

ONE-LOOP QCD CORRECTED DISTRIBUTION IN $B \rightarrow X e \bar{\nu}_e$ * **

M. JEŻABEK^{a,b} AND P. URBAN^a

^a Department of Field Theory and Particle Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland

^b Institute of Nuclear Physics
Kawory 26a, 30-055 Cracow, Poland

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We give an analytic formula for the double distribution of hadronic invariant mass and charged lepton energy to one-loop order of the perturbative QCD. Although infrared singular, this quantity is closely related to physical observables that can be obtained thereof through proper convolution.

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1. Introduction

Today, the interest in bottom meson decays has never been stronger. Of those, the semileptonic kind are particularly fit for precision measurements of the Standard Model parameters such as the Cabibbo–Kobayashi–Maskawa matrix elements $|V_{cb}|$ and $|V_{ub}|$ [1, 2] or the quark masses. The theoretical predictions rely on perturbation theory as well as the effective theory of heavy quarks [3–6] for inclusion of QCD effects. The present techniques are able to provide successful description of the semileptonic decays and it has been known that significant contributions to the B decays come from both perturbative [7–11] and nonperturbative [12–14] corrections.

In this paper, we give the one-loop perturbative correction to the double differential distribution in terms of the hadron system mass and the charged lepton energy. The masses of the final particles are set to zero throughout the present calculation.

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The double differential distribution contains an explicit dependence on the fictitious gluon mass as it must due to the infrared singularities present both in the virtual and the real radiation corrections. It is not until after one integrates over the invariant mass of hadrons that the apparent divergence cancels to leave a finite, measurable result. The distribution we give can be integrated over any desired range of the hadronic mass to yield an experimentally valid quantity. Hence, arbitrary cuts on the hadronic mass may easily be imposed.

The result we obtain is consistent with the published formulae for the single differential distribution in both variables considered. In particular, we have checked they agree with the charged lepton energy distribution, first published by Ježabek and Kühn [7], and the hadronic mass distribution, as found in [11, 15].

2. Kinematics

2.1. Variables

In our treatment of the decay of a B meson, we need to consider both three- and four-body final states. The latter includes a gluon radiated by the initial or final quark. Denote the four-momenta of the involved particles as Q, q, l, ν, G for the b quark, final state quark, electron, neutrino and gluon, respectively. Also define the hadronic four-momentum $P = q + G$. We are working in the rest frame of the B meson, which in the present approximation reduces to that of the b quark.

The variables we use are x for the electron energy, t denoting the invariant mass of the leptons, and z for the invariant mass of the hadron system, all scaled according to the following formulae:

$$x = \frac{2(Q \cdot l)}{m_b^2}, \quad t = \frac{(l + \nu)^2}{m_b^2}, \quad z = \frac{(q + G)^2}{m_b^2}, \quad (1)$$

with m_b the bottom quark mass. Also we will use the scaled masses of the gluon and the final u quark, which both pop up in the intermediate formulae in the course of calculating the contributions from three- and four-body processes.

$$\lambda_G = \frac{m_G}{m_b}, \quad \varepsilon = \sqrt{\rho} = \frac{m_u}{m_b}. \quad (2)$$

The system of the quark in the final state and the real gluon is described by the following quantities:

$$P_0 = \frac{1}{2}(1 - t + z) \quad (3)$$

$$P_3 = \sqrt{P_0^2 - z} = \frac{1}{2}[1 + t^2 + z^2 - 2(t + z + tz)]^{1/2}, \quad (4)$$

$$P_{\pm}(z) = P_0(z) \pm P_3(z), \quad (5)$$

$$\mathcal{Y}_p = \frac{1}{2} \ln \frac{P_+(z)}{P_-(z)} = \ln \frac{P_+(z)}{\sqrt{z}}, \quad (6)$$

where $P_0(z)$ and $P_3(z)$ are the energy and the length of the momentum vector of the system in the b quark rest frame, and $\mathcal{Y}_p(z)$ is the corresponding rapidity. Similarly for the virtual boson W :

