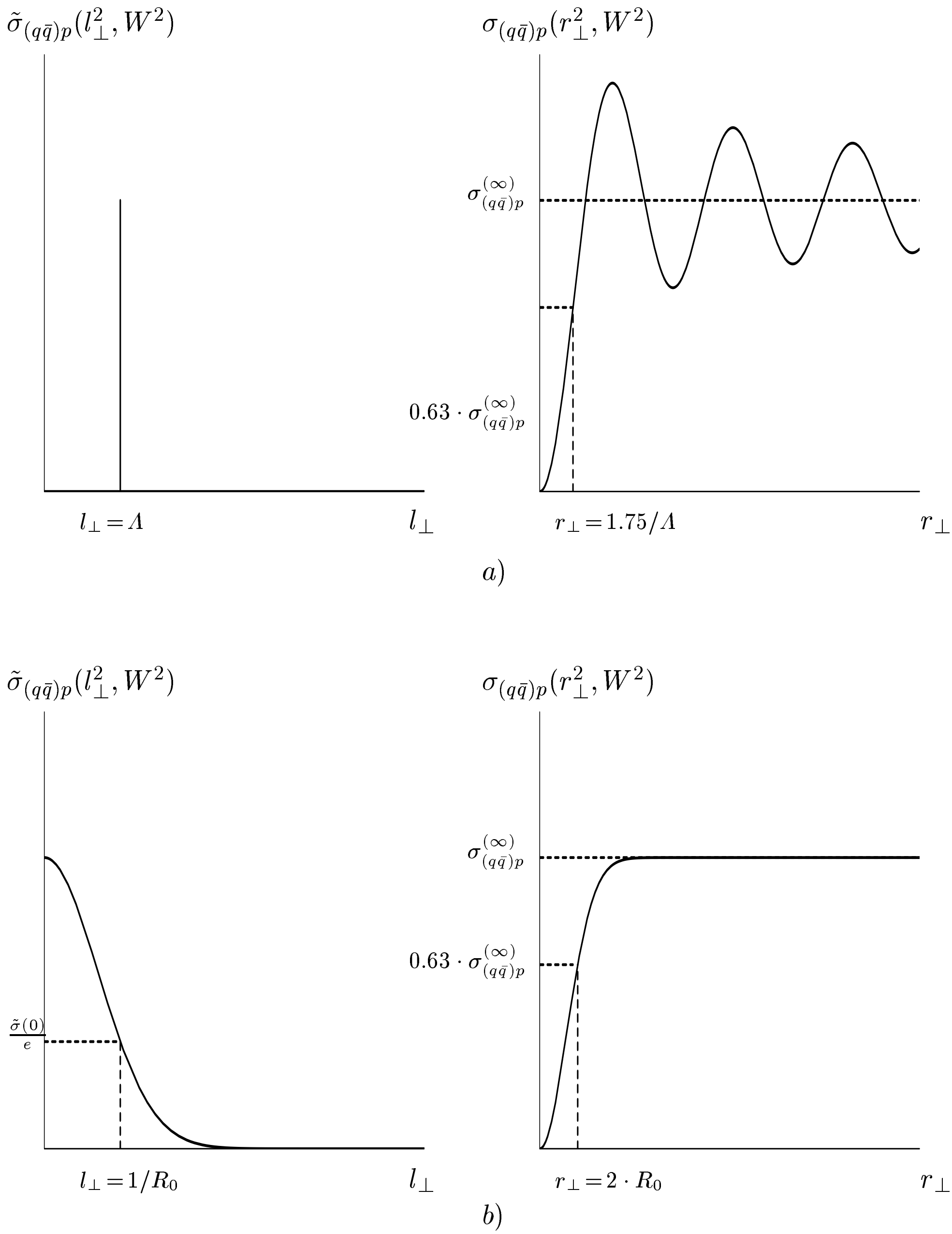

Fig. 5. The transverse-position-space dipole cross section $\sigma_{(q\bar{q})p}(r_\perp^2, W^2)$ and its Fourier transform $\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2)$ for two simple choices in transverse momentum space, a) for a δ–function and b) for a Gaussian.

