PROPAGATION OF ENERGETIC PARTONS IN MATTER*

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We review the properties of energetic parton propagation in hot or cold QCD matter, as obtained in recent work. The medium induced energy loss is studied. It has the remarkable feature to grow as L^2, the length of the traversed matter squared. Numerical estimates suggest that it may be significantly enhanced in hot matter compared to cold matter, thus pointing towards a possible signal for quark–gluon plasma formation. The more realistic case of an expanding (longitudinally) QCD plasma is studied. The resulting radiative energy loss can be as large as 6 times the corresponding one in a static plasma at the reference temperature $T(L)$ which is reached after the parton propagates on a distance L. Finally, the spectrum of soft radiated gluons is studied, leading to the calculation of the medium dependent energy lost by a jet with opening angle $\theta_{\rm cone}$. It is shown that the fraction of this energy loss to the integrated one exhibits a universal behavior in terms of $\theta^2_{\rm cone}L^3\widehat{q}$ where $\widehat{q}$ is the transport coefficient characterizing the medium. Phenomenological implications for the difference between hot and cold matter are discussed.

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1. Introduction

Over the past few years, a lot of work has been devoted to the propagation of high energy partons (jets) through hot and cold QCD matter. The jet $p_\perp$-broadening and the gluon radiation induced by multiple scattering, together with the resulting radiative energy loss of the jet have been studied [1–14]. These studies are extensions to QCD of the analogous QED problem considered long ago by Landau, Pomeranchuk and Migdal [16]. A number of interesting results have been found: When a high energy parton

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transverses a length L of hot or cold matter, the induced radiative energy loss is proportional to L^2. The energy loss of a high energy jet in a hot QCD plasma appears to be much larger than in cold nuclear matter even at moderate temperatures of the plasma, $T \sim 200$ MeV.

The order of magnitude of the effect in hot matter compared to the case of cold nuclear matter may be expected to be large enough to lead to an observable and remarkable signal of the production of the quark gluon plasma (QGP). Indeed, it has been proposed to measure the magnitude of "jet quenching" in the transverse momentum spectrum of hard jets produced in heavy-ion-collisions [17, 18]. Jet quenching is the manifestation of energy loss as seen in the suppression and change of shape of the jet spectrum compared with hadronic data. The angular distribution of radiated gluons which are the main source of energy loss has been studied [10–13, 15]. This allows for quantitative predictions of the energy lost outside the cone defining the jet. As a result, the energy loss is quite collimated in the case of a hot QCD medium .

In Section 2, we give the basic elements of the equations and describe the coherent pattern of the gluon radiative spectrum induced by multiple scattering.

In Section 3, we derive the induced energy loss and the jet transverse momentum broadening in terms of phenomenologically significant quantities. Orders of magnitude are given.

In section 4, the more realistic case of an expanding QCD plasma is considered and the corresponding energy loss calculated.

In Section 5, we indicate the elements of derivation of the angular dependence of the radiative gluon spectrum and the phenomenological implications for measuring the energy loss in hot QCD matter and cold matter.

2. The coherent pattern of the gluon radiative spectrum induced by multiple scattering

We consider a high energy parton say a quark of energy E traversing hot or cold matter under a length L. The main assumption [1] is that the scattering centers are static and uncorrelated (in the spirit of the Glauber picture). We focus on purely radiative processes since the collisional energy loss vanishes in the case of static centers.

We define a normalized quark–"particle" cross-section

$$V(Q^2) = \frac{1}{\pi\sigma}\,\frac{d\sigma}{dQ^2}\,, \tag{1}$$

where Q is the 2-dimensional transverse momentum transfer scaled by an appropriate scale:

$$\vec{Q} = \frac{\vec{q}}{\mu}$$

and

$$\sigma = \int \frac{d\sigma}{d^2Q} \, d^2Q \,. \tag{2}$$

In the case of a hot QCD plasma, the "particle" is a quark or gluon and it is a nucleon in the case of cold matter. To guarantee that $d\sigma/dQ^2$ depends only on $\vec{Q}$, we assume that the energy transfer from the quark to a particle in the medium be small compared to the incident energy. The scale μ characteristic of the medium is conveniently taken as the Debye screening mass in the hot case and as a typical momentum transfer in a quark–nucleon collision. The condition that the independent scattering picture be valid may be expressed as:

$$\mu^{-1} \ll \lambda \,, \tag{3}$$

where λ is the parton mean free path in the medium $\lambda = 1/\rho\sigma$. We assume that a large number of scatterings takes place, that is

$$L \gg \lambda \,. \tag{4}$$

The general argument which allows us to understand the coherent pattern of the radiation induced by multiple scattering in the medium is the following. Let us define the formation time of the emitted gluon:

$$t_{\rm f} = \frac{2\omega}{k_\perp^2} \,, \tag{5}$$

where ω and $k_\perp$ are respectively the energy and the transverse momentum of the gluon. We compare $t_{\rm f}$ with the gluon mean free path λ (leaving aside for the moment color factors). When $t_{\rm f} \gg \lambda$, radiation takes place in a coherent fashion with many scattering centers acting as a single one. The typical $k_\perp$ being of order μ, we may specify the gluon frequency range for which this coherent regime takes place:

$$\lambda\mu^2 \ll \omega < E \,. \tag{6}$$

Radiation is suppressed with respect to summing incoherent emission on each site. It is easy to understand that in this regime, the energy loss on a distance L is proportional to L^2. Indeed, ΔE is roughly determined by the maximum energy a radiated gluon can have still maintaining a coherence length $\leq L$. The formation time of the radiated gluon is $2\omega/k^2_{\perp\rm max}$

with $k_{\perp\max}$ the maximum transverse momentum that the gluon gets in the medium as it is being produced. Taking $k^2_{\perp\max} = \mu^2 \frac{L}{\lambda}$ where $\frac{L}{\lambda}$ is the number of scattering centers and setting $t_{\rm f} \le L$ one finds $\omega \lesssim {}^1\!/_2 \frac{\mu^2}{\lambda} L^2$. The proportionality of ΔE to L^2 as understood here in a heuristic way will be confirmed by the careful derivation outlined in the following pages. It is worth noticing that this form of ΔE is valid as long as the length $L < L_{\rm cr} \equiv \sqrt{\lambda E/\mu^2}$. This corresponds to imposing $\omega < E$.

2.1. Jet $p_\perp$-broadening

On the way to deriving the gluon radiative spectrum, let us start with the classical diffusion equation satisfied by the transverse momentum distribution of a high energy parton which encounters multiple scattering in a medium. Suppose the parton is produced in a hard collision with an initial transverse momentum distribution $f_0(U^2)$; U is the dimensionless transverse momentum

$$\vec{U} \equiv \frac{\vec{p}}{\mu} \qquad \text{and} \qquad \int d^2U f_0(U^2) = 1\,.$$

Neglecting the transverse momentum given to the parton by induced gluon emission, one can derive a classical kinetic equation for the transverse momentum distribution $f(U^2, z)$ after a distance z in the medium [5].

In terms of the variable $t = \frac{z}{\lambda_R}$ with λ_R the mean free path for a parton of color representation R, one finds the following gain–loss equation

$$\frac{\partial f(U^2,t)}{\partial t} = + \int f(U'^2,t)\; V((\vec{U}' - \vec{U})^2) d^2U' - \int f(U^2,t)\; V((\vec{U} - \vec{U}')^2) d^2U' \tag{7}$$

with
$$f(U^2,0) = f_0(U^2)\,. \tag{8}$$

Defining the Bessel transform $\widetilde{f}(B^2,t)$ as

$$\widetilde{f}(B^2,t) = \int d^2U\ \mathrm{e}^{-i\vec{B}\cdot\vec{U}}\ f(U^2,t) \tag{9}$$

and
$$\widetilde{V}(B^2) = \int d^2Q\ \mathrm{e}^{-i\vec{B}\cdot\vec{Q}}\ V(Q^2) \tag{10}$$

we find
$$\frac{\partial \widetilde{f}(B^2,t)}{\partial t} = -\frac{1}{4} B^2 \widetilde{v}(B^2) \widetilde{f}(B^2,t) \tag{11}$$

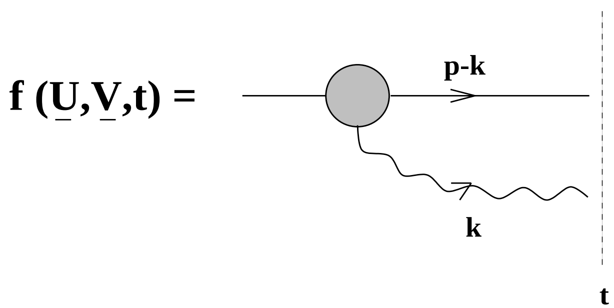
with

$$\widetilde{v}(B^2) \equiv \frac{4}{B^2}(1 - \widetilde{V}(B^2)) \,. \tag{12}$$

As discussed in [5], Eq. (11) is valid for hot and cold QCD media.

