TESTS OF CHIRAL PERTURBATION THEORY AT DAΦNE*

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DAΦNE will offer an opportunity to check Chiral Perturbation Theory predictions at higher order. In this talk I have selected a few topics for which it is expected that the lowest order calculation will not be sufficient in order to compare with the experimental results. In particular I will discuss pion pair production in two photon collisions, K_{l_3} and K_{l_4} decays.

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1. Introduction

Quantum Chromodynamics is by now established as the proper theory to describe Strong Interactions and, consequently, it is the theory behind hadronic physics. Unfortunately, the QCD coupling constant at the energies needed to bind quarks and antiquarks into mesons and baryons is too large to allow for a sensible perturbative expansion. Alternative ways to describe hadron interactions are needed. Lattice QCD with MonteCarlo simulations provides a very promising way to study hadronic properties. However, only masses and simple processes have been calculated with this method. A huge increase in the computer power (which is not foreseeable in the near future) is needed in order to deal with more involved quantities, such as cross-sections and decay widths. These quantities are better studied in the framework of effective theories. Chiral Perturbation Theory (ChPT) is such an effective theory in which the quark and gluon degrees of freedom are replaced by the pseudoscalar fields. The Lagrangian of the theory is written in terms of

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these fields, taking care that all the symmetries of the QCD Lagrangian are included in the effective theory. The perturbative expansion is performed in terms of the momenta involved in the processes, with the lowest order being $O(p^2)$. In the second section of this talk I will present a short introduction to Chiral Perturbation Theory with a view on what will be needed in the rest of the talk. More complete reviews of the status of ChPT can be found in Ref. [1,2].

The new Φ-factory, DAΦNE (Double Accelerator for Nice Experiments), will offer the possibility of experimentally studying many processes that can be described by ChPT. The design luminosity of DAΦNE, $\mathcal{L} = 5 \times 10^{32}$ cm^{-2}sec^{-1}, will allow to reach an experimental precision that can only be matched with the precision obtained at Next to Leading Order (and in some cases Next to Next to Leading Order) theoretical calculations. DAΦNE has recently started operating although the luminosity is only 1.5×10^{30} cm^{-2}sec^{-1}. The physics possibilities of this accelerator are very wide and they are covered by three experiments: KLOE (K Long Observation Experiment) for Particle Physics, FINUDA (FIsica NUcleare at DAphne) for Nuclear Physics and DEAR (Daphne Exotic Atom Research) for Atomic Physics.

The main goal of DAΦNE is the study of CP violation. With the nominal luminosity one expects 7.5×10^9 $K_S^0 K_L^0$ pairs a year. This large number of K pairs will allow to measure $\text{Re}(\varepsilon'/\varepsilon)$ with an error $\delta(\varepsilon'/\varepsilon) = 1 \times 10^{-4}$ through the measurement of the double ratio

$$\frac{N_L^{+-}/N_S^{+-}}{N_L^{00}/N_S^{00}} = \left| \frac{A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-)}{A(K_L^0 \to \pi^0\pi^0)/A(K_S^0 \to \pi^0\pi^0)} \right|^2 = 1 + 6\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right). \quad (1)$$

The present experimental values for ε'/ε have been obtained with the study of the same ratio and are summarized in Table I. The errors are still larger than the expected sensitivity at DAΦNE, although it is expected that they will also be able to reduce their errors down to $\delta(\varepsilon'/\varepsilon) = 1 \times 10^{-4}$. However, it is important to notice that all the present experiments are performed in hadronic colliders. Thus, the systematic errors will be very different at DAΦNE than in the other experiments. Hopefully, with the combined efforts of all the experimental groups we will soon have a clear situation concerning the experimental value of ε'/ε. The present world average shown in Table I is compatible with the Standard Model prediction, although one has to go into one corner of the allowed region in the parameter space [8].

The measurement of ε'/ε can also be performed with a new method. The idea is to identify the $\pi^+\pi^-$ and $\pi^0\pi^0$ production points in the process $\Phi \to \overline{K}K \to \pi^+\pi^-\pi^0\pi^0$ and build the asymmetry:

$$A(d) = \frac{N(d>0) - N(d<0)}{N(d>0) + N(d<0)} = A_R(d)\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) + A_I(d)\text{Im}\left(\frac{\varepsilon'}{\varepsilon}\right). \quad (2)$$

TABLE I

Experimental results for ε'/ε

ε'/ε	Group
$(23 \pm 7) \times 10^{-4}$	NA31 [3]
$(7.4 \pm 5.9) \times 10^{-4}$	E731 [4]
$(28.0 \pm 4.1) \times 10^{-4}$	KTeV [5]
$(18.5 \pm 7.3) \times 10^{-4}$	NA48 [6]
$(21.2 \pm 2.8) \times 10^{-4}$	World Average [7]