2.3. A remark on the two-gluon exchange

In the above treatment, the two-gluon exchange only entered in terms of a generic structure of the interaction of the fermion-antifermion ($q\bar{q}$) state with the nucleon. As a cross-check, one may alternatively evaluate the two-gluon exchange diagrams in the low-x limit of DIS as a more concrete QCD model. In order to treat the low-x limit, the introduction of Sudakov variables proves useful [19, 24]. As long as no detailed model assumptions are introduced for the lower vertex in Fig. 4, the final result coincides with the one obtained above, even though $\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2)$ now contains the quark-gluon coupling, colour factors and the gluon propagator. The treatment is of interest, nevertheless, as in this covariant approach, one does not rely on the propagator rule for the $q\bar{q}$ system or the colour transparency assumption. In the sense of leading to the same result as the $q\bar{q}$-state propagator rule, the structure of off-diagonal GVD is realized by the pQCD model of two-gluon exchange. The validity of the expression (13) for $\sigma_{\gamma^*_{\mathrm{T,L}}p}$ is more general, however, not strictly bound to the two-gluon exchange ansatz.

3. The Q^2 dependence explicitly

To obtain the Q^2 dependence contained in the expression for $\sigma_{\gamma^*_{\mathrm{T}}p}$ (W^2, Q^2) in (13) explicitly, $\tilde{\sigma}_{(q\bar{q})p}$ must be specified, and a fivefold integration is to be carried out. I will restrict myself to the main steps and refer to the original paper [4] for a detailed presentation of the procedure.

Carrying out the angular integration over the direction of the transverse momentum of the incoming quark, $\vec{k}_\perp$, in (13), and introducing the masses of the $q\bar{q}$ system appearing in the propagators, one remains with a fourfold integration over $dz dl_\perp^2 dM_{q\bar{q}}^2 dM_{q\bar{q}}'^2$. Concerning $\tilde{\sigma}_{(q\bar{q})p}(l_\perp^2, W^2)$, for simplicity, in the recent paper, we refrained from introducing an appropriate W dependence, surely necessary for a precise description of the experimental data. We rather carried out simple model calculations based on the δ–function and the Gaussian ansatz in (18) and (19). The essential conclusions on the origin of the Q^2 dependence from $q\bar{q}$- vector-state propagation, arrived at with this procedure, will remain valid in a more complete treatment, also taking into account the energy dependence.

Even upon specializing (13) to the δ–function ansatz (18), a complete analytical treatment of the remaining threefold integral cannot be carried out for arbitrary values of Q^2. Nevertheless, simple and practically exact expressions for the Q^2 dependence of $\sigma_{\gamma^*_{\mathrm{T,L}}p}(W^2, Q^2)$ can be derived by applying the mean-value theorem to the integrations over the configuration variable z and the mass $M_{q\bar{q}}'^2$.

In the expressions for the transverse cross section (13), we replace z, or rather $z(1-z)$, by the mean value

$$\kappa_{\rm T} = \kappa_{\rm T}(Q^2) = \bar{z}_{\rm T}(Q^2)(1-\bar{z}_{\rm T}(Q^2))\,. \tag{20}$$

We have indicated a potential Q^2 dependence of $\bar{z}_{\rm T}(Q^2)$. Likewise, we replace M'^2 by an average, conveniently parametrized by the parameter $\delta_{\rm T}$ according to

$$\overline{M}'^2 = M^2 + \frac{l_\perp^2}{\bar{z}(1-\bar{z})}\frac{1}{(1+2\delta_{\rm T})}\,. \tag{21}$$

