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LOW x QCD AND PENETRATION OF ULTRAHIGH ENERGY NEUTRINOS THROUGH THE EARTH*

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In this talk we present a calculation for the cross sections for neutrino interaction with the nucleon. Parton distributions are calculated using the unified BFKL/DGLAP formalism which embodies also important subleading $\ln 1/x$ effects. It is shown that this calculation is consistent with the other based on GRV and CTEQ parton distributions up to 40% for the highest energies: $\sim 10^{12}$ GeV. We also calculate the attenuation of neutrinos on their way through the Earth to the detector. We solve the transport equation which also embodies the regeneration due to neutral current interactions besides attenuation. We present the results for different angles and fluxes originating from different sources like active galactic nuclei, gamma ray bursts and top–down models.

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1. Introduction

The penetration of ultrahigh energy neutrinos through the earth can be strongly affected by the increase of the neutrino–nucleon interactions with matter. It has been shown [1, 2] that the increase of the cross section at the energies of 10^{12} GeV causes that the Earth becomes opaque to these neutrinos. Clearly the attenuation rate strongly depends on the parton distributions inside the nucleon especially at low values of Bjorken x. This happens because at high energies (up to 10^{12} GeV) one probes very small values of x.

$$x \sim \frac{M_W^2}{2M_p\nu} \sim 10^{-8}\,, \tag{1}$$

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where ν is the energy of the exchanged vector-boson in the frame where nucleon is at rest and M_W and M_p are the W and proton masses correspondingly. The present region explored by HERA extends at most to 10^{-5} in Bjorken x at quite small values of $Q^2 \sim 1\,\mathrm{GeV}^2$. Therefore one can question whether it is satisfactory to use the standard parton distributions without possible novel $\ln 1/x$ effects at the region of very low values of Bjorken x. We propose to use the so called BFKL/DGLAP unified evolution equation system [3] where all the $\ln 1/x$ resummation has been taken into account and treated on equal footing with the $\ln Q^2$ effects. Additionally we have included very important subleading effects in $\ln 1/x$ by imposing the so called consistency constraint [4, 5] which limits the virtualities of the exchanged gluon momenta in the ladder to their transverse parts.

We then used the calculated parton distributions as the input to the transport equation [6, 7] for the neutrino flux. This equation contains the regular attenuation term as well as the regeneration term due to neutral current interaction. Neutrinos which interact via Z exchange at higher energy will pile up at lower energies and give rise to the flux.

We have solved this equation for different incident angles as well as for different models for the fluxes. We consider neutrino fluxes corresponding to active galactic nuclei, gamma ray bursts and the top–down models, together with atmospheric neutrino background. For details of the calculation see [8].

2. Parton distributions at small x

We consider here the usual deep inelastic lepton–hadron scattering process $lN \to l'X$ where l, l' is the incoming and the outgoing lepton correspondingly, N is the nucleon and X is the arbitrary hadronic final state. We use standard DIS variables, $Q^2 = -q^2$ the squared four-momentum of the exchanged gauge boson, and the scaling variable $x = Q^2/2p\cdot q$, where p is the four momentum of the incoming nucleon.

We shall start at first with the BFKL equation in the leading order [9] which is an evolution equation for the unintegrated gluon distribution function $f(x,k^2)$,

$$f(x,k^2) = f^{(0)}(x,k^2) + \overline{\alpha}_S k^2 \int_x^1 \frac{dz}{z} \int \frac{dk'^2}{k'^2} \times \left\{ \frac{f(x/z,k'^2) - f(x/z,k^2)}{|k'^2 - k^2|} + \frac{f(x/z,k^2)}{[4k'^4 + k^4]^{\frac{1}{2}}} \right\}, \qquad (2)$$

where $\overline{\alpha}_S = N_c \alpha_S/\pi$ and $k = k_T, k' = k'_T$ denote the transverse momenta of the gluons, see Fig. 1. The term in the integrand containing $f(x/z,k'^2)$