2.2. Radiative gluon spectrum

Let us now turn to the gluon spectrum. We shall from now on specialize to a quark jet. The general case is given in [6]. After the gluon is emitted from the high energy quark, the quark–gluon system moves through the medium and carries out multiple scatterings. As derived in [3, 4, 6], the spectrum for the radiated gluon is calculated in terms of the interference term between the quark-gluon amplitude at time t and the complex conjugate Born amplitude. For simplicity, we restrict here to the case where the quark enters the medium from outside. (An additional term is needed [6] in the case when the quark is produced via a hard scattering at $t = 0$ in the medium). We denote by $f(\vec{U}, \vec{V}, t)$ the quark gluon amplitude at time t. $\vec{U}$ is the scaled gluon momentum $\vec{U} \equiv \frac{\vec{k}}{\mu}$ and $\vec{V} - \vec{U}$ the scaled quark momentum as illustrated below.



To account for the gluon polarization, f is a 2-dimensional vector which will be implied hereafter. The dependence on $\vec{U}$ and $\vec{V}$ is actually only in the combination $\vec{U} - x\vec{V}$ with $x = \frac{k}{p}$. $f(\vec{U}, \vec{V}, t)$ satisfies the initial condition $f(\vec{U}, \vec{V}, 0) = f_0(\vec{U}, \vec{V})$ where f_0 is the Born terms amplitude to be described shortly.

The induced gluon spectrum is written as:

$$\begin{aligned} \frac{\omega\ dI}{d\omega\ dz} &= \frac{\alpha_s\ C_F}{\pi^2 L} 2\mathrm{Re} \int d^2U \left\{ \int_0^L dt_2 \int_0^{t_2} dt_1 \right. \\ &\left. \times \left[\rho\sigma \frac{N_C}{2C_F} f(\vec{U} - x\vec{V}, t_2 - t_1) \right] \left[\rho\sigma \frac{N_C}{2C_F} f_0^*(\vec{U} - x\vec{V}) \right] \right\}_{\omega}^{\omega=\infty} \end{aligned} \tag{13}$$

The various terms in (13) have simple interpretations. The $\frac{\alpha_S C_F}{\pi^2}$ is the coupling of a gluon to a quark. The $1/L$ comes because we calculate the spectrum per unit length of the medium. The factor $\frac{N_c}{2C_F} f(\vec{U}-x\vec{V},t_2-t_1)\rho\sigma dt_1$ is the number of scatterers in the medium, $\rho\sigma dt_1$, times the amplitude with gluon emission at t_1, evolved in time up to t_2, the time of emission in the complex conjugate amplitude. The factor $\frac{N_c}{2C_F} f_0^*(\vec{U}-x\vec{V})\rho\sigma dt_2$ gives the number of scatterers times gluon emission in the complex conjugate Born amplitude. The subtraction of the value of the integrals at $\omega=\infty$ eliminates the medium independent contribution. Eq. (13) may be simplified using $t \equiv \left(\frac{2C_F}{N_c}\lambda\right)\tau$. Defining $\tau_0 = \frac{N_c}{2C_F}\frac{L}{\lambda}$, we obtain

$$\omega\frac{dI}{d\omega\ dz} = \frac{\alpha_s\ N_c}{\pi^2\lambda}\mathrm{Re}\int d^2Q \times\left\{\int_0^{\tau_0} d\tau\left(1-\frac{\tau}{\tau_0}\right) f(\vec{U}-x\vec{V},\tau)\cdot f_0^*(\vec{U}-x\vec{V})\right\}_{\omega}^{\omega=\infty}. \quad (14)$$

Due to the specific dependence of f and f_0 in $\vec{U}$ and $\vec{V}$, it is possible to express them in terms of a single impact parameter as:

$$f(\vec{U}-x\vec{V},\tau) = \int \frac{d^2B}{(2\pi)^2}\ \mathrm{e}^{i\vec{B}\cdot(\vec{U}-x\vec{V})}\widetilde{f}(\vec{B},\tau)\,,$$
$$f_0(\vec{U}-x\vec{V}) = \int \frac{d^2B}{(2\pi)^2}\ \mathrm{e}^{i\vec{B}\cdot(\vec{U}-x\vec{V})}\widetilde{f}_0(\vec{B})\,, \quad (15)$$

allowing us to obtain the following expression for the spectrum in impact parameter space:

$$\frac{\omega dI}{d\omega\ dz} = \frac{\alpha_s N_c}{2\pi^3\lambda}\mathrm{Re}\int\frac{d^2B}{2\pi}\left\{\int_0^{\tau_0} d\tau\left(1-\frac{\tau}{\tau_0}\right)\cdot\widetilde{f}(\vec{B},\tau)\cdot\widetilde{f}_0^*(\vec{B})\right\}_{\omega}^{\omega=\infty}. \quad (16)$$

2.2.1. The Born amplitude $f_0(\vec{U}-x\vec{V})$

The diagrams describing the Born amplitude are shown in Fig. 1. Graphs a–c correspond to inelastic reactions with the medium while graphs d–g correspond to forward scattering in the medium. For terms a–c there are corresponding inelastic reactions in the complex conjugate amplitude. In the approximation that the forward elastic amplitude for quark scattering off particles in the medium is purely imaginary, the elastic and inelastic terms are proportional to $V(Q^2)$. The color factors and the expression of each graph contribution are derived in [6].

