POSITRONIUM PROPERTIES*

Andrzej Czarnecki

Physics Department, Brookhaven National Laboratory
Upton, New York 11973, USA

(Received November 4, 1999)

This talk gives an elementary introduction to the basic properties of positronium. Recent progress in theoretical studies of the hyperfine splitting and lifetime of the ground state is reviewed. Sensitivity of these precisely measured quantities to some New Physics effects is discussed.

PACS numbers: 36.10.Dr, 12.20.Ds

1. Introduction

Positronium, an electron-positron bound state, is a particularly simple system which offers unique opportunities for testing our understanding of bound-states in the framework of Quantum Electrodynamics (QED). Because its constituents are much lighter (by a factor of about 200) than any other known charged particles (muons or pions), the spectrum and lifetimes of positronium states can be understood with very high precision within an effective theory of electrons and photons. Comparison of theoretical predictions with experiments constrains a variety of New Physics phenomena, such as axions, millicharged particles, paraphotons, etc. In addition, the opportunity of testing the predictions with high accuracy stimulates development of theoretical tools which can also be applied in other areas of physics, such as QCD.

This talk gives an elementary introduction to basic properties of positronium, reviews some recent improvements in their theoretical description, and briefly touches upon implications for New Physics searches.

* Presented at the XXXIX Cracow School of Theoretical Physics, Zakopane, Poland, May 29–June 8, 1999.

2. Positronium spectrum

Positronium (Ps) is an atom resembling hydrogen in many respects. To first approximation its mass differs from the sum of its constituents' masses ($2m_e \simeq 1.02$ MeV) by the small binding energy,

$$E_B = -\frac{m_e\alpha^2}{4} \simeq -6.8\,\mathrm{eV}, \tag{1}$$

which is about half that of hydrogen, because the reduced mass in positronium is $m_e/2$. The ground state and its radial excitations form the so-called gross spectrum, $E_n = E_B/n^2$ $(n = 1\ldots)$.

This picture, based on the non-relativistic Schrödinger equation, has to be corrected for relativistic effects. In this talk we will be concerned entirely with the lowest radial state with zero angular momentum (S wave). It comes in two varieties, depending on the sum of spins of the electron and positron: para-positronium (p-Ps) with total spin 0, and ortho-positronium (o-Ps), which is a triplet with total spin 1. p-Ps is slightly lighter, due to the spin-spin interaction. This difference of masses, called the hyperfine splitting (HFS), offers one of the most accurate tests of bound-state theory based on QED. (For a review of other precisely measured energy intervals in Ps see *e.g.* [1,2].)

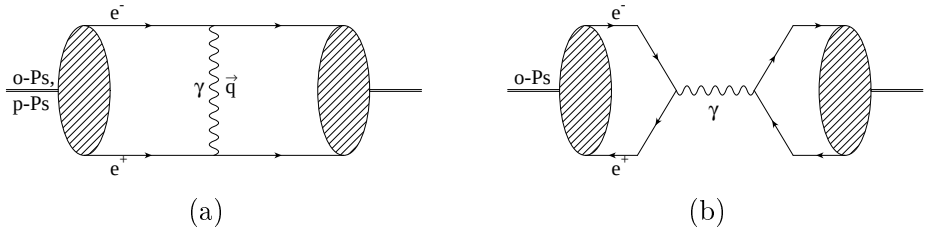

Fig. 1. Lowest order contributions to HFS in positronium.

The origin of HFS can be best explained with the lowest-order diagrams contributing to this effect, shown in Fig. 1. They represent two types of interactions: direct and annihilation. These two effects together give the following difference of energy levels of o-Ps and p-Ps. (That difference is conventionally expressed in terms of the corresponding frequency $\Delta\nu = \Delta E/2\pi\hbar$. In this paper we will often use $\Delta\nu$ to denote the energy intervals):

$$\Delta\nu^{(0)} = (\Delta\nu)^D + (\Delta\nu)^A. \tag{2}$$

The direct interaction diagram 1(a) represents a magnetic photon exchange which induces a spin-dependent potential

$$V_M = -\frac{\pi\alpha}{4m_e^2\vec{q}^2}[\sigma_i, \vec{\sigma}\cdot\vec{q}][\sigma_i', \vec{\sigma}'\cdot\vec{q}], \tag{3}$$

where un-primed and primed σ-matrices act on the electron and positron spinors, respectively. The difference of expectation values of this potential in o-Ps and p-Ps states gives

$$(\Delta\nu)^D = \frac{m_e\alpha^4}{3}. \tag{4}$$

The annihilation contribution (Fig. 1(b)) shifts only the o-Ps energy and we can evaluate it directly from the product of amplitudes of o-Ps becoming a photon and of the reverse process. Projection on the triplet state can be obtained by taking the trace with the o-Ps spin wave function, $\frac{1+\gamma_0}{2\sqrt{2}}\not{\xi}$, where ξ is the o-Ps polarization vector ($\xi^2 = -1$):