After these replacements, the integration over dM^2 can be carried out to yield [4]

$$\begin{aligned}\sigma_{\gamma^*_{\rm T}p}(W^2,Q^2;\kappa_{\rm T}(Q^2),\delta_{\rm T}) &= \frac{\alpha}{2\pi}\left(\frac{e_q}{e_0}\right)^2\sigma^{(\infty)}_{(q\bar{q})p}(1-2\kappa_{\rm T}(Q^2))\\ \times\Bigg[\left((1+2\delta_{\rm T})\frac{Q^2}{\Lambda^2}\kappa_{\rm T}(Q^2)+(1+\delta_{\rm T})\right)&\ln\left(1+\frac{\Lambda^2}{\kappa_{\rm T}(Q^2)(1+2\delta_{\rm T})(Q^2+M_0^2(\kappa_{\rm T}(Q^2)))}\right)\\ -\frac{Q^2}{(Q^2+M_0^2(\kappa_{\rm T}(Q^2)))}\Bigg]\,.\end{aligned} \tag{22}$$

The mass $M_0(\kappa_{\rm T}(Q^2)) \equiv k^2_{\perp 0}/\kappa_{\rm T}(Q^2)$ corresponds to the threshold $k^2_{\perp 0}$ introduced in (9). In addition to the basic physical parameters, $\sigma^{(\infty)}_{(q\bar{q})p}$, normalizing the cross section, and Λ^2 determining the value of the three-momentum, $l^2_\perp$, transferred to the $q\bar{q}$ system, the expression for $\sigma_{\gamma^*_{\rm T}p}$ in (22) now contains the theoretical quantities $\kappa_{\rm T}(Q^2)$ and $\delta_{\rm T}$. Their numerical values are obtained by comparing the mean-value result (22) with the exact result obtained by numerical integration of the basic expression (13). While $\delta_{\rm T}$ turns out to be independent of Q^2, the configuration, $\kappa_{\rm T}(Q^2)$, is indeed found to depend on Q^2 logarithmically,

$$\kappa_{\rm T}(Q^2) = \frac{3}{6+4\ln\left(c_1\frac{Q^2}{\Lambda^2}+\exp(c_2)\right)}\,. \tag{23}$$

The constant c_2 is uniquely determined by the mean-value evaluation of the photoproduction cross section, that yields $\kappa_{\rm T}(0)$, while c_1 is deduced from the asymptotic Q^2 behaviour. Asymptotically, from (22) with $\kappa_{\rm T}(Q^2)$ from (23), one obtains

$$\begin{aligned}\sigma_{\gamma^*_{\rm T}p}(W^2,Q^2\to\infty) &= \frac{\alpha}{3\pi}\left(\frac{e_q}{e_0}\right)^2\sigma^{(\infty)}_{(q\bar{q})p}\\ &\times\left[\frac{\Lambda^2}{Q^2}\ln\left(\frac{Q^2}{\Lambda^2}\right)+(\ln c_1)\frac{\Lambda^2}{Q^2}+\mathcal{O}\left(\frac{\ln Q^2}{Q^4}\right)\right]\,.\end{aligned} \tag{24}$$

For the ensuing discussion of the $(1/Q^2)\log Q^2$ dependence in (24), it is useful to present also the asymptotic result obtained from (22), if we ignore the Q^2 dependence of $\kappa_{\rm T}(Q^2)$ by putting $\kappa_{\rm T}(Q^2) \equiv \kappa_{\rm T}(Q^2=0)$. In this case, one finds,

$$\sigma_{\gamma^*_{\rm T}p}(W^2, Q^2 \to \infty; \kappa_{\rm T}(0)) = \frac{\alpha}{3\pi}\left(\frac{e_q}{e_0}\right)^2 \sigma^{(\infty)}_{(q\bar{q})p}\frac{3}{4}\frac{(1-2\kappa_{\rm T}(0))}{\kappa_{\rm T}(0)}\left[\frac{\Lambda^2}{Q^2}+\mathcal{O}\left(\frac{1}{Q^4}\right)\right], \tag{25}$$