$$W_0(z) = \frac{1}{2}(1+t-z), \quad (7)$$

$$W_3(z) = \sqrt{W_0^2 - t} = \frac{1}{2}[1+t^2+z^2-2(t+z+tz)]^{1/2}, \quad (8)$$

$$W_{\pm}(z) = W_0(z) \pm W_3(z), \quad (9)$$

$$\mathcal{Y}_w(z) = \frac{1}{2} \ln \frac{W_+(z)}{W_-(z)} = \ln \frac{W_+(z)}{\sqrt{t}}. \quad (10)$$

Kinematically, the three body decay is a special case of the four body one, with the four-momentum of the gluon set to zero, thus resulting in simply replacing $z = \rho$. The following variables are then useful:

$$p_0 = P_0(\rho) = \frac{1}{2}(1-t+\rho), \quad p_3 = P_3(\rho) = \sqrt{p_0^2 - \rho}, \quad (11)$$

$$p_{\pm} = P_{\pm}(\rho) = p_0 \pm p_3, \quad w_{\pm} = W_{\pm}(\rho) = 1 - p_{\mp}, \quad (12)$$

$$Y_p = \mathcal{Y}_p(\rho) = \frac{1}{2} \ln \frac{p_+}{p_-}, \quad Y_w = \mathcal{Y}_w(\rho) = \frac{1}{2} \ln \frac{w_+}{w_-}. \quad (13)$$

2.2. Kinematical boundaries

The electron energy may vary within the range of

$$0 \leq x \leq 1, \quad (14)$$

while for a fixed value of x , the allowed interval of z is

$$0 \leq z \leq 1 - x. \quad (15)$$

On the other hand, without fixing x , the hadronic mass may vary as

$$0 \leq z \leq 1, \quad (16)$$

and then the following boundaries on x are found for a specified z :

$$0 \leq x \leq 1 - z. \quad (17)$$

Given values of x and z , the intermediate boson invariant mass t varies within the following limits,

$$0 \leq t \leq x \left(1 - \frac{z}{1-x} \right) \equiv t_m. \quad (18)$$

3. Evaluation of the QCD corrections

The differential decay rate for the process $b \rightarrow u l \bar{\nu}_l$ can be written as

$$d\Gamma = d\Gamma_0 + d\Gamma_{1,3} + d\Gamma_{1,4}, \quad (19)$$

where

$$d\Gamma_0 = G_F^2 m_b^5 |V_{ub}|^2 \mathcal{M}_{1,3} d\mathcal{R}_3(Q; q, l, \nu) / \pi^5 \quad (20)$$

stands for the Born approximation,

$$d\Gamma_{1,3} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{ub}|^2 \mathcal{M}_{1,3} d\mathcal{R}_3(Q; q, l, \nu) / \pi^6 \quad (21)$$

comes from the interference between the virtual gluon and Born amplitudes, and

$$d\Gamma_{1,4} = \frac{2}{3} \alpha_s G_F^2 m_b^5 |V_{ub}|^2 \mathcal{M}_{1,4} d\mathcal{R}_4(Q; q, G, l, \nu) / \pi^7 \quad (22)$$

is due to the real gluon emission. The Lorentz invariant n -body phase space is defined as

$$d\mathcal{R}_n(P; p_1, \dots, p_n) = \delta^{(4)}(P - \sum p_i) \prod_i \frac{d^3 \mathbf{p}_i}{2E_i}. \quad (23)$$