$$\begin{aligned}(\Delta\nu)^A &= -\frac{4\pi\alpha}{(2m_e)^2}|\psi(0)|^2 \left(\mathrm{Tr}\,\frac{1+\gamma_0}{2\sqrt{2}}\not{\xi}\frac{\not{p}-m_e}{2m_e}\gamma^\mu\frac{\not{p}+m_e}{2m_e}\right)^2 \\ &= \frac{m_e\alpha^4}{4}.\end{aligned} \tag{5}$$

$|\psi(0)|^2 = \frac{m_e^3\alpha^3}{8\pi}$ is the square of the Ps wave function at the origin and p denotes the electron (or positron) four-momentum, in which we neglect the spatial components at this level of accuracy ($p.\xi = 0$).

Adding the two contributions, we find the lowest-order HFS:

$$\Delta\nu^{(0)} = (\Delta\nu)^D + (\Delta\nu)^A = \frac{7m_e\alpha^4}{12} \simeq 204\,387\ \mathrm{MHz}. \tag{6}$$

This result was obtained in [3–5]. Present experimental accuracy requires computing one- and two-loop corrections to this formula.

The one-loop diagrams are shown in Fig. 2. They were calculated by Karplus and Klein [6], and found to give the following contributions to the HFS:

$$\begin{aligned}2a + 2d &= -\frac{1}{6}\frac{m_e\alpha^5}{\pi}, \\ 2b &= -\frac{m_e\alpha^5}{\pi}, \\ 2c &= -\frac{2}{9}\frac{m_e\alpha^5}{\pi}, \\ 2e &= \frac{1-\ln 2}{2}\frac{m_e\alpha^5}{\pi}.\end{aligned} \tag{7}$$

Together with the lowest order result (6), they lead to the following corrected formula for the HFS:

$$\Delta\nu^{(0)+(1)} = m_e\alpha^4\left[\frac{7}{12} - \left(\frac{8}{9} + \frac{\ln 2}{2}\right)\frac{\alpha}{\pi}\right]. \tag{8}$$

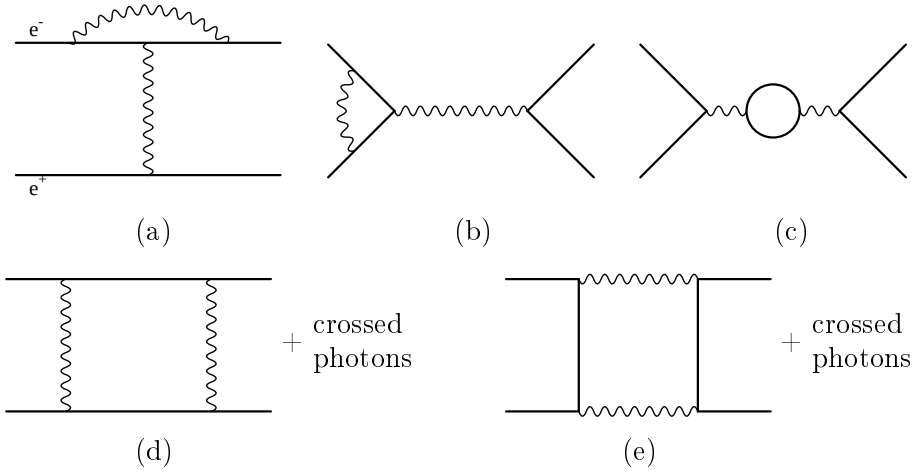

Fig. 2. One-loop corrections to the positronium HFS.

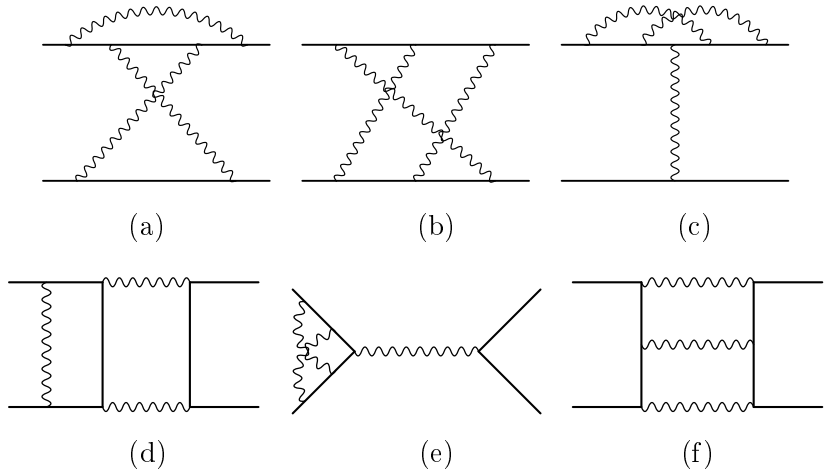

Fig. 3. Examples of two-loop contributions to the positronium HFS: (a) radiative-recoil, (b) pure recoil, (c) non-recoil, (d) correction from the virtual p-Ps annihilation, (e,f) the same with the o-Ps.