i.e. by ignoring the Q^2 dependence of the configuration, one loses the $\log Q^2$ factor present[5] in (24). We conclude that the logarithmic Q^2 dependence of the $q\bar{q}$ configuration, implicitly contained in (13), is essential for the $\log Q^2$ dependence in (24) that corresponds to a logarithmic violation of scaling for the structure function F_2. The example of putting $\kappa_{\rm T}(Q^2) \equiv \kappa_{\rm T}(0)$ demonstrates that the $1/Q^2$ (scaling) dependence is unaffected by ignoring the Q^2 dependence of the configuration. In fact, as repeatedly stressed, it is the destructive interference between diagonal and off-diagonal transitions, characteristic for off-diagonal GVD, that leads to the $1/Q^2$ scaling behaviour. This conclusion is substantiated also by the results for the longitudinal cross section to be given below. In the longitudinal case, the configuration variable turns out to be independent of Q^2 in conjunction with a $1/Q^2$ scaling of the cross section.

In order to explicitly demonstrate the validity of the mean-value evaluation (22), in Fig. 6, we present the ratio $r_{\rm T}(Q^2, \kappa_{\rm T}(Q^2))$,

$$r_{\rm T}(Q^2, \kappa_{\rm T}(Q^2)) \equiv \frac{\sigma_{\gamma^*_{\rm T}p(W^2,Q^2)}}{\sigma_{\gamma^*_{\rm T}p(W^2,Q^2;\kappa_{\rm T}(Q^2),\delta_{\rm T})}}\,. \tag{26}$$

The numerator in (26), the exact numerical result for $\sigma_{\gamma^*_{\rm T,L}p}$, is obtained by numerical integration of (13). The denominator is based on the mean-value result (22). For the numerical evaluation, we chose $\Lambda^2/k^2_{\perp 0} = 1$ and $\Lambda^2 = 0.05$. For this choice of Λ^2 we found for the theoretical parameters $\delta_{\rm T}$ in (21) and c_1, c_2 in (23),

$$\delta_{\rm T} = 0.52, \quad c_1 = 1.50, \quad c_2 = 3.65\,. \tag{27}$$

The value of $c_2 = 3.65$ corresponds to $\kappa_{\rm T}(0) = 0.1455$. As seen in Fig. 6, with $\kappa_{\rm T}(Q^2)$ from (23), the mean-value evaluation practically coincides with the exact result: $r_{\rm T}(Q^2, \kappa_{\rm T}(Q^2)) = 1$ is well fulfilled for all $Q^2 \geq 0$.

Dropping the Q^2 dependence by replacing $\kappa_{\rm T}(Q^2)$ by $\kappa_{\rm T}(0)$ in (22) and (26), as expected from the above discussion, as a consequence of the

[5] The asymptotic Q^2 dependence (24) was also verified analytically [4] directly from (13) without application of the mean-value theorem.

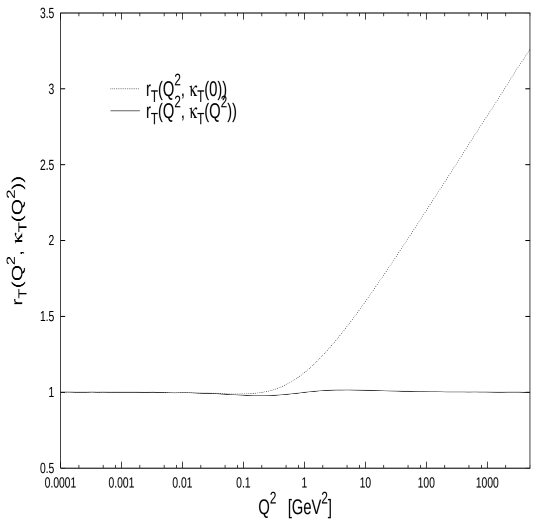

Fig. 6. The lines show the ratio $\frac{r_{\rm T}(Q^2,\kappa_{\rm T}(Q^2))=\sigma_{\gamma^*_{\rm T}p}(W^2,Q^2)/}{\sigma_{\gamma^*_{\rm T}p}(W^2,Q^2;\kappa_{\rm T}(Q^2),\delta_{\rm T})}$ from (26). The numerator is obtained by numerical integration of (13), the denominator by evaluating the mean-value expression (22). The solid line is based on $\kappa_{\rm T}(Q^2)$ from (23). The dotted line shows the result of ignoring the Q^2 dependence of $\kappa_{\rm T}(Q^2)$ by putting $\kappa_{\rm T}(Q^2) \equiv \kappa_{\rm T}(0)$ in (22).