N is the number of events with positive or negative values of $d = d_c - d_n$ where d_c and d_n are the distances between the interaction point and the charged and neutral pion pair production points, respectively. The functions $A_{\mathrm{R}}(d)$ and $A_I(d)$ have a very simple asymptotic behavior for large values of d:

$$A_{\mathrm{R}}(d) \to 3 \quad A_I(d) \to 0 \,. \tag{3}$$

Thus, a measurement of the asymmetry (2) for large values of d provides a new measurement of $\mathrm{Re}\,(\varepsilon'/\varepsilon)$. The sensitivity of this method is, however, a bit worse than the one of the double ratio (1): $\delta(\mathrm{Re}\,(\varepsilon'/\varepsilon)) \sim 1.8 \times 10^{-4}$. Moreover, a fit to the whole dependence of the asymmetry on d also provides a value for $\mathrm{Im}\,(\varepsilon'/\varepsilon)$ although the expected error will be rather large: $\delta(\mathrm{Im}\,(\varepsilon'/\varepsilon)) \sim 3.4 \times 10^{-3}$ [9].

2. Chiral perturbation theory: a brief introduction

The QCD Lagrangian can be written in terms of $q = \mathrm{column}(u\ d\ s)$ in the form:

$$\mathcal{L}_{\mathrm{QCD}} = i\overline{q}_{\mathrm{L}}\gamma^{\mu}D_{\mu}q_{\mathrm{L}} + i\overline{q}_{\mathrm{R}}\gamma^{\mu}D_{\mu}q_{\mathrm{R}} + \overline{q}_{\mathrm{L}}m_q q_{\mathrm{R}} + \overline{q}_{\mathrm{R}}m_q q_{\mathrm{L}} + \mathcal{L}^{HF} + \mathcal{L}^{G} \,, \tag{4}$$

where the term $\mathcal{L}^{HF}$ includes the contribution from heavy quarks, $\mathcal{L}^{G}$ contains only gluonic terms and $m_q = \mathrm{diag}(m_u\ m_d\ m_s)$ contains the quark masses. It is clear that in the limit where $m_q = 0$ the Lagrangian is invariant under independent transformations of the left and right-handed quark fields, *i.e.* under the group $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$:

$$q_{\mathrm{L}} \to g_{\mathrm{L}}q_{\mathrm{L}} \qquad q_{\mathrm{R}} \to g_{\mathrm{R}}q_{\mathrm{R}} \qquad \text{with} \qquad g_{\mathrm{L}}, g_{\mathrm{R}} \in \mathrm{SU}(3)_{\mathrm{L,R}} \,. \tag{5}$$

In view of this symmetry, one would expect that all hadrons appear in multiplets of opposite parity. However, there is no evidence for such a symmetry. We cannot blame the small quark masses this big effect. Instead, before

the appearance of QCD it was already recognized that SU(3) was a rather good symmetry [10]. The chiral symmetry is, thus, spontaneously broken: $\mathrm{SU(3)_L \times SU(3)_R \to SU(3)_V}$ through the non-zero value of the quark condensate:

$$\langle 0|\overline{u}u|0\rangle = \langle 0|\overline{d}d|0\rangle = \langle 0|\overline{s}s|0\rangle \sim -(250 \text{ MeV})^{3/2}, \tag{6}$$

which becomes the order parameter of the spontaneous chiral symmetry breaking. The Goldstone theorem assures that in this process eight goldstone bosons appear (one for each broken generator) [11]. These bosons are massless in the limit of massless quarks, but the small explicit chiral symmetry breaking through the quark masses gives a small mass to the goldstone bosons.

The idea of ChPT is to write down an effective Lagrangian where the quarks and gluons have been replaced by the goldstone bosons appearing in the spontaneous chiral symmetry breaking. A convenient parametrization is in terms of a 3×3 unitary matrix:

$$\Sigma = \mathrm{e}^{2iM/f} \quad \text{with} \quad M = \begin{pmatrix} \dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\[2mm] \pi^- & -\dfrac{\pi^0}{\sqrt{2}} + \dfrac{\eta}{\sqrt{6}} & K^0 \\[2mm] K^- & \overline{K}^0 & -2\dfrac{\eta}{\sqrt{6}} \end{pmatrix}, \tag{7}$$

and f is a free constant. This matrix transforms under $\mathrm{SU(3)_L \times SU(3)_R}$ as:

$$\Sigma \to g_\mathrm{L} \Sigma g_\mathrm{R}^\dagger . \tag{8}$$

The effective Lagrangian contains an infinite number of terms, but it can be expanded according to the number of derivatives. This is something more than a convenient classification. Physically, it means an expansion in terms of powers of momenta that have to be small compared with the chiral symmetry breaking scale, which is ~ 1 GeV. Lorentz invariance requires the number of derivatives to be even. Thus, the first term is:

$$\mathcal{L}_2 = \frac{f^2}{8} \mathrm{tr}\, \partial_\mu \Sigma \partial^\mu \Sigma^\dagger . \tag{9}$$