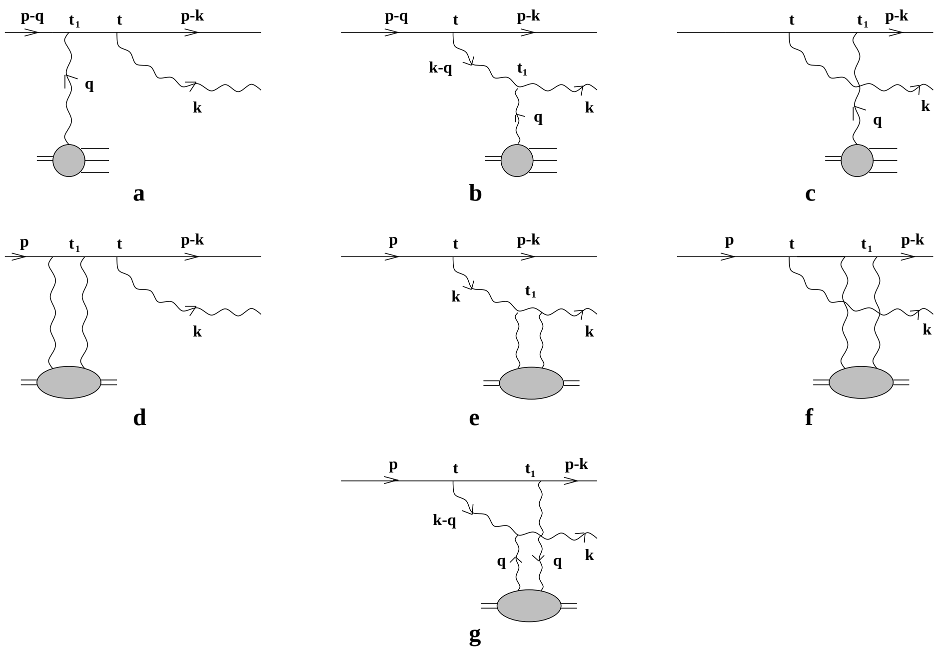


Fig. 1.

2.2.2. The time evolved amplitude

The quark–gluon amplitude $f(\vec{U},\vec{V},t)$ obeys an integral evolution equation derived in Refs. [3, 4, 6]. In impact parameter space and in the small x-limit, this equation takes the simple form

$$\frac{\partial}{\partial \tau}\widetilde{f}(\vec{B},\tau) = i\widetilde{\kappa}\ \nabla_B^2\ \widetilde{f}(\vec{B},\tau) - 2(1-\widetilde{V}(B))\widetilde{f}(\vec{B},\tau) \tag{17}$$

with $\widetilde{\kappa} = \frac{2C_F}{N_C}\left(\frac{\lambda\mu^2}{2\omega}\right)$ and $\widetilde{f}(\vec{B},0) = \widetilde{f}_0(\vec{B})$. This equation is a Schrödinger-type evolution equation for the propagation of the quark–gluon system in a QCD medium. Comparing Eq. (17) to Eq. (11) is instructive. The term proportional to $\widetilde{\kappa}$ in (17) is clearly of quantum origin associated to the phase of the amplitude whereas Eq. (11) is a classical diffusion equation. The contributions entering the expression of the spectrum (14) are depicted in Fig. 2.