missing $\log Q^2$ term in (25), implies a linear asymptotic rise in $\log Q^2$ for $r_{\rm T}(Q^2, \kappa_{\rm T}(0))$. Compare Fig. 6. In Table I, we show a few numerical values for $\kappa_{\rm T}(Q^2)$. Its slow logarithmic dependence is responsible for the $\log Q^2$ factor in $\sigma_{\gamma^*_{\rm T}p}$, corresponding to a logarithmic violation of scaling for F_2.

TABLE I

The parameter $\kappa_{\rm T}(Q^2)$ from (23) and the related angular dependences as a function of Q^2. We used $\Lambda^2/k^2_{\perp 0} = 1$, or $k^2_{\perp 0} = 0.05$ GeV^2.

Q^2 [GeV^2]	$\kappa_{\rm T}(Q^2)$	$\sin\vartheta$	ϑ
0.01	0.1453	0.76	49.46^0
1	0.1309	0.72	41.25^0
100	0.0788	0.56	32.09^0

An analogous procedure may be applied to the longitudinal cross section, $\sigma_{\gamma^*_{\rm L}p}$. In distinction from the transverse case, it turns out that both, $\kappa_{\rm L} \equiv \bar{z}_{\rm L}(1-\bar{z}_{\rm L})$ and $\delta_{\rm L}$ are independent of Q^2. The longitudinal cross section in mean-value evaluation becomes

$$\sigma_{\gamma_L^* p}(W^2, Q^2; \kappa_L, \delta_L) = \frac{2\alpha}{\pi}\left(\frac{e_q}{e_0}\right)^2 \sigma^{(\infty)}_{(q\bar{q})p} Q^2 \kappa_L \left[\frac{1}{(Q^2 + M_0^2(\kappa_L))} - \frac{(1+2\delta_L)\kappa_L}{\Lambda'^2} \ln\left(1 + \frac{\Lambda'^2}{\kappa_L(1+2\delta_L)(Q^2 + M_0^2(\kappa_L))}\right)\right]. \tag{28}$$

For $Q^2 \to \infty$ we have a $1/Q^2$ dependence, corresponding to a scaling contribution to F_2,

$$\sigma_{\gamma_L^* p}(W^2, Q^2 \to \infty; \delta_L) = \frac{2\alpha}{\pi}\left(\frac{e_q}{e_0}\right)^2 \sigma^{(\infty)}_{(q\bar{q})p} \left[\frac{\Lambda^2}{2(1+2\delta_L)Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)\right], \tag{29}$$

while for $Q^2 \to 0$ the longitudinal cross section rises linearly with Q^2. The numerical values for κ_L and δ_L are given by

$$\kappa_L = 0.1714, \quad \delta_L = -0.1767\,. \tag{30}$$

As in the transverse case, the mean-value evaluation (28) provides an excellent approximation of the exact result obtained by numerical integration of (13). Compare the original paper [4].

In Fig. 7, we show the transverse and longitudinal cross sections normalized by (transverse) photoproduction.