This is the only relevant term with two derivatives, because other possible terms one can think off, such as $\Sigma \partial_\mu \partial^\mu \Sigma^\dagger$, differ from (9) only in a total derivative. Expanding Σ it is obvious that the Lagrangian in Eq. (9) contains the kinetic terms for all the pseudoscalar mesons and interaction terms involving 4, 6 and a larger number of pseudoscalars. Moreover, taking the axial current, one has:

$$\langle 0|J_\mu^{L_{1+i2}}|\pi^+\rangle = -\frac{i}{\sqrt{2}} f_\pi P_{\pi_\mu} \quad \text{with} \quad J_\mu^{L_a} = -\frac{if^2}{4} \mathrm{tr}\,(T^a \partial_\mu \Sigma \Sigma^\dagger), \tag{10}$$

leading to the identification at this order of the free constant f with the well-known pion decay constant $f_\pi = f = 132$ MeV. Note that at this point there is a complete SU(3) symmetry among the three decay constants: $f_\pi = f_\eta = f_K$.

The effects of the explicit chiral symmetry breaking through the non-vanishing values of the quark masses can be included in the Lagrangian (9) adding some new terms:

$$\mathcal{L}_2 = \frac{f^2}{8}\mathrm{tr}\,\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger + (\Sigma\chi^\dagger + \chi\Sigma^\dagger)\right),\tag{11}$$

where χ contains the external scalar and pseudoscalar fields in the following way:

$$\chi = B(s - ip),\qquad \text{where}\qquad s = m + \cdots.\tag{12}$$

B is again a free constant that can be calculated in terms of the pseudoscalar and quark masses:

$$B = \frac{2m_\pi^2}{m_u + m_d} = \frac{2m_K^2}{m_u + m_s} = \frac{6m_\eta^2}{m_u + m_d + m_s}.\tag{13}$$

From this relations, eliminating the quark masses, one can obtain the Gell-Mann–Okubo mass relation [12]

$$4m_K^2 - m_\pi^2 = 3m_\eta^2.\tag{14}$$

The new term in the Lagrangian also contains more interaction terms, which are proportional to the pseudoscalar masses. The expansion, thus, is not only in powers of the momenta, but also in powers of the pseudoscalar masses.

External vector fields can be introduced in the theory converting the derivatives appearing in the Lagrangian in covariant derivatives:

$$\mathcal{L}_2 = \frac{f^2}{8}\mathrm{tr}\,\left(D_\mu\Sigma D^\mu\Sigma^\dagger + (\Sigma\chi^\dagger + \chi\Sigma^\dagger)\right);$$
$$D_\mu\Sigma = \partial_\mu\Sigma + iL_\mu\Sigma - i\Sigma R_\mu\tag{15}$$

and adding the appropriate kinetic terms for the vector fields L_μ and R_μ. These fields transform under SU(3)$_\mathrm{L}$ × SU(3)$_\mathrm{R}$ as:

$$L_\mu \rightarrow g_\mathrm{L}L_\mu g_\mathrm{L}^\dagger - ig_\mathrm{L}\partial_\mu g_\mathrm{L}^\dagger,$$
$$R_\mu \rightarrow g_\mathrm{R}R_\mu g_\mathrm{R}^\dagger - ig_\mathrm{R}\partial_\mu g_\mathrm{R}^\dagger.\tag{16}$$

In particular, we can introduce electromagnetic interactions involving photons and pseudoscalars with the identification $L_\mu = R_\mu = eA_\mu Q$, where Q

is the quark charge matrix:

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \tag{17}$$

Weak processes with external W fields can also be introduced with the identification: $R_\mu = 0$ and

$$L_\mu = \frac{e}{\sqrt{2} \sin \theta_W} (W_\mu^+ T_+ + \text{h.c.}) \quad T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

We will use this form in our discussion of K_{l3} and K_{l4} decays.

In this way we complete the description of the lowest order ChPT Lagrangian. With this Lagrangian we can reproduce all the current algebra results obtained in the sixties. The advantage now is that we have a tool that allows to calculate in a systematic way corrections to these results.

The next order corrections are $O(p^4)$. It is interesting to notice that one loop diagrams contribute to terms at this order. Indeed, in any one loop diagram the number of vertices must be the same as the number of internal lines. Since each internal line contributes at $O(p^{-2})$ (they are pseudoscalar propagators), the total dimension of the loop contribution is given by the momentum integral, $i.e.$ $O(p^4)$. This result can be easily generalized for any L-loop diagram containing N_d vertices of dimension d [14]

$$D = 2L + 2 + \sum_d (d-2)N_d. \tag{19}$$

Thus, any loop diagram contributes to terms with a dimension larger than the dimensions of the vertices involved in the diagram.