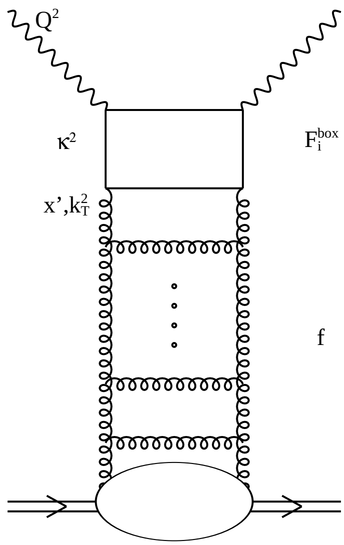

Fig. 1. Diagrammatic representation of the k_T-factorization formula. At lowest order in α_S the gauge boson–gluon fusion processes, $Vg \to q\bar{q}$, are given by the quark box shown (together with the crossed box). The variables κ and k denote the transverse momenta of the indicated virtual particles.

corresponds to real gluon emission, whereas the terms involving $f(x/z, k^2)$ represent the virtual contributions and lead to the Reggeization of the t-channel exchanged gluons. The inhomogeneous driving term $f^{(0)}$ is the input function and it will be specified later. If we define the Mellin transform to be:

$$\tilde{f}^{\omega} = \int_0^1 dx x^{\omega} f(x) \tag{3}$$

then the leading order BFKL equation performs the summation of the $(\frac{\alpha_s}{\omega})^n$ terms in the anomalous dimension expansion for the gluon. The solution of the LO BFKL equation for fixed α_S gives a QCD or hard pomeron with intercept $\alpha(0) = 1 + \lambda$ with $\lambda = \overline{\alpha}_S 4 \ln 2$. The $\ln(1/x)$ resummation has recently been carried out [10] at next-to-leading order (NLO). It is found to give a very large $O(\alpha_S^2)$ correction to λ

$$\lambda \simeq \overline{\alpha}_S \, 4 \ln 2(1 - 6\overline{\alpha}_S), \tag{4}$$

which implies that the NLO approximation is unreliable for realistic values of α_S.

Rather we must use a formalism which contains an estimate of an all-order resummation. Clearly it would be desirable to identify physical effects which could be resummed to all orders and which at the same time yield a NLO value of λ that is comparable to (4). As it happens the imposition of the consistency constraint [4, 5]

$$k'^2 < \frac{k^2}{z} \tag{5}$$

on the real gluon emission term gives just such an effect. The origin of the constraint is the requirement that the virtuality of the exchanged gluon is dominated by its transverse momentum $|k'^2| \simeq k_T'^2$. For clarity we have restored the subscript T in this equation.

If condition (5) is imposed on the BFKL equation it can be still solved analytically. The result is an all-order effect, which at NLO gives the large modification

$$\lambda \simeq \overline{\alpha}_S \, 4 \ln 2 (1 - 4.2 \overline{\alpha}_S) \tag{6}$$

of the LO value. However it is found that the all-order correction is a much milder modification, although still significant. A related result can be found in Ref. [11]. It can be shown that the imposition of this constraint results in the resummation of the major part of the subleading $\ln 1/x$ terms . We can therefore make the BFKL equation (2) for the gluon much more realistic by imposing the consistency condition (5), as well as by allowing the coupling α_S to run which is also a subleading effect.

Moreover we can extend its validity to cover the full range of x. To do this we note that the BFKL equation embodies the important double leading log part of DGLAP evolution which is driven just by the singular $1/z$ part of the splitting function P_{gg}. To obtain a reliable description throughout the full x range (and not just at small x) we must include the remaining terms in P_{gg}, together with the quark to gluon transitions. We also introduce in Eq. (2) the parameter k_0^2 ($k_0^2 \simeq 1\,\mathrm{GeV}^2$) which divides the non-perturbative ($k'^2 < k_0^2$) from the perturbative ($k'^2 > k_0^2$) region. Finally we note that the contribution from the infrared region $k'^2 < k_0^2$ in (2) may be expressed [3] in terms of the integrated gluon distribution at scale k_0^2, that is $g(x, k_0^2)$. All the above modifications of (2) are encapsulated in a unified BFKL/DGLAP equation of the form

$$f(x,k^2) = \tilde{f}^{(0)}(x,k^2) + \overline{\alpha}_S(k^2) k^2$$
$$\times \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk'^2}{k'^2} \left\{ \frac{f\left(\frac{x}{z}, k'^2\right) \Theta\left(\frac{k^2}{z} - k'^2\right) - f\left(\frac{x}{z}, k^2\right)}{|k'^2 - k^2|} + \frac{f\left(\frac{x}{z}, k^2\right)}{[4k'^4 + k^4]^{\frac{1}{2}}} \right\}$$

$$+\overline{\alpha}_S(k^2)\int\limits_x^1 \frac{dz}{z}\left(\frac{z}{6}P_{gg}(z)-1\right)\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}f\left(\frac{x}{z},k'^2\right)$$

$$+\frac{\alpha_S(k^2)}{2\pi}\int\limits_x^1 dzP_{gq}(z)\Sigma\left(\frac{x}{z},k^2\right)\,. \tag{7}$$