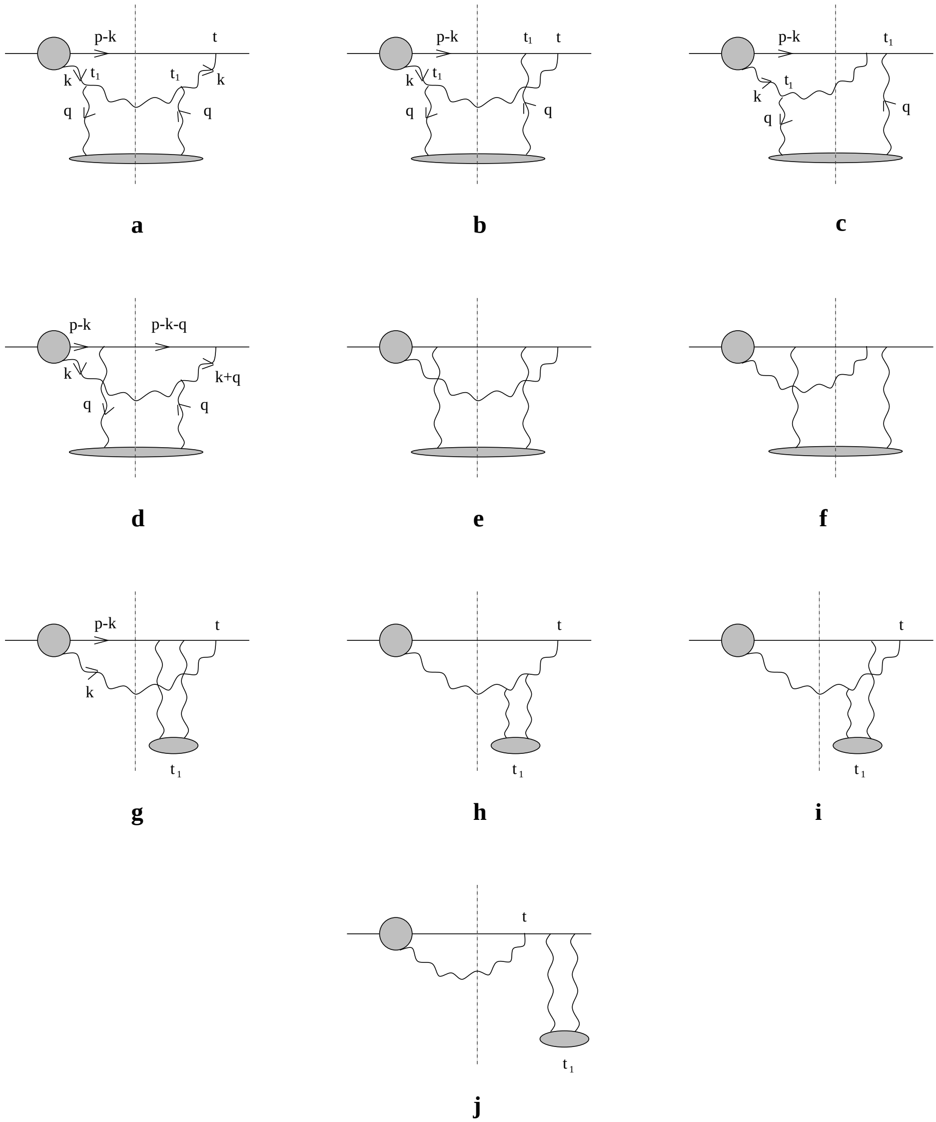


Fig. 2.

3. The induced energy loss

3.1. Solving the equations

So long as $\widetilde{v}(B^2) \equiv 4(1 - \widetilde{V}(B)/B^2$ can be treated as a constant, the solution of Eq. (17) proceeds in analogy with that of the 2-dimensional harmonic oscillator with imaginary frequency. We expect that the behavior of $\widetilde{v}(B^2)$ is close in general to the Coulomb case i.e. $\approx \ell n 1/B^2$ at small B^2. The solution of (17) is worked out in [4] and is thus valid to logarithmic accuracy. The energy loss per unit length

$$-\frac{dE}{dz} = \int_0^\infty \frac{\omega\ dI}{d\omega\ dz} \tag{18}$$

can be evaluated in the small x approximation leading to

$$-\frac{dE}{dz} = \frac{\alpha_s\ N_\mathrm{c}}{12}\ \frac{\mu^2 L}{\lambda}\widetilde{v}(\tau_0^{-1})\,. \tag{19}$$

In the case where the jet is produced in matter, one finds

$$-\frac{dE}{dz} = \frac{\alpha_s\ N_\mathrm{c}}{4}\ \frac{\mu^2 L}{\lambda}\widetilde{v}(\tau_0^{-1})\,. \tag{20}$$

As for jet $p_\perp$-broadening, it is possible to define a characteristic width of the distributions $f(U^2, t)$ which is found to be [5]:

$$p_{\perp W}^2 = \frac{\mu^2}{\lambda} L\ \widetilde{v}(\tau_0^{-1})\,. \tag{21}$$

The linear growth with L is expected and has been used to discuss $p_\perp$-broadening of high energy partons in nuclei. The coefficient $\widehat{q} = \frac{\mu^2}{\lambda}\widetilde{v}(\tau_0^{-1})$ plays the role of a transport coefficient as encountered in classical diffusion equations. Notice the remarkable relation between energy loss and jet $p_\perp$-broadening:

$$-\frac{dE}{dz} = \frac{\alpha_s\ N_c}{12} p_{\perp W}^2 \tag{22}$$

in the case of a quark entering the medium from outside.