In order to be consistent in the chiral expansion we have to consider the higher dimension terms in the Lagrangian together with the loop calculations. In particular, since one loop calculations give a result $O(p^4)$ we have to add to the Lagrangian the terms of this order that have been neglected up to now. The complete set of $O(p^4)$ terms invariant under parity, charge conjugation and chiral transformations were first written by Gasser and Leutwyler [13]:

$$\mathcal{L}_4 = L_1 \left[\text{tr} \left(D_\mu \Sigma^\dagger D^\mu \Sigma \right) \right]^2 + L_2 \text{tr} \left(D_\mu \Sigma^\dagger D_\nu \Sigma \right) \text{tr} \left(D^\mu \Sigma^\dagger D^\nu \Sigma \right)$$

$$+ L_3 \text{tr} \left(D_\mu \Sigma^\dagger D^\mu \Sigma D_\nu \Sigma^\dagger D^\nu \Sigma \right) + L_4 \text{tr} \left(D_\mu \Sigma^\dagger D^\mu \Sigma \right) \text{tr} \left(\Sigma^\dagger \chi + \chi^\dagger \Sigma \right)$$

$$+ L_5 \text{tr} \left[D_\mu \Sigma^\dagger D^\mu \Sigma \left(\Sigma^\dagger \chi + \chi^\dagger \Sigma \right) \right] + L_6 \left[\text{tr} \left(\Sigma^\dagger \chi + \chi^\dagger \Sigma \right) \right]^2$$

$$+L_7 \left[\mathrm{tr} \left(\Sigma^\dagger \chi - \chi^\dagger \Sigma \right) \right]^2 + L_8 \mathrm{tr} \left(\chi^\dagger \Sigma \chi^\dagger \Sigma + \Sigma^\dagger \chi \Sigma^\dagger \chi \right)$$

$$-iL_9 \mathrm{tr} \left(R^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger + L^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma \right) + L_{10} \mathrm{tr} \left(\Sigma^\dagger R^{\mu\nu} \Sigma L_{\mu\nu} \right)$$

$$+H_1 \mathrm{tr} \left(R_{\mu\nu} R^{\mu\nu} + L_{\mu\nu} L^{\mu\nu} \right) + H_2 \mathrm{tr} \left(\chi^\dagger \chi \right), \tag{20}$$

where

$$
\begin{aligned}
R_{\mu\nu} &= \partial_\mu R_\nu - \partial_\nu R_\mu - i \left[R_\mu, R_\nu \right], \\
L_{\mu\nu} &= \partial_\mu L_\nu - \partial_\nu L_\mu - i \left[L_\mu, L_\nu \right].
\end{aligned}
\tag{21}
$$

The equations of motion of the $O(p^2)$ have been used to remove the terms than contain second derivatives. In this Lagrangian we have introduced 12 new free constants.

A second consequence of the previous dimension counting for the one loop diagrams is that their result will, in general, be divergent. These divergences can, however, be removed with the help of the 12 constants appearing in Eq. (20). Since this Lagrangian contains *all* the terms at $O(p^4)$, the divergent terms appearing in the loop calculation have to be contained in Eq. (20). We can define the constants in terms of a finite, measurable part and an infinite part: [1]

$$
\begin{aligned}
L_i &= L_i^r(\mu^2) + \lambda \Gamma_i & i &= 1, 10, \\
H_i &= \lambda \Delta_i & i &= 1, 2.
\end{aligned}
\tag{22}
$$

Using dimensional regularization,

$$\lambda = \frac{1}{32\pi^2} \left[\frac{1}{\varepsilon} + 1 - \gamma - \log \mu^2 + \log(4\pi) \right], \tag{23}$$

where ε is related to the number of dimensions through the usual expression: $d = 4 - 2\varepsilon$ and γ is the Euler constant. The Γ_i and Δ_i in Eq. (22) can be determined to cancel all the divergences appearing in one loop calculations [13]:

$$\Gamma_1 = \frac{3}{32} \qquad \Gamma_2 = \frac{3}{16} \qquad \Gamma_3 = 0 \qquad \Gamma_4 = \frac{1}{8} \qquad \Gamma_5 = \frac{3}{8}$$

$$\Gamma_6 = \frac{11}{144} \qquad \Gamma_7 = 0 \qquad \Gamma_8 = \frac{5}{48} \qquad \Gamma_9 = \frac{1}{4} \qquad \Gamma_{10} = -\frac{1}{4} \tag{24}$$

$$\Delta_1 = -\frac{1}{8} \qquad\qquad \Delta_2 = \frac{5}{24}.$$

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[1] The constants H_1, H_2 do not have a physical relevance. They are only needed to cancel the divergences appearing in the loop calculations.

In this way we can get finite results for all the physical amplitudes in spite of the fact that we are dealing with a non-renormalizable theory. The price we have to pay is the introduction of 10 free constants. This procedure can be repeated at any order with the introduction of new free constants at each order in the expansion. The theory is non-renormalizable because we need an infinite number of constants to renormalize the whole theory, in contrast to what happens in a renormalizable theory where a finite number of constants allows to absorb all the divergences appearing at any order in the perturbative expansion.