3.1. Three-body contributions

The Born approximation is evaluated according to (20) with the matrix element

$$\mathcal{M}_{0,3} = q l Q \nu, \quad (24)$$

and the Dalitz parametrization of the phase space,

$$d\mathcal{R}_3(Q; q, l, \nu) = \frac{\pi^2}{4} dx dt. \quad (25)$$

Subsequently, the dot products appearing in $\mathcal{M}_{0,3}$ are put in terms of the variables x and t ,

$$\begin{aligned} Qq &= \frac{m_b^2}{2}(1 + \rho - t), & lq &= \frac{m_b^2}{2}(x - t) \\ Q\nu &= \frac{m_b^2}{2}(1 - \rho - x + t), & l\nu &= \frac{m_b^2}{2}t \\ Ql &= \frac{m_b^2}{2}x, & \nu q &= \frac{m_b^2}{2}(1 - \rho - x). \end{aligned} \quad (26)$$

which readily yields the double differential distribution. On integration over t the latter yields the Born approximation to the electron energy spectrum (see Eqs (39), (41)).

The virtual correction is calculated along the same lines, now with the matrix element

$$\begin{aligned} \mathcal{M}_{1,3} = & - \left[H_0 q l Q \nu + H_+ \rho Q \nu Q l + H_- q \nu q l \right. \\ & + \frac{1}{2} \rho (H_+ + H_-) \nu l + \frac{1}{2} \rho (H_+ - H_- + H_L) [l (Q - q - \nu)] (Q \nu) \\ & \left. - \frac{1}{2} H_L [l (Q - q - \nu)] (q \nu) \right], \end{aligned} \quad (27)$$

substituted into (21). In (27), the form factors are as follows,

$$\begin{aligned} H_0 = & 4 \left(1 - \frac{Y_p p_0}{p_3} \right) \ln \lambda_G + \frac{2 p_0}{p_3} \left[\text{Li}_2 \left(1 - \frac{p_- w_-}{p_+ w_+} \right) \right. \\ & - \text{Li}_2 \left(1 - \frac{w_-}{w_+} \right) - Y_p (Y_p + 1) + 2 (\ln \sqrt{\rho} + Y_p) (Y_w + Y_p) \Big] \\ & + \frac{2 p_3 Y_p + (1 - \rho - 2t) \ln \sqrt{\rho}}{t} + 4, \end{aligned} \quad (28)$$

$$H_{\pm} = \frac{1}{2} \left[1 \pm \frac{(1 - \rho)}{t} \right] \frac{Y_p}{p_3} \pm \frac{1}{t} \ln \sqrt{\rho}, \quad (29)$$

$$H_L = \frac{1}{t} (1 - \ln \sqrt{\rho}) + \frac{1 - \rho}{t^2} \ln \sqrt{\rho} + \frac{2}{t^2} Y_p p_3 + \frac{\rho Y_p}{t p_3}. \quad (30)$$

Integration over the invariant mass of the intermediate W boson gives the desired contribution to the double differential distribution,

$$\begin{aligned} \frac{d\Gamma_{1,3}}{12\Gamma_0 dx dz} = & \delta(z) \left\{ \frac{1}{3} \log \lambda_G [10x - 25x^2 + \frac{34}{3}x^3 + (10 - 24x + 18x^2 - 4x^3) \log \bar{x} \right. \\ & - 6x^2 \log \varepsilon + 4x^3 \log \varepsilon] + \frac{1}{3} (2x^3 - 3x^2) \text{Li}_2(x) + \frac{1}{18} \log \bar{x} [121 \\ & - 276x + 195x^2 - 40x^3 - \log \bar{x} (30 - 72x + 54x^2 - 12x^3)] \\ & \left. + \frac{121}{18}x - \frac{443}{36}x^2 + \frac{128}{27}x^3 + \frac{1}{6} \log \varepsilon (-3x^2 + 2x^3 + 6x^2 \log \varepsilon - 4x^3 \log \varepsilon) \right\}, \end{aligned} \quad (31)$$

where we have denoted

$$\bar{x} = 1 - x. \quad (32)$$

While we present the decay rate assuming massless final particles, it is not possible to eliminate the final quark as well as gluon mass dependence out of the matrix element alone. This is removed once the integrated real radiation term is added.