3.2. Estimates

The parameter controlling the magnitude of the energy loss is $\widehat{q}$. Estimates can be provided for its value, allowing us to give orders of magnitude for the energy loss.

3.2.1. Hot matter

Taking $T = 250$ MeV, $\frac{\mu^2}{\lambda} \sim 1$ GeV/fm^2 taken from perturbative estimates at finite T, a typical value for $\widetilde{v} \approx 2.5$, we find $\widehat{q} \simeq 0.1$ GeV^3 [5, 15]. With $\alpha_s = {}^1\!/_3$, this leads for the total induced energy loss to

$$-\Delta E \approx 30\ \mathrm{GeV}\ \left(\frac{L}{10\ \mathrm{fm}}\right)^2 . \tag{23}$$

3.2.2. Cold nuclear matter

In Ref. [5], we show that it is possible to relate $\widehat{q}$ to the gluon structure function G evaluated at an average scale $\frac{\mu^2}{B^2} \sim \mu^2 \frac{\lambda}{L}$. Taking the nuclear density $\rho \sim 0.15\ \text{fm}^{-3}$, $\alpha_s = 1/2$, $xG \sim 1$, it is found that

$$-\Delta E \approx 2\ \text{GeV}\ \left(\frac{L}{10\ \text{fm}}\right)^2 . \tag{24}$$

One should not take the exact values too seriously. However they do suggest that hot matter may be effective in stimulating radiative energy loss of high energy partons.

4. How to take into account the expansion of the hot QCD plasma

In the above the properties of the medium and the interactions of the parton/jet, characterized by the basic parameter $\widehat{q}$, have been supposed constant in time. It is interesting to consider the case of a high energy quark entering an expanding hot medium. We imagine the medium to be a quark gluon plasma produced in a relativistic central A-A collision which occurs at (proper) time $t = 0$. At time t_0, the quark enters the homogeneous plasma at high temperature T_0 which expands longitudinally with respect to the collision axis. t_0 may be considered as being the thermalization time. It turns out that for most of the results the limit $t_0 \to 0$ can be taken. The quark is supposed to propagate in the transverse direction at $y = 0$ so that the distance on which it propagates is equal to the proper time t. On its way it hits layers of matter which are cooled down due to the longitudinal expansion of the hadrons. We assume that the plasma lived long enough so that the quark is able to propagate on a distance L within the gluon plasma phase of matter. The properties of the expanding plasma are described by the hydrodynamical model proposed by Bjorken [19] with the scaling law

$$T^3\ t^\alpha = \text{const}\,. \tag{25}$$

The parameters μ and λ depend on T and thus on t. The power α which is approximated by a constant may take values between 0 and 1. The value $\alpha = 1$ corresponds to the ideal fluid. The natural generalization of Eq. (16) for the induced spectrum is obtained by properly specifying the time dependence. The gluon emission amplitude at t_1, evolved in time to $t_2 > t_1$ is now $f(\vec{b}; t_2, t_1)$ ($\vec{b}$ is the unscaled 2-dimensional impact parameter) and $f_0^*(\vec{b}; t_2)$ denotes the complex conjugate Born amplitude for emission at t_2. We find (in the large N_c limit) [14]

$$\frac{\omega\ dI}{d\omega\ dz} = \frac{\alpha_s\ N_c}{2\pi^2}\ \frac{1}{L}2\mathrm{Re}\left\{ \int_{t_0}^{t_0+L} \frac{dt_2}{\lambda(t_2)} \int_{t_0}^{t_2} \frac{dt_1}{\lambda(t_1)} \right. \\ \left. \times \int \frac{d^2b}{(2\pi)^2}\ f(\vec{b};t_2,t_1)\cdot f_0^*(\vec{b};t_2) \right\}\Bigg|_{\omega}^{\omega=\infty} \tag{26}$$

the initial condition is

$$f(\vec{b};t_1,t_1) = f_0(\vec{b};t_1)\,. \tag{27}$$

The amplitude $f(\vec{b};t_2,t_1)$ satisfies a Schrödinger-like evolution equation. At fixed t_1 in the logarithmic approximation, taking $\widetilde{v}(b^2) \equiv \widetilde{v}$, it is a 2-dimensional harmonic oscillator equation which takes the following form

$$i\frac{\partial}{\partial t_2} f(\vec{b};t_2,t_1) = \left[\frac{1}{2\omega}\vec{\nabla}_b^2 - \frac{1}{2}\omega\ \omega_0^2(t_2)\vec{b}^2\right] f(\vec{b};t_2,t_1)\,. \tag{28}$$