The finite parts of the constants $L_1^r, \ldots, L_{10}^r$ depend of the renormalization scale μ as it was explicitly noted in Eq. (22). Indeed, since the physical amplitudes are renormalization scale independent the μ dependence of the finite part of the one loop diagrams will cancel renormalization scale dependence of the constants. This dependence can, thus, be easily expressed using the definition (22):

$$L_i^r(\mu_2) = L_i^r(\mu_1) + \frac{\Gamma_i}{(4\pi)^2} \log \frac{\mu_1}{\mu_2}. \tag{25}$$

The values of these constants have to be fixed by experiment. In Table II we show their values at the scale $\mu = m_\rho$ together with an indication of the experimental data from which these numbers have been obtained [2]

TABLE II

The values of the L_i coefficients and the input used to determine them, they are quoted at a scale $\mu = m_\rho$.

L_i	Value $\cdot 10^3$	Input
1	0.4 ± 0.3	K_{e4} and $\pi\pi \to \pi\pi$
2	1.35 ± 0.3	K_{e4} and $\pi\pi \to \pi\pi$
3	-3.5 ± 1.1	K_{e4} and $\pi\pi \to \pi\pi$
4	-0.3 ± 0.5	$1/N_c$ arguments
5	1.4 ± 0.5	F_K/F_π
6	-0.2 ± 0.3	$1/N_c$ arguments
7	-0.4 ± 0.2	Gell-Mann–Okubo, L_5, L_8
8	0.9 ± 0.3	$m_{K^0} - m_{K^+}$, L_5, baryon mass ratios
9	6.9 ± 0.7	pion electromagnetic charge radius
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$

Let us discuss, as an example, the situation in $\pi\pi$ scattering. It is a simple exercise to obtain from the Lagrangian in Eq. (11) the Weinberg amplitude [14]:

$$A(s, t, u) = \frac{s - m_\pi^2}{f_\pi^2} \tag{26}$$

that fixes the scattering amplitude for $\pi^a(p_a)\pi^b(p_b) \to \pi^c(p_c)\pi^d(p_d)$ through the isospin decomposition:

$$T_{ab,cd} = \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,s,u) + \delta_{ad}\delta_{bc}A(u,t,s), \qquad (27)$$

with $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$ and $u = (p_a - p_d)^2$. The amplitudes of definite isospin can be expanded in partial wave amplitudes according to:

$$A^I(s, \cos\theta) = i\frac{32\pi\sqrt{s}}{\sqrt{s - 4m_\pi^2}} \sum_{l=0}^{\infty} (2l+1)P_l(\cos\theta)\left(1 - e^{2i\delta_l^I(s)}\right), \qquad (28)$$

where δ_l^I are the phase shifts. The corresponding scattering lengths, a_l^I, are defined as the slopes of the phase shifts at threshold. The lowest order predictions from Eq. (26) are:

$$a_0^0 = 0.156 \qquad\qquad a_0^0 - a_2^0 = 0.201, \qquad (29)$$

to be compared with the experimental data [15]:

$$a_0^0 = 0.26 \pm 0.05 \qquad\qquad a_0^0 - a_2^0 = 0.29 \pm 0.04. \qquad (30)$$

It is clearly important to evaluate the correction to these results.

The $\pi\pi$ scattering amplitude at $O(p^4)$ receive contributions from one loop diagrams involving vertices from $\mathcal{L}_2$, tree diagrams involving vertices from $\mathcal{L}_4$ and wave function renormalization. In Fig. 1 we show the experimental values of the phase shift difference $\delta_0^0 - \delta_1^1$ as a function of the $\pi\pi$ center of mass energy compared to the $O(p^2)$ (dashed line) and $O(p^4)$ (dot-dashed line) results [13]. The solid line is the result of an $O(p^6)$ calculation [16]. The results for the scattering lengths are:

$$\begin{aligned} a_0^0 &= 0.156 + 0.044 + 0.017 = 0.217, \\ a_0^0 - a_2^0 &= 0.201 + 0.042 + 0.016 = 0.258. \end{aligned} \qquad (31)$$

The three terms in the right hand side correspond to the $O(p^2)$, $O(p^4)$ and $O(p^6)$, respectively. In these results we can see that the inclusion of the higher order terms tend to improve the agreement between the theoretical values and the experimental data. We should also note the nice convergence shown by the perturbative expansion, even though the first correction is quite large.