Now the driving term has the form

$$\tilde{f}^{(0)}(x,k^2)=\frac{\alpha_S(k^2)}{2\pi}\int\limits_x^1 dzP_{gg}(z)\frac{x}{z}g\left(\frac{x}{z},k_0^2\right)\,. \tag{8}$$

It is important to note that (7) only involves $f(x,k^2)$ in the perturbative region $k^2>k_0^2$. The input (8) is provided by the conventional gluon at scale k_0^2, just as in pure DGLAP evolution.

The last term in (7) is the contribution of the singlet quark distribution to the gluon, with

$$\Sigma=\sum_q x(q+\bar{q})=\sum_q(S_q+V_q)\,, \tag{9}$$

where S and V denote the sea and valence quark momentum distributions. The gluon, in turn, helps to drive the sea quark distribution through the $g\to q\bar{q}$ transition. Thus equation (7) has to be solved simultaneously with an equivalent equation for $\Sigma(x,k^2)$.

In order to calculate the quark distribution and the structure function we use the high energy or k_T factorization formula [12] of the following form,

$$S_q(x,Q^2)=\int\limits_x^1\frac{dz}{z}\int\frac{dk^2}{k^2}S^q_{\rm box}(z,k^2,Q^2)f\left(\frac{x}{z},k^2\right)\,, \tag{10}$$

where $S^{\rm box}$ describes the quark box (and crossed-box) contribution shown in Fig. 1 and can be interpreted as a partonic structure function. The exact formulae for the $S^q_{\rm box}$ read:

$$\begin{aligned}S_q^{\rm box}(z,k^2,Q^2) &= \frac{Q^2}{4\pi^2k^2}\int\limits_0^1 d\beta\int d^2\kappa'\alpha_S\left\{[\beta^2+(1-\beta)^2]\left(\frac{\boldsymbol{\kappa}}{D_{1q}}-\frac{\boldsymbol{\kappa}-\boldsymbol{k}}{D_{2q}}\right)^2\right.\\ &+ \left.[m_q^2+4Q^2\beta^2(1-\beta)^2]\left(\frac{1}{D_{1q}}-\frac{1}{D_{2q}}\right)^2\right\}\delta(z-z_0)\,, \end{aligned}\tag{11}$$

where $\boldsymbol{\kappa}' = \boldsymbol{\kappa} - (1-\beta)\boldsymbol{k}$ and

$$\begin{aligned} D_{1q} &= \kappa^2 + \beta(1-\beta)Q^2 + m_q^2\,, \\ D_{2q} &= (\boldsymbol{\kappa} - \boldsymbol{k})^2 + \beta(1-\beta)Q^2 + m_q^2\,, \\ z_0 &= \left[1 + \frac{\kappa'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k^2}{Q^2}\right]^{-1}. \end{aligned} \tag{12}$$

We have to specify the input function for the quark distribution when the momenta of the quarks and gluons are below the cutoff k_0^2. After this our full equation for the quark distribution is as follows:

$$\begin{aligned} \Sigma(x,k^2) &= S_{\mathrm{non}-p}(x) + \sum_q \int_x^a \frac{dz}{z} S_q^{\mathrm{box}}(z, k'^2 = 0, k^2) \frac{x}{z} g\left(\frac{x}{z}, k_0^2\right) \\ &+ \sum_q \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \int_x^1 \frac{dz}{z} S_q^{\mathrm{box}}(z, k'^2, k^2) f\left(\frac{x}{z}, k'^2\right) + V(x,k^2) \\ &+ \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} \frac{\alpha_S(k'^2)}{2\pi} \int_x^1 dz P_{qq}(z) S_{uds}\left(\frac{x}{z}, k'^2\right)\,, \end{aligned} \tag{13}$$