With $\omega_0^2(t_2) = i\widehat{q}(t)/\omega$ where $\widehat{q}(t)$ is the t dependent transport coefficient

$$\widehat{q}(t) = \frac{\mu^2(t)}{\lambda(t)}\widetilde{v} = \widehat{q}(t_0)\left(\frac{T}{T_0}\right)^3 \equiv \widehat{q}(t_0)\left(\frac{t_0}{t}\right)^\alpha\,. \tag{29}$$

The derivation given in Ref. [14] leads to an analytic expression for the induced energy loss

$$-\frac{dE}{dz} = \frac{6}{(2-\alpha)(3-2\alpha)}\left(\frac{-dE}{dz}\right)_{\mathrm{static}}, \tag{30}$$

where $-\left(\frac{dE}{dz}\right)_{\mathrm{static}}$ is the energy lost by a quark traversing a medium at fixed temperature $T(L)$. It is remarkable that the result is independent of T_0. For the ideal gas case when $\alpha = 1$, the resulting enhancement factor is large equal to 6. Notice that in QCD $\alpha \sim 1 - O(\alpha_s^2)$ [20].

5. Angular dependence of the radiative gluon spectrum

As a consequence of the above, the energy loss is a huge effect in hot matter and may constitute a remarkable signal of production of the QGP.

The natural observable to measure is the transverse momentum spectrum of hard jets produced in heavy ion collisions. The consequence of a large energy loss is the attenuation of the spectrum usually denoted as jet quenching. It is necessary to study the angular distribution of radiated gluons in order to give quantitative predictions for the energy lost by a jet traversing hot matter. Only the gluons which are radiated outside the cone defining the jet contribute to the energy loss.

In [15] a complete calculation of the angular distribution is given for the realistic case of a hard jet produced in the medium.

5.1. Gluon transverse momentum spectrum

In contrast to the $\vec{U}$-integrated spectrum, given in Eq. (16), it is necessary to take into account the possibility that the emitted gluon may rescatter in the medium after time t_2. For example, the contribution shown in Fig. 2a should be decorated with final state interactions as illustrated in Fig. 3.

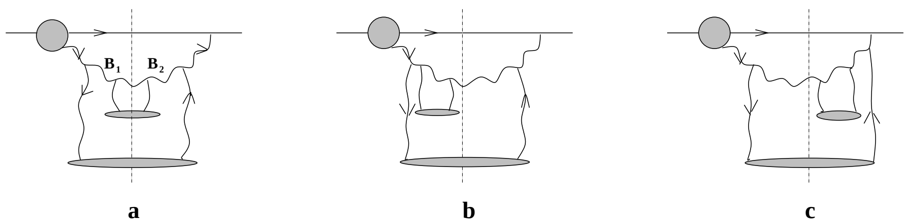


Fig. 3. Final state interactions contributing to Fig. 2a: (a) real, (b) and (c) virtual interactions.

The analysis of $p_\perp$-broadening given in [5] leads to define a final state interaction amplitude in impact parameter space as [15]:

$$\mathcal{F}_{fsi}(\vec{B}_1 - \vec{B}_2, \tau) = \exp\left[-\frac{1}{2}(\vec{B}_1 - \vec{B}_2)^2 \tau \widetilde{v}\right], \tag{31}$$

where $\vec{B}_1$ and $\vec{B}_2$ are respectively the impact parameters in the amplitude and the complex conjugate amplitude. As a consequence the spectrum is worked out to be, in the small x limit:

$$\begin{aligned} \omega \frac{dI}{d\omega\ dz\ d^2U} = & \frac{\alpha_s\ C_F}{\pi^2 L} 2\mathrm{Re}\left\{ \int_0^{\tau_0} d\tau_2 \int \frac{d^2 B_1}{(2\pi)^2} \frac{d^2 B_2}{(2\pi)^2}\ \mathrm{e}^{i(\vec{B}_1 - \vec{B}_2)\cdot\vec{U}} \right. \\ & \times \left[\int_0^{\tau_2} d\tau_1\ \widetilde{f}(\vec{B}_1, \tau_2 - \tau_1)\right] \cdot \frac{4\pi_i \vec{B}_2}{B_2^2} \left[\widetilde{V}(\vec{B}_1 - \vec{B}_2) - \widetilde{V}(\vec{B}_1)\right] \\ & \left. \times \mathrm{e}^{-\widetilde{v}/2(\vec{B}_1 - \vec{B}_2)^2(\tau_0 - \tau_2)} \right\}_{\omega}^{\omega=\infty}. \end{aligned} \tag{32}$$

An easy check of this formula, is that it gives back Eq. (16) when integrating on $\vec{U}$ (in the small x-limit, the Born amplitude $f_0(\vec{B}) = -\frac{4\pi_i}{B^2}\vec{B}(1 - \widetilde{V}(\vec{B}))$.