3.2. Four-body contributions

The four-body phase-space is decomposed as follows:

$$d\mathcal{R}_4(Q; q, G, l, \nu) = dz d\mathcal{R}_3(Q; P, l, \nu) d\mathcal{R}_2(P; q, G). \quad (33)$$

Discarding terms vanishing along with the gluon mass $\lambda_G \rightarrow 0$, the matrix element representing the real gluon radiation may be written as

$$\mathcal{M}_{1,4} = \frac{\mathcal{B}_1}{(Q G)^2} - \frac{\mathcal{B}_2}{Q G P G} + \frac{\mathcal{B}_3}{(P G)^2}, \quad (34)$$

where

$$\mathcal{B}_1 = q l [Q \nu (Q G - 1) + G \nu - Q \nu Q G], \quad (35)$$

$$\begin{aligned} \mathcal{B}_2 = q l [G \nu - q \nu Q G + Q \nu (q G - Q G - 2q Q)] \\ + Q \nu (Q l q G - G l q Q), \end{aligned} \quad (36)$$

$$\mathcal{B}_3 = Q \nu (G l q G - \rho l P). \quad (37)$$

We refer the reader to [9] for the details of the integration that leads to the triple differential rate in terms of x, t and z . This rate is conveniently split into terms according as they are infrared convergent or divergent. Hence we can write,

$$\frac{d\Gamma_{1,4}}{12\Gamma_0 dx dt dz} = \mathcal{F}_{\text{conv}} + \mathcal{F}_{\text{div}}. \quad (38)$$

This expression is subsequently integrated over t . The infrared finite part has been integrated with the help of FORM. The formulae obtained in this way suffer from infrared and collinear divergences. Thence the gluon and final quark masses subsist as regulators in spite of the limit we have taken. Of course, both remnant dependences vanish after the integration over the hadronic system mass is performed, which involves summing the virtual and real contributions.

4. Results

The first order QCD corrected double differential decay rate can be written in the form,

$$\frac{d\Gamma}{12\Gamma_0 dx dz} = f_0(x)\delta(z) + \frac{2\alpha_s}{3\pi} [f_1^s\delta(z) + f_1^c(x, z)\theta(z - \lambda_G^2)], \quad (39)$$

with

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}, \quad (40)$$

where the first term on the right hand side is the Born approximation, given by

$$f_0(x) = \frac{1}{6}x^2(3 - 2x), \quad (41)$$

while

$$\begin{aligned} f_1^s(x) = & -4f_0(x)\log^2\lambda_G + \frac{1}{18}(120x - 291x^2 + 130x^3)\log\lambda_G \\ & + \frac{1}{3}(-10 + 24x - 18x^2 + 4x^3)\log\bar{x}\log\left(\frac{\bar{x}}{\lambda_G^2}\right) \\ & + \frac{1}{6}(83 - 196x + 145x^2 - 32x^3)\log\bar{x} \\ & + \frac{1}{18}(249x - 426x^2 + 155x^3) - \frac{2}{3}f_0(x) [\pi^2 + 3\text{Li}_2(x)], \end{aligned} \quad (42)$$

and

$$\begin{aligned} f_1^c(x, z) = & \frac{1}{z}[-2f_0(x)\log z + \frac{1}{36}(120x - 291x^2 + 130x^3) \\ & + \frac{1}{3}(10 - 24x + 18x^2 - 4x^3)\log\bar{x}] + \left[\frac{10z+3z^2}{\bar{x}} - \frac{z+2z^2}{\bar{x}^2} + \frac{2}{3}\frac{z^2}{\bar{x}^3}\right]\log\frac{z}{\bar{x}^2} \\ & + \frac{1}{2}\bar{x}^{-1}(-69z - 25z^2) + \frac{1}{4}\bar{x}^{-2}(11z + 25z^2) \\ & + \bar{x}^{-3}\left(-\frac{29}{18}z^2\right) + \left(\frac{101}{2} - 17xz - 81x + \frac{45}{2}x^2 - 31z - \frac{3}{2}z^2\right)\log\bar{x} \\ & + (32 - 6xz - 30x + 4x^2 + 29z + z^2)\log\bar{x}\log\frac{z}{\bar{x}} \\ & + \log z(21xz + 32x - 16x^2 - 9z - \frac{5}{3}z^2) \\ & - 20xz + \frac{101}{2}x - \frac{97}{4}x^2 + \frac{127}{4}z + \frac{283}{36}z^2. \end{aligned} \quad (43)$$