where $a = (1 + 4m_q^2/Q^2)^{-1}$ and $V = x(u_v + d_v)$. We have separated off the non-perturbative contributions. $S_{\mathrm{non}-p}$ is the contribution from the region $k^2, \kappa'^2 < k_0^2$ and the next term is the contribution from the region $k^2 < k_0^2 < \kappa'^2$. An $S \to S$ contribution (from the light u, d, s quarks) is also included. For the light u, d, s quarks $S_q^{\mathrm{box}}(z, k'^2 = 0, k^2)$ in (13) is defined with the κ'^2 integration restricted to the region $\kappa'^2 > k_0^2$.

Equations (7) and (13) form a set of coupled integral equations for the unknown functions $f(x,k^2)$ and $\Sigma(x,k^2)$. We solve them assuming simple parametric form of the inputs:

$$\begin{aligned} xg(x,k_0^2) &= N(1-x)^{\beta}\,, \\ S_{\mathrm{non}-p}(x) &= C_p(1-x)^8 x^{-0.08}\,. \end{aligned} \tag{14}$$

We have fixed the free parameters by fitting the resulting structure functions to the HERA data [3]. The resulting parton distributions will be extrapolated in order to estimate the cross section for the neutrino interaction at

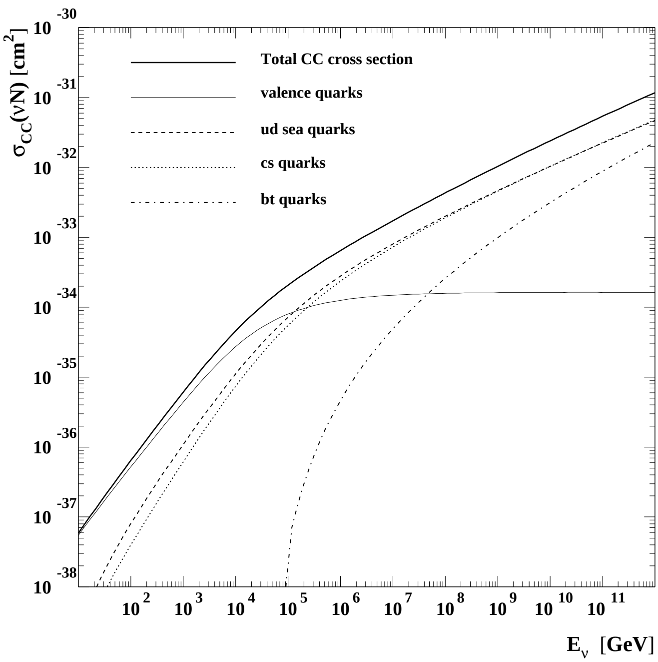

Fig. 2. The total νN charged current cross section and its decomposition into components of different origin as a function of the laboratory neutrino energy.

very high energies. We calculate the cross sections from the usual formula using the results of our unified coupled evolution equations,

$$\frac{d^2\sigma^{\nu,\overline{\nu}}}{dxdy} = \frac{G_F M E}{\pi}\left(\frac{M_i^2}{Q^2+M_i^2}\right)^2 \left\{\frac{1+(1-y)^2}{2}F_2^\nu(x,Q^2) - \frac{y^2}{2}F_L^\nu(x,Q^2) \pm y\left(1-\frac{y}{2}\right)xF_3^\nu(x,Q^2)\right\}, \tag{15}$$

where G_F is the Fermi coupling constant, M is the proton mass, E is the laboratory energy of the neutrino and $y = Q^2/xs$. The mass M_i is either M_W or M_Z according to whether we are calculating charged current (CC) or neutral current (NC) neutrino interactions. We show the results for the neutrino–nucleon cross section in Fig. 2, where we have plotted the total charged current cross section and its decomposition into different ingredients. The dominant contribution at low energies (and correspondingly for large values of Bjorken x) are of course valence quarks. For higher energies $E > 10^5$GeV the light sea quarks start to dominate. The overall contribution from bt quarks is small, it reaches however $\sim 15\%$ for the highest energies. We have compared our calculation with the ones based on the ordinary NLO DGLAP evolution [1, 2, 14] and we have found the agreement to be up to 40% for the highest possible energies. In this way the values of the cross section for the ultrahigh energies have been strongly constrained.