The example we just discussed is also relevant for DAΦNE because the experimental data for the $\pi\pi$ phase shifts and scattering lengths are extracted from K_{l4} decays [15,17]. Since the agreement between the theoretical and experimental results is at the 1σ level for a_0^0 (and just a bit better for $a_0^0 - a_2^0$), it is clear that a significant reduction of the experimental error at DAΦNE will provide an stringent test of the ChPT result.
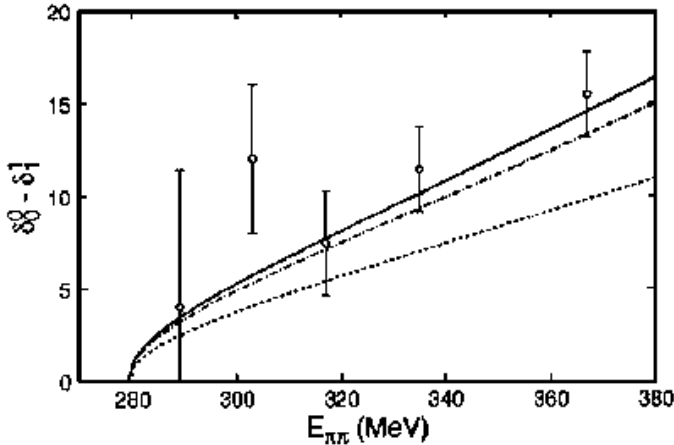
Fig. 1. Phase shift difference as a function of the center of mass energy. The dashed line stands for the lowest order calculation, the dot-dashed line for the $O(p^4)$ and the solid line for the $O(p^6)$, assuming that the constants of the $O(p^6)$ Lagrangian vanish.

3. $\gamma\gamma \to \pi\pi$

We will first discuss charged pion pair production in two photon processes, where the photons are assumed to be real. The lowest order amplitude is easily calculated from the Feynman rules for scalar electrodynamics that can be found in any textbook on Quantum Field Theory. The total cross section as a function of the $\gamma\gamma$ center of mass energy is:

$$\sigma(s) = \frac{\pi\alpha^2}{2s}\beta\left(2|a|^2 - 4(2-\delta)\mathrm{Re}\,a + 4\left[2 - 2\delta + (1-\beta^2)\left(1 + \frac{1}{2}\delta\right)\right]\right),\tag{32}$$

where

$$\delta = \frac{1-\beta^2}{\beta}\ln\frac{1+\beta}{1-\beta}\tag{33}$$

and β is the velocity of the pions in their center of mass system. The lowest order cross section is obtained from Eq. (32) with the identification $a = 1$ and it is plotted in Fig. 2 compared to the experimental data from Ref. [18]. The $O(p^4)$ result can also be obtained from Eq. (32) with [19]

$$a = 1 + \frac{4s}{f^2}(L_9^r + L_{10}^r) - \frac{1}{16\pi^2 f^2}\left(\frac{3}{2}s + m_\pi^2\ln^2 Q_\pi + \frac{1}{2}m_K^2\ln^2 Q_K\right)\tag{34}$$
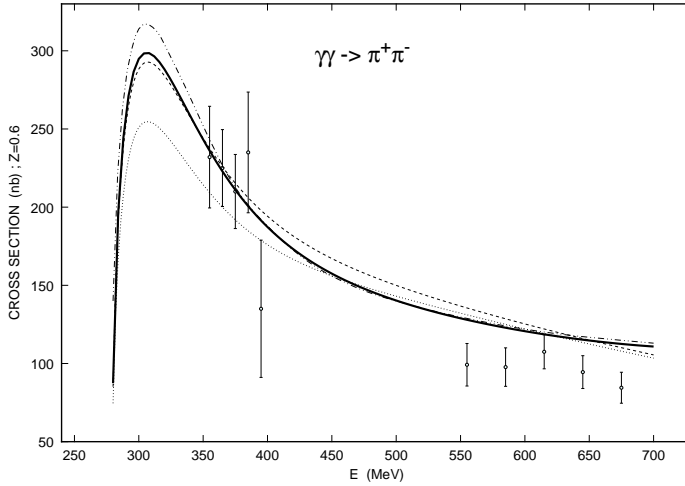
Fig. 2. $\gamma\gamma \to \pi^+\pi^-$ cross-section as a function of the center of mass energy. The dotted line stands for the lowest order prediction, the dashed line is the result of the $O(p^4)$ calculation and the solid line is the result of the full, $O(p^6)$, calculation. The dashed- double dotted line is the result of a dispersive calculation [21].

where Q_i is given by:

$$Q_i = \frac{\sqrt{s - 4m_i^2} + \sqrt{s}}{\sqrt{s - 4m_i^2} - \sqrt{s}}. \tag{35}$$

The second and third term on this expression correspond to the $O(p^4)$ tree contributions and the loop contributions, respectively. In Eq. (34) there is a dependence on the sum of two unknown constants from the $O(p^6)$ Lagrangian, namely L_9^r and L_{10}^r. This sum also contributes to the structure dependent term in $\pi \to e\nu\gamma$. From the experimental data (see Ref. [13]) one obtains

$$L_9^r + L_{10}^r = (1.4 \pm 0.4) \times 10^{-3}. \tag{36}$$

Thus, at this order we also have a parameter free prediction for the total cross section, which is shown in Fig. 2 (dashed line). Comparing it with the lowest order prediction we can see that there is a 13% increase in the value of the cross section at the peak. The $O(p^6)$ corrections to this cross-section have also been calculated [20] and they are also shown in Fig. 2 (solid line).