The above formula is easily integrated over either of the variables to give the single differential distributions in hadronic system mass or charged lepton energy. Then expressions confirming previous calculations [7, 11, 15] are found. The evident singularity of this distribution disappears after integration over the hadronic system mass. That this indeed is so, can be seen by expressing it in terms of the following distributions,

$$\left(\frac{1}{z}\right)_+ = \lim_{\lambda \rightarrow 0} \left(\frac{1}{z}\theta(z - \lambda) + \log\lambda\delta(z)\right), \quad (44)$$

$$\left(\frac{\log z}{z}\right)_+ = \lim_{\lambda \rightarrow 0} \left(\frac{\log z}{z}\theta(z - \lambda) + \frac{1}{2}\log^2\lambda\delta(z)\right). \quad (45)$$

The substitution of these to Eqs (42), (43) results in the formal identification,

$$\theta(z - \lambda_G^2)\frac{1}{z} = \theta(z - \lambda_G^2)\left(\frac{1}{z}\right)_+ - \delta(z)\log\lambda_G^2, \quad (46)$$

$$\theta(z - \lambda_G^2)\frac{\log z}{z} = \theta(z - \lambda_G^2)\left(\frac{\log z}{z}\right)_+ - \frac{1}{2}\log^2\lambda_G^2\delta(z). \quad (47)$$

Upon application of Eqs (46), (47) to the correction terms, the latter take on the following form:

$$\begin{aligned} f_1^s(x) = & \frac{1}{3}(-10 + 24x - 18x^2 + 4x^3) \log^2 \bar{x} \\ & + \frac{1}{6}(83 - 196x + 145x^2 - 32x^3) \log \bar{x} \\ & + \frac{1}{18}(249x - 426x^2 + 155x^3) - \frac{2}{3}f_0(x) [\pi^2 + 3\text{Li}_2(x)], \quad (48) \end{aligned}$$

$$\begin{aligned} f_1^c(x, z) = & -2f_0(x) \left(\frac{\log z}{z} \right)_+ \\ & + \left(\frac{1}{z} \right)_+ \left[\frac{1}{36}(120x - 291x^2 + 130x^3) + \left(\frac{10}{3} - 8x + 6x^2 - \frac{4}{3}x^3 \right) \log \bar{x} \right] \\ & + \left[\frac{10z + 3z^2}{\bar{x}} - \frac{z + 2z^2}{\bar{x}^2} + \frac{2z^2}{3\bar{x}^3} \right] \log \frac{z}{\bar{x}^2} \\ & + \frac{1}{2}\bar{x}^{-1}(-69z - 25z^2) + \frac{1}{4}\bar{x}^{-2}(11z + 25z^2) + \bar{x}^{-3} \left(-\frac{29}{18}z^2 \right) \\ & + \left(\frac{101}{2} - 17xz - 81x + \frac{45}{2}x^2 - 31z - \frac{3}{2}z^2 \right) \log \bar{x} \\ & + (32 - 6xz - 30x + 4x^2 + 29z + z^2) \log \bar{x} \log \frac{z}{\bar{x}} \\ & + \log z \left(21xz + 32x - 16x^2 - 9z - \frac{5}{3}z^2 \right) \\ & - 20xz + \frac{101}{2}x - \frac{97}{4}x^2 + \frac{127}{4}z + \frac{283}{36}z^2. \quad (49) \end{aligned}$$