3. Transport equation

In order to calculate the absorption of the neutrino flux when penetrating through the Earth we use the transport equation [6, 7]. This equation contains the usual absorption term as well as the regeneration term. The regeneration term originates from the simple fact that the neutrinos which do interact via neutral current will be removed from the spectrum at higher energies but will regenerate at lower energies. Both these effects are included in the following transport equation for the neutrino flux $I(E,\tau)$,

$$\frac{dI(E,\tau)}{d\tau} = -\sigma_{\rm TOT}(E)I(E,\tau) + \int \frac{dy}{1-y}\frac{d\sigma_{\rm NC}(E',y)}{dy}I(E',\tau)\,, \tag{16}$$

where $\sigma_{\rm TOT} = \sigma_{\rm CC} + \sigma_{\rm NC}$ and where y is, as usual, the fractional energy loss such that

$$E' = \frac{E}{1-y}\,. \tag{17}$$

The variable τ is the number density of nucleons n integrated along a path of length z through the Earth

$$\tau \;=\; \int\limits_0^z dz' n(z')\,. \tag{18}$$

The number density $n(z)$ is defined as $n(z) = N_A\rho(z)$ where $\rho(z)$ is the density of Earth along the neutrino path length z and N_A is the Avogadro number. Clearly the number of nucleons τ encountered along the path z depends upon the nadir angle θ between the normal to the Earth's surface (passing through the detector) and the direction of the neutrino beam incident on the detector. For example $\theta = 0°$ corresponds to a beam transversing the diameter of the Earth. To compute the variation of τ with the angle θ we need to know the density profile of the Earth. We use the preliminary Earth model [13]. We have solved this equation for different fluxes and for different incident angles, and studied the effect of the regeneration term. In Fig. 3 we show the shadowing factor which is defined to be,

$$S(E,\tau) = \frac{I(E,\tau)}{I^0(E)}\,, \tag{19}$$

where $I^0(E) = I(E,\tau)$ is the initial flux at the surface of the Earth. We observe two main facts:

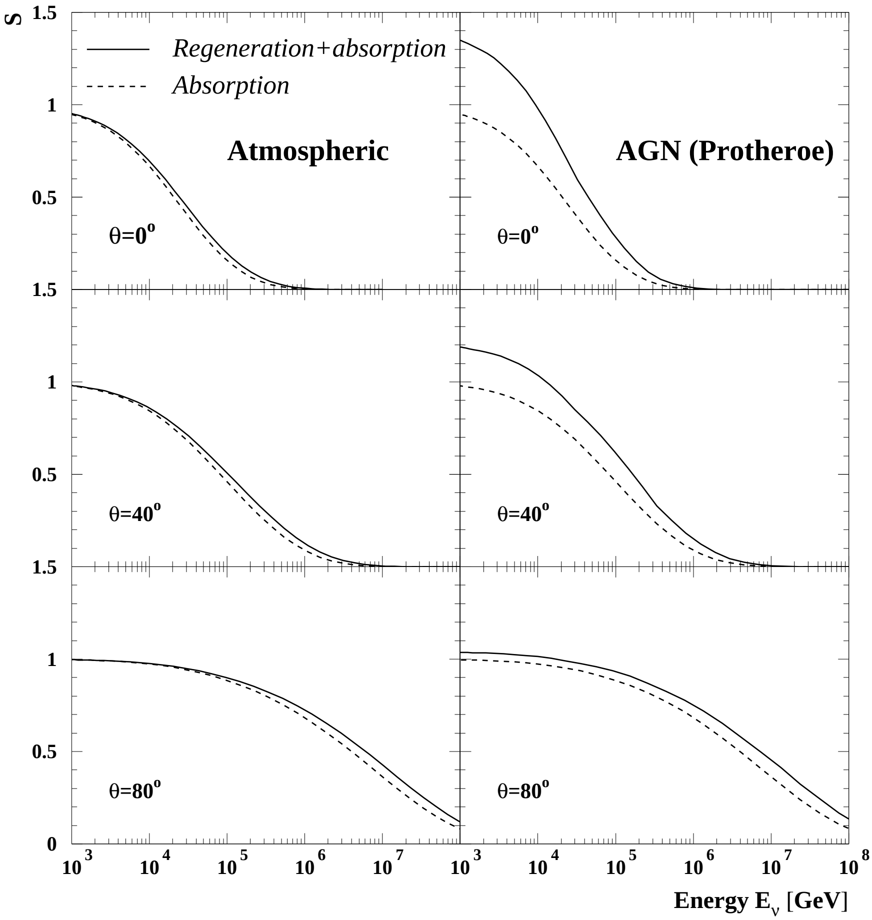

Fig. 3. The shadowing factor S of (19) for two different initial neutrino fluxes incident at three different nadir angles on a detector. The angle $\theta = 0°$ corresponds to penetration right through the Earth's diameter. The two curves on each plot show the shadowing factor with and without NC regeneration included.

- first: for small nadir angles the effect of absorbtion is very large especially for large energies, this will lead to the decrease of the number of arriving neutrinos at the detector.

- second: the regeneration term for the flux from active galactic nuclei is much more important than for the atmospheric neutrino background.

This is connected with the fact that the regeneration is more important for flat fluxes, such as AGN fluxes than for the steeply falling flux like the atmospheric one. In Fig. 4 and 5 we show the results for the active galactic nuclei fluxes [16–18], gamma ray burst [19] and the sample top–down model [20] together with the atmospheric neutrinos background [15] for different nadir angles. One can observe that the absorbtion is indeed very strong for energies above 10^8GeV.

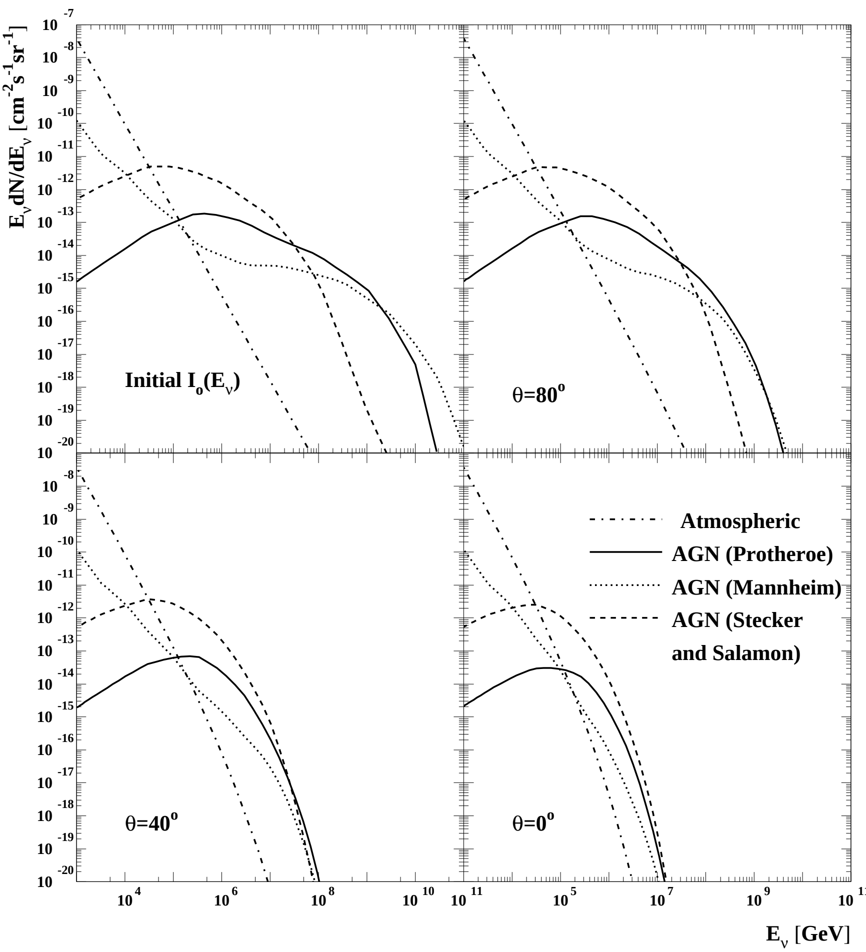

Fig. 4. The initial flux $I_0(E)$ and the flux at the detector $I(E)$ for three different nadir angles corresponding to three models for AGN neutrinos. The background atmospheric neutrino flux is also shown. All the fluxes are given for muon neutrinos.