The process with neutral pion pair production is very interesting from the theoretical point of view. Inspecting the Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_4$ one can easily realize that there are no tree level contributions to this process. Thus, the lowest order contribution is $O(p^4)$ and is given by one loop contributions. Since there are no $O(p^4)$ terms in the Lagrangian contributing to this

process, one knows that the one loop calculation must be finite (otherwise, there would be no way to cancel the divergence from the loop calculation) and contain no free parameter. The result of this calculation is shown in Fig. 3 [19] (dashed line), compared to the experimental data from Crystall Ball [22]. In this case, the $O(p^6)$ corrections have also been calculated (solid line) and turn out to be important [23]. Indeed, in this case they are the Next to Leading Order corrections!
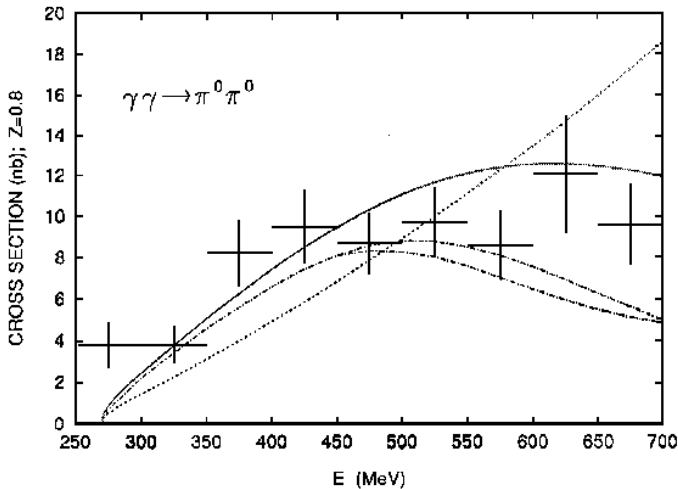

Fig. 3. $\gamma\gamma \to \pi^0\pi^0$ cross-section as a function of the center of mass energy at $O(p^4)$ (dashed line) and $O(p^6)$ (solid line). The result of a dispersive calculation from [24] is shown with a dashed-dotted line.

The discussion above on pion pair production have been performed assuming that the incoming photons are real, which is a good approximation for a no-tag experiment. However, this is not a feasible experimental situation at DAΦNE, where both, electrons and positrons, must be tagged in order to have a reliable measurement. The minimum deflection angle for the incoming electron is $10°$. One should, thus, extend the previous calculations to the whole $e^+e^- \to e^+e^-\pi^+\pi^-$ process. This is work in progress. The expected number of events with a $\pi\pi$ invariant mass lower than 600 MeV at DAΦNE is $O(10^4)$ for charged pions and $O(10)$ for neutral pions. Since the two photon effective luminosity is larger for lower $\gamma\gamma$ center of mass energy, one can expect to be able to discriminate the $O(p^4)$ effects in the charged pion channel. For neutral pion pair production, the number of events is too low to allow for any precision measurements.

4. Semileptonic K decays: K_{l3} and K_{l4}

The subject of semileptonic K decays have been extensively studied by Bijnens and collaborators [25]. In this talk I will only cover two channels. The first one I want to discuss consists of K_{l3} decays, *i.e.*

$$
\begin{aligned}
K^+ &\to \pi^0 l^+ \nu_l \,, \\
K^0 &\to \pi^- l^+ \nu_l \,,
\end{aligned}
$$

where l can be an electron or a muon. The matrix element for K^+ decay has the general structure:

$$
T = \frac{G_F}{\sqrt{2}} V_{us}^* l^\mu F_\mu^+(p, p') \,, \tag{37}
$$

where G_F is the Fermi coupling constant, V_{us} is the corresponding matrix element of the CKM matrix, l_μ is the leptonic current and $F_\mu^+(p, p')$ is the hadronic current that depends on the K^+ and π^0 momenta, respectively. The general form of this current involves two form factors:

$$
F_\mu^+(p, p') = (p + p')_\mu f_+^{K\pi}(t) + (p - p')_\mu f_-^{K\pi}(t) \,, \tag{38}
$$

where $t = (p - p')^2$. Similar expressions can be obtained for K^0 decays. Instead of $f_+^{K\pi}$ and $f_-^{K\pi}$ one normally uses $f_+^{K\pi}$ and the scalar form factor

$$
f_0^{K\pi} = f_+^{K\pi} + \frac{1}{m_K^2 - m_\pi^2} f_-^{K\pi} \tag{39}
$$

and, for simplicity, the dependence on t of both form factors is assumed to be linear:

$$
f_{+,0}^{K\pi}(t) = f_+^{K\pi}(0) \left(1 + \lambda_{+,0} \frac{t}{m_\pi^2} \right). \tag{40}
$$