Clearly, the gluon mass does not enter the integrated distribution, defined as

$$F(x, z) = \frac{1}{12\Gamma_0} \int_0^z dz' \frac{d\Gamma}{dx dz'} = f_0(x) + \frac{2\alpha_s}{3\pi} F_1(x, z), \quad (50)$$

for which we obtain,

$$F_1(x, z) = (c_1 + c_2 z + c_3 z^2 + c_4 z^3) \log z + c_5 \log^2 z + c_6 z + c_7 z^2 + c_8 z^3 + c_9. \quad (51)$$

The coefficients c_1 to c_9 are as follows,

$$\begin{aligned} c_1 &= \frac{1}{36}[(120 - 288x + 216x^2 - 48x^3) \log \bar{x} + 120x - 291x^2 + 130x^3], \\ c_2 &= (-30x + 4x^2 + 32) \log \bar{x} + 32x - 16x^2, \\ c_3 &= 5\bar{x}^{-1} - \frac{1}{2}\bar{x}^{-2} + \frac{1}{2}(-6x + 29) \log \bar{x} + \frac{21x-9}{2}, \\ c_4 &= \bar{x}^{-1} - \frac{2}{3}\bar{x}^{-2} + \frac{2}{9}\bar{x}^{-3} + \frac{1}{3} \log \bar{x} - \frac{5}{9}, \\ c_5 &= -\frac{1}{2}x^2 + \frac{1}{3}x^3, \\ c_6 &= \frac{1}{2}(-102x + 37x^2 + 37) \log \bar{x} \end{aligned}$$

$$\begin{aligned}
& + (30x - 4x^2 - 32) \log^2 \bar{x} + \frac{74x - 33x^2}{4}, \\
c_7 = & \frac{1}{4}(-28x - 91) \log \bar{x} + \frac{1}{2}(6x - 29) \log^2 \bar{x} - 10\bar{x}^{-1} \log \bar{x} - \frac{79}{4}\bar{x}^{-1} \\
& + \bar{x}^{-2} \left(\log \bar{x} + \frac{13}{8} \right) - \frac{61}{4}x + \frac{145}{8}, \\
c_8 = & \frac{1}{36} \{ -22 \log \bar{x} - 12 \log^2 \bar{x} - 72\bar{x}^{-1} \log \bar{x} - 162\bar{x}^{-1} \\
& + 48\bar{x}^{-2} \log \bar{x} + 83\bar{x}^{-2} - 16\bar{x}^{-3} \log \bar{x} - 22\bar{x}^{-3} + 101 \}, \\
c_9 = & \frac{1}{36} \{ 6(83 - 196x + 145x^2 - 32x^3) \log \bar{x} + 12(-10 + 24x \\
& - 18x^2 + 4x^3) \log^2 \bar{x} - 12\pi^2 x^2 + 8\pi^2 x^3 + 498x \\
& - 852x^2 + 310x^3 - 36x^2 \text{Li}_2(x) + 24x^3 \text{Li}_2(x) \}. \tag{52}
\end{aligned}$$

One way of making comparison between the parton model predictions and the resonance ridden experimental data is to consider moments of distribution. We define those as

$$M_n(x) = \frac{1}{12F_0} \int_0^{1-x} dz z^n \frac{d\Gamma}{dx dz}. \tag{53}$$

While the zeroth moment corresponds to the electron energy distribution itself, the singular part of the double distribution leaves no trace on the higher moments. In fact, they are then vanishing in the Born approximation. The first five moments are expressed in terms of the following functions,

$$M_n(x) = \frac{2\alpha_s}{3\pi} m_n(x), \quad n \geq 1, \tag{54}$$

which are given by the formulae beneath (see also Fig. 1):

$$m_1 = \left(-\frac{35}{144} + \frac{5}{8}x^2 - \frac{4}{9}x^3 + \frac{x^4}{16} \right) \log \bar{x} - \frac{35}{144}x + \frac{19}{72}x^2 - \frac{x^3}{48}, \tag{55}$$