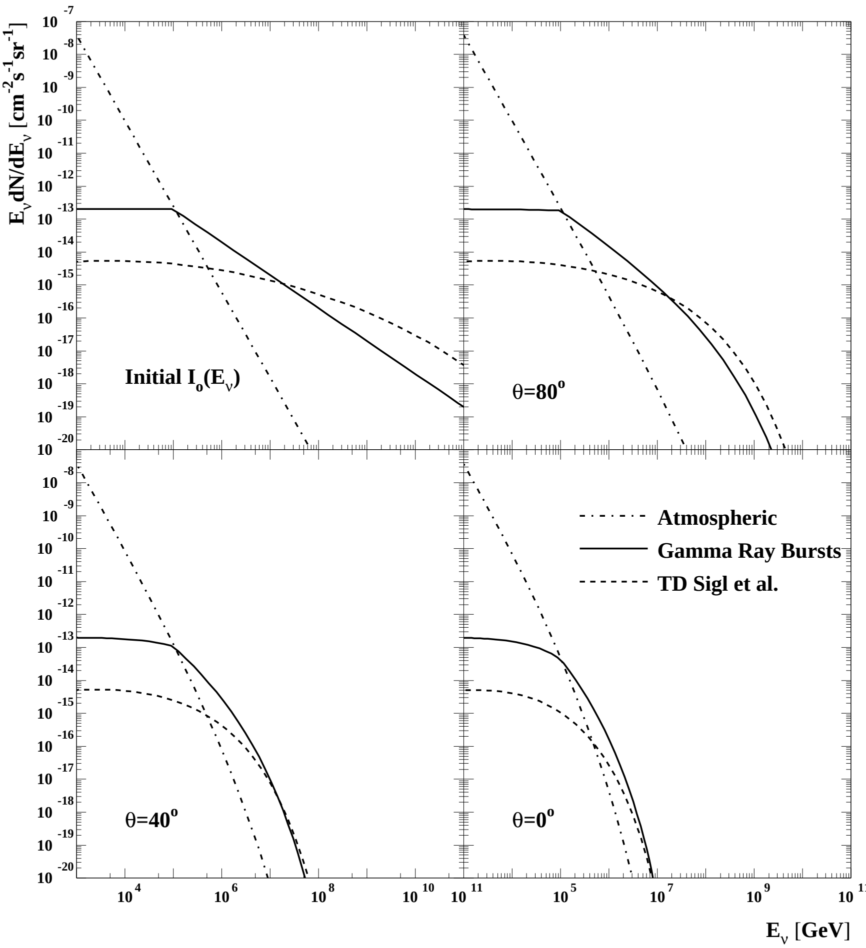

Fig. 5. As for Fig. 4, but showing neutrino fluxes from gamma ray bursts and from a top–down model.All the fluxes are given for muon neutrinos.

4. Conclusions

We have calculated the cross section for the ultrahigh energy neutrino interactions with nucleons using the unified BFKL/DGLAP framework. This system of evolution equations treats the leading $\ln Q^2$ and $\ln 1/x$ on equal footing. The important subleading effects have been resummed via consistency constraint. Such procedure gives much more reliable results for the gluon density than the LO BFKL equation. The results have been compared with the ones based on standard DGLAP evolution with the CTEQ and GRV parton distributions. They all agree very well up to 40% for the highest energies. Therefore we were able to diminish the uncertainty of the cross section calculation. Next we have used the parton distributions obtained from the unified BFKL/DGLAP system as an input to the transport equation. We have solved this equation for different fluxes such as active galactic nuclei, gamma ray bursts and top–down models for various incident angles. We have showed that the regeneration term can be very important for large depth lengths and for flat fluxes. Due to large values of the cross sections the attenuation effects reduce the fluxes of ultrahigh energy neutrinos particularly at small nadir angles. Nevertheless there is a window for the observation of AGN by km^3 underground detectors of the energetic decay muons. We have found that the AGN flux exceeds the atmospheric neutrino background for neutrinos energies $E \gtrsim 10^5$ GeV. One should note however that the muon rates observed at the detectors will be enhanced for large cross sections at high energies. Therefore the whole simulation of the process beginning from the neutrino flux at the surface to the muon detection is needed.

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