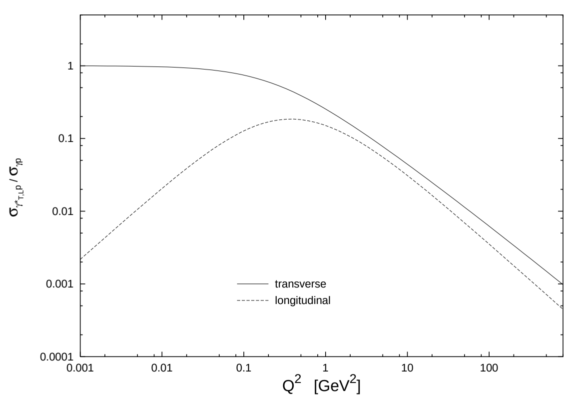

Fig. 7. Numerical results for $\sigma_{\gamma_T^* p}$ (solid line), and $\sigma_{\gamma_L^* p}$ (dotted line) from (13), normalized by the photoproduction cross section $\sigma_{\gamma p}$. The results shown are obtained by numerical integration of (13). The mean-value results (22) (with $\kappa_T(Q^2)$ from (23)) and (28) coincide with the ones shown, apart from a minor deviation in the longitudinal cross section around $Q^2 \approx 1$ GeV^2.

For a detailed comparison with experiment, the theory has to be extended by incorporating the W dependence. Compare Refs. [21, 23] for interesting work in this direction. Nevertheless, it is worth noting that the drop by almost 2 orders of magnitude from $Q^2 = 0$ GeV^2 to $Q^2 \approx 100$ GeV^2 seen in the experimental data in Fig. 1 is also present in the present theoretical results.

Finally, in Fig. 8 we show the longitudinal to transverse ratio R. It rises linearly for $Q^2 \to 0$ and drops as $1/\log Q^2$ for $Q^2 \to \infty$.

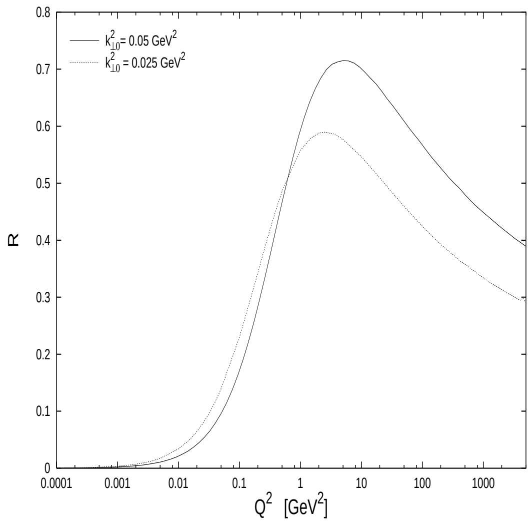

Fig. 8. The longitudinal-to-transverse ratio R. The solid curve corresponds to $\Lambda^2 = k^2_{\perp 0} = 0.05$ GeV^2 as used in Fig. 7. The dotted curve is obtained for $\Lambda^2 = 0.05$ GeV^2 and $k^2_{\perp 0} = 0.025$ GeV^2, as indicated.

4. Off-diagonal GVD in vector–meson production

Reformulating and extending the off-diagonal GVD ansatz [25] for elastic vector meson production, recent work [26] by Schuler, Surrow and myself yields a satisfactory representation of the transverse cross section and the longitudinal-to-transverse ratio, R, for elastic ρ^0, ϕ and $J/$Psi-production [26, 27]. The theoretical prediction for $\sigma_{\mathrm{T},\gamma^* p \to Vp}$ is based on

$$\sigma_{\gamma^*_{\mathrm{T}} p \to Vp} = \frac{m^4_{V,\mathrm{T}}}{(Q^2 + m^2_{V,\mathrm{T}})^2} \sigma_{\gamma p \to Vp}(W^2) \,. \tag{31}$$

I refer to Ref. [26] for the prediction for R. The inclusion of off-diagonal transitions with destructive interference yields $m^2_{V,\mathrm{T}} < m^2_V$, where m_V