$$\begin{aligned}
m_2 = & \left(-\frac{449}{3600} + \frac{7x}{24} - \frac{61}{360}x^2 - \frac{2}{45}x^3 + \frac{13}{240}x^4 - \frac{13}{1800}x^5 \right) \log \bar{x} \\
& - \frac{449}{3600}x + \frac{533}{1800}x^2 - \frac{9}{40}x^3 + \frac{109}{1800}x^4 - \frac{x^5}{144}, \tag{56}
\end{aligned}$$

$$\begin{aligned}
m_3 = & \left(-\frac{103}{1800} + \frac{119}{600}x - \frac{29}{120x^2} + \frac{19}{180}x^3 + \frac{x^4}{120} - \frac{3}{200}x^5 + \frac{x^6}{600} \right) \log \bar{x} \\
& - \frac{103}{1800}x + \frac{697}{3600}x^2 - \frac{653}{2700}x^3 + \frac{49}{360}x^4 - \frac{7}{200}x^5 + \frac{47}{10800}x^6, \tag{57}
\end{aligned}$$

$$\begin{aligned}
m_4 = & \log \bar{x} \left(-\frac{1313}{44100} + \frac{2}{15}x - \frac{81}{350}x^2 + \frac{59}{315}x^3 - \frac{5}{84}x^4 - \frac{x^5}{175} + \frac{2}{315}x^6 - \frac{2}{3675}x^7 \right) \\
& - \frac{1313}{44100}x + \frac{5729}{44100}x^2 - \frac{11}{49}x^3 + \frac{857}{4410}x^4 - \frac{781}{8820}x^5 + \frac{311}{14700}x^6 - \frac{19}{7350}x^7, \tag{58}
\end{aligned}$$

$$\begin{aligned}
m_5 = & \log \bar{x} \left(-\frac{485}{28224} + \frac{23}{245}x - \frac{211}{1008}x^2 + \frac{151}{630}x^3 - \frac{95}{672}x^4 + \frac{2}{63}x^5 + \frac{29}{5040}x^6 \right. \\
& \left. - \frac{x^7}{294} + \frac{31}{141120}x^8 \right) - \frac{485}{28224}x + \frac{1321}{14400}x^2 - \frac{15879}{78400}x^3 + \frac{33599}{141120}x^4 \\
& - \frac{4523}{28224}x^5 + \frac{979}{15680}x^6 - \frac{9809}{705600}x^7 + \frac{163}{100800}x^8.
\end{aligned} \tag{59}$$

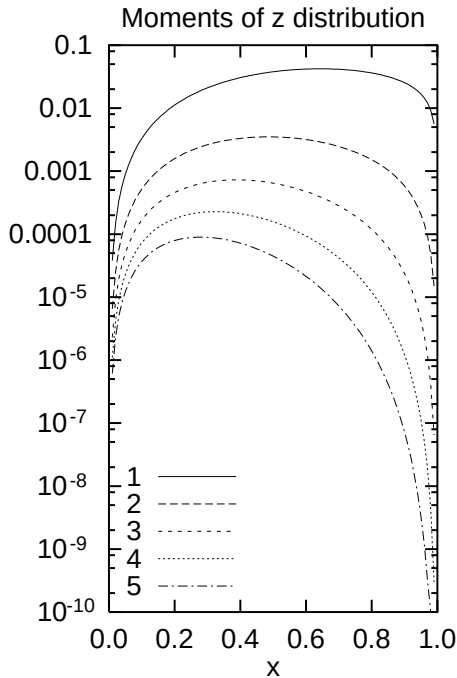


Fig. 1. Moments of hadron mass distribution m_n as a function of the scaled electron energy x . The first five moments are shown as indicated by the legend.

5. Summary

One-loop QCD correction has been found to the electron energy and hadronic system mass double distribution in semileptonic B decays. The result has not been published before. It agrees with the known single distributions in both variables. Although infrared singular, it is of use for experimental analysis with appropriate cuts imposed.

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