5.2. Angular induced radiative energy loss

Let us define the integrated loss outside an angular cone of opening angle $\theta_{\rm cone}$ given by

$$\Delta E(\theta_{\rm cone}) = L \int_0^\infty d\omega \int_{\theta_{\rm cone}}^{\pi} \frac{\omega dI}{d\omega\ dz\ d\theta}\ d\theta\,. \tag{33}$$

For $\theta_{\rm cone} = 0$, the total loss ΔE (Eq. (20) is recovered. One can find in Ref. [15] the details of the calculation of $\Delta E(\theta_{\rm cone})$. The integral on θ may be performed analytically in the approximation of small angles but the ω, τ_2 and τ_1 (Eq. (32)) integrals are done numerically.

Defining $R(\theta_{\rm cone})$ as the ratio $\frac{\Delta E(\theta_{\rm cone})}{\Delta E}$, one can show the remarkable feature that R depends on a single dimensionless variable

$$R = R(c(L)\theta_{\rm cone})\,, \tag{34}$$

where

$$c^2(L) = \frac{N_C}{2C_F}\widehat{q}\left(\frac{L}{2}\right)^3\,. \tag{35}$$

The medium and the size dependence of R is universally contained in $c(L)$. This "scaling" behavior may be understood using the following heuristic arguments: the radiated energy loss is dominated by gluons having $\omega \simeq \frac{\mu^2}{\lambda}L^2$. The angle that the emitted gluon makes with the quark is $\theta = k/\omega$ with $k^2 \sim \mu^2\frac{L}{\lambda}$ so that the typical gluon angle is such that

$$\theta^2 \sim \frac{\lambda}{\mu^2}\ \frac{1}{L^3} \sim \frac{1}{\widehat{q}L^3}\,.$$

5.3. Estimates

We may use the estimates of section 3 to give orders of magnitude for $c(L)$ in the case of a hot/cold medium:

$$c(L)_{\rm hot} \simeq 40\ (L/10\ {\rm fm})^{3/2}\,.$$

A much smaller value is found in the cold nuclear matter case:

$$c(L)_{\rm cold} \sim 10\ (L/10\ {\rm fm})^{3/2}\,.$$

In Fig. 4, we show the variation of R with $\theta_{\rm cone}$. As expected from the fact that R depends universally on $c(L)\theta_{\rm cone}$, the curve is more collimated

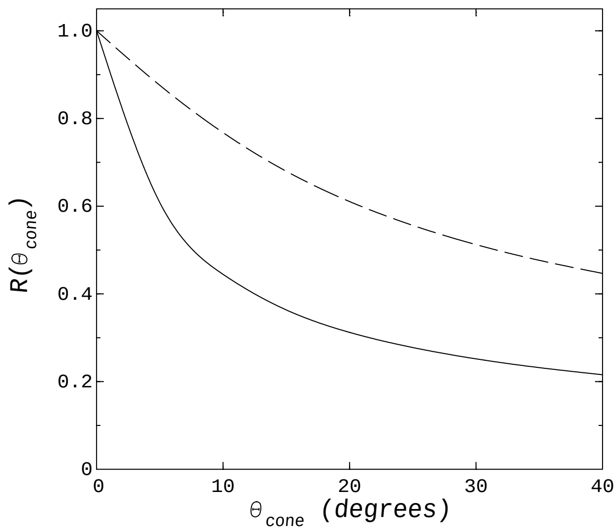


Fig. 4. Medium induced (normalized) energy loss distribution as a function of cone angle $\theta_{\rm cone}$ for hot ($T = 250$ MeV) (solid curve) and cold matter (dashed curve) at fixed length $L = 10 fm$.

in the hot medium than in the cold medium case. In the hot QCD case, taking $\theta_{\rm cone} = 10^\circ$ (40°), the fraction R is reduced to 40 % (20 %). When comparing to the cold matter case, $\Delta E(40^\circ)$ is still quite large, however: $\Delta E_{\rm hot}(40^\circ) \simeq 12$ GeV $\left(\frac{L}{10\ {\rm fm}}\right)^2$, compared to $\Delta E_{\rm cold}(40^\circ) \sim 3$ GeV $\left(\frac{L}{10\ {\rm fm}}\right)^2$.

As a conclusion, we expect that the medium induced angular energy loss $\Delta E(\theta_{\rm cone})$ for energetic jets can be large in hot QCD matter and although collimated still appreciably larger than in cold matter for current cone sizes: $\theta_{\rm cone} \sim 30^\circ - 40^\circ$.

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