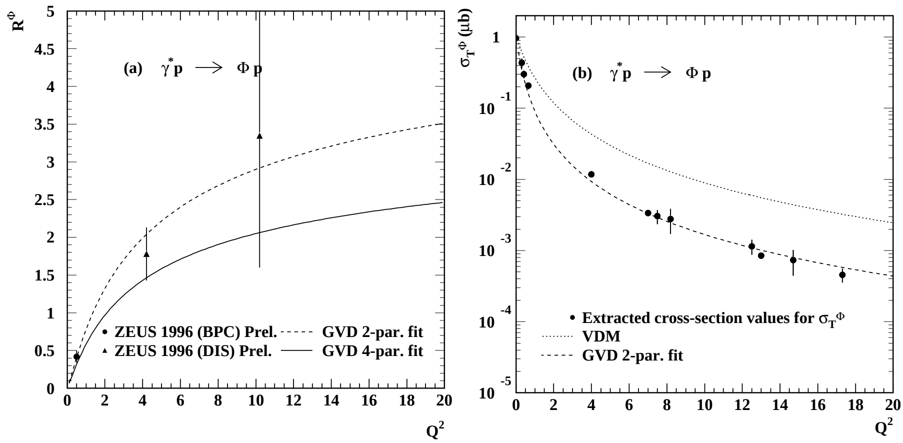

Fig. 9. GVD in $\gamma^* p \to \phi\ p$ [26].

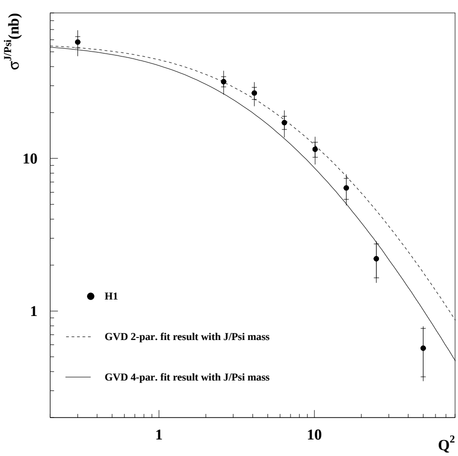

Fig. 10. GVD in $\gamma^* p \to J/\mathrm{Psi}\ p$ [27].

stands for the mass of the vector meson being produced. As an example, in Fig. 9, I show ϕ production. The curves are based on $m^2_{\phi,\mathrm{T}} = 0.40 m^2_\phi$ and $\sigma_{\gamma p \to \phi p} = 1.0 \mu b$. The theoretical curves for J/Psi production in Fig. 10 were obtained by the replacement $m^2_\phi \to m^2_{J/\mathrm{Psi}}$ and $\sigma_{\gamma p \to \phi p} \to \sigma_{\gamma p \to J/\mathrm{Psi}\ p}$. A more direct connection between the result of Ref. [4] and the off-diagonal GVD treatment in Ref. [26] needs to be established.

5. Conclusions

I will be brief in my conclusions. Taking into account the $q\bar{q}$ configuration in the $\gamma^* \to q\bar{q}$ transition and the opposite signs of the fermion and the antifermion interaction with the nucleon target in the formulation of GVD, one arrives at a representation of the photoabsorption cross section containing destructive interference between diagonal and off-diagonal transitions characteristic for off-diagonal GVD. The destructive interference is responsible for scaling, the implicit dependence on the $q\bar{q}$ configuration is responsible for the logarithmic violation of scaling. The longitudinal-to-transverse ratio decreases logarithmically with Q^2 for $Q^2 \to \infty$. The classical momentum-space formulation has been shown to lead to results identical to the position-space treatment based on the notion of colour transparency. Needless to be stressed again, off-diagonal GVD is compatible with QCD, it is in fact contained in QCD. The two-gluon exchange provides the simplest QCD model showing the features of the more general GVD-momentum-space or the transverse-position-space formulation. Off-diagonal GVD also yields the experimentally observed Q^2 dependence for (elastic) vector–meson forward production. A careful treatment of the energy dependence, and further work on the diffractively produced final states will be indispensable in order to approach a detailed understanding of deep-inelastic scattering in the low-x diffraction region.

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