The experimental situation for the values of the slope parameters is very clear for λ_+ [28] and is shown in Table III. The situation, however, is much more confusing for λ_0 [28], but the most recent measurement was performed 18 years ago. At lowest order in ChPT, both slope parameters vanish (*i.e.* the form factors are independent of t) but they receive contributions at $O(p^4)$. These contributions are given by a typical function for loop calculations, but this function approximates in a very nice way to a straight line, justifying the empirical approximation made in (40). It depends on the parameter L_9^r. This parameter also appears in the pion electromagnetic form factor. From a fit to this quantity we obtain $L_9^r = (6.8 \pm 0.2) \times 10^{-3}$ [19]

and we have again a parameter free prediction for the slope parameters. In this way we obtain:

$$\lambda_+ = 0.031 \qquad\qquad \lambda_0 = 0.017. \qquad (41)$$

The result for λ_+ is in very good agreement with the experimental data and it will be interesting to compare the value obtained for λ_0 at DAΦNE with this prediction. In order to asses the improvement one can expect at DAΦNE in the precision of the measurements we should take into account that the number of K^+ and K^0 decays in the previous measurements are 10^5 and 4×10^6, respectively, while at DAΦNE one expects 3×10^8 events per year in each channel.

TABLE III

Experimental results for the slope parameter in K_{l3} decays, λ_+

λ_+	Decay Channel
0.0286 ± 0.0022	$K_{e\ 3}^+$
0.032 ± 0.008	$K_{\mu\ 3}^+$
0.0300 ± 0.0016	$K_{e\ 3}^0$
0.034 ± 0.005	$K_{\mu\ 3}^+$

Let us finally turn our attention into the K_{l4} decays, $i.e.$;

$$
\begin{aligned}
K^+ &\to \pi^+ \pi^- l^+ \nu_l \,, \\
K^+ &\to \pi^0 \pi^0 l^+ \nu_l \,, \\
K^0 &\to \pi^0 \pi^- l^+ \nu_l \,.
\end{aligned}
\qquad (42)
$$

In this case there are four independent form factors. F, G, R, and H. The form factor R cancels for $m_e = 0$ and has not been measured up to now, so we will neglect it in our discussion. The other form factors can be parametrized in the form

$$
\begin{aligned}
F &= f_s \mathrm{e}^{i\delta_0^0} + f_p \mathrm{e}^{i\delta_1^1} \cos\theta_\pi + D\text{-wave}\,, \\
G &= g \mathrm{e}^{i\delta_1^1} + D\text{-wave}\,, \\
H &= h \mathrm{e}^{i\delta_1^1} + D\text{-wave}\,,
\end{aligned}
\qquad (43)
$$

where the phase shifts are the same ones as in $\pi\pi$ scattering discussed in Section 2. θ_π is the angle between the π^+ in the two pion rest frame and the dipion line of flight in the K rest frame. Neglecting the D-wave contribution

and assuming $f_p = 0$ and a linear dependence of the form factors:

$$
\begin{aligned}
f_s(q^2) &= f_s(0)(1 + \lambda q^2)\,, \\
g(q^2) &= g(0)(1 + \lambda q^2)\,, \\
h(q^2) &= h(0)(1 + \lambda q^2),
\end{aligned}
\tag{44}
$$

with $q^2 = \dfrac{s_\pi - 4m_\pi^2}{4m_\pi^2}$ and s_π being the two pion invariant mass, Rosselet and collaborators have obtained the following experimental values [15]:

$$
\begin{aligned}
f_s(0) &= 5.59 \pm 0.14\,, \quad g(0) = 4.77 \pm 0.27\,, \\
h(0) &= -2.68 \pm 0.68\,, \quad \lambda = 0.08 \pm 0.02\,.
\end{aligned}
\tag{45}
$$

The first non-vanishing contribution to the form factor H is due to the chiral anomaly [26] and one obtains $H = -2.66$, in good agreement with the experimental data. The next order corrections have also been evaluated and turns out to be very small [27].

The theoretical calculation of the F and G form factors is now much more involved than in the previous cases. The lowest order result for both form factors is 3.74, but the $O(p^4)$ corrections receive contributions from many of the unknown constants. In particular, they receive contributions from L_1^r, L_2^r and L_3^r that cannot be determined from any other process. One should use data on these decays to precisely determine the values of these parameters. One could use the form factors and slope parameters to obtain values for these constants and in this way obtain a parameter free prediction for the low energy parameters in $\pi\pi$ scattering. Alternatively, one could also perform a fit to the whole set of data [25]. The authors of Refs. [25] and [29] have also estimated the higher order corrections and they have found that their effects might be sizeable.

5. Conclusions

The main purpose of this talk was to show that the experiments that will be performed at DAΦNE will be sensitive to higher order corrections in Chiral Perturbation Theory. I have not gone through the whole list processes that have been studied. They can be found in [30]. In all the cases we have discussed (except in two neutral pion production in two photon processes) we have seen that the great improvement that can be expected in the experimental data from DAΦNE with respect to previous experiments will require Next to Leading Order Calculations ($O(p^4)$) and, as it looks in K_{l4} decays Next to Next to Leading Order ($O(p^6)$). It should also be noted that ChPT provides clear expressions that can be used to directly compare

with the experimental data, instead of using a number of assumptions (as the linear dependence of the form factors).

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