At the two-loop level the number of diagrams and their complexity increase dramatically. In fact, their analytical evaluation was completed only this year. Examples of various types of effects are shown in Fig. 3. Their evaluation took (with breaks) more than 40 years of efforts by many groups (see *e.g.* [7] for references). Particularly unclear was the status of recoil

effects, pictured in Fig. 3(b). Until recently there were 3 disagreeing numerical evaluations [8–10]. The difference between the extreme results was about 5.7 MHz, almost 8 times larger than the present accuracy of the HFS measurement [11].

In view of that discrepancy, we recomputed the two-loop recoil corrections [2, 7], using Non-Relativistic QED (NRQED) [8]. In contrast to the earlier studies, dimensional regularization was employed. In this way we were able to completely separate the different characteristic energy scales in the problem and, for the first time, find an analytical result for the recoil effects. Together with the analytical results obtained earlier for the remaining radiative-recoil, non-recoil, and various annihilation contributions, one obtains the following formula for the positronium HFS, including complete $\mathcal{O}\left(m_e\alpha^6\right)$ and leading-logarithmic [12] $\mathcal{O}\left(m_e\alpha^7\ln^2\alpha\right)$ effects:

$$\begin{aligned}\Delta\nu &= m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right.\\ &\quad+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right]\\ &\quad\left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\mathcal{O}\left(\alpha^3\ln\alpha\right)\right\}\\ &= 203\,392.01(46)\ \text{MHz}. \end{aligned} \tag{9}$$

The theoretical error quoted has been estimated by taking half of the last calculated term, the leading-logarithmic $\mathcal{O}\left(m_e\alpha^7\ln^2\alpha\right)$ contribution. The theoretical prediction in (9) differs by 2.91(74)(46) MHz from the most accurate measurement [11], where the first error is experimental and the second theoretical. If these errors are combined in quadrature, this corresponds to a 3.3σ deviation. It would be very interesting to compute further corrections, especially the next-to-leading $\mathcal{O}\left(m_e\alpha^7\ln\alpha\right)$ effects and, even more important, to re-measure the HFS splitting.

3. Positronium decays

In addition to various energy intervals in the Ps spectrum, other quantities which can be measured with high precision are the lifetimes of the singlet and triplet ground-states, p-Ps and o-Ps (because of the possibility of e^+e^- annihilation, both states are unstable). p-Ps has total spin 0 and can decay into two photons, with a short lifetime of about $0.125\times10^{-3}\,\mu$s. On the other hand, o-Ps must decay into at least three photons, because a spin 1 state cannot decay into two photons. Its lifetime, $\simeq 0.14\,\mu$s, is about 1000 times longer than that of p-Ps, and is somewhat easier to accurately determine.

3.1. Parapositronium decays

Barring C-violating effects (*e.g.* caused by the weak interactions), p-Ps can annihilate into only even number of photons (see Fig. 4).

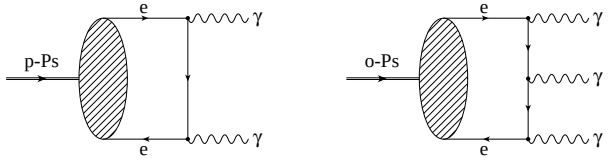

Fig. 4. Lowest order decay channels of p-Ps and o-Ps.

The decay rate of the p-Ps ground state, 1^1S_0, can be calculated as a series in α. The two-photon decay rate is

$$\Gamma(\text{p-Ps} \to \gamma\gamma) = \frac{m_e\alpha^5}{2}\left[1 - \left(5 - \frac{\pi^2}{4}\right)\frac{\alpha}{\pi} + 2\alpha^2 \ln\frac{1}{\alpha} + 1.75(30)\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + \mathcal{O}\left(\alpha^3 \ln\frac{1}{\alpha}\right)\right] = 7989.50(2)\ \mu\text{s}^{-1}, \quad (10)$$

where the non-logarithmic terms $\mathcal{O}\left(\alpha^2\right)$ [13, 14] and leading-logarithmic terms $\mathcal{O}\left(\alpha^3 \ln^2\alpha\right)$ [12] have been obtained only recently.

The four–photon branching ratio is of relative order α^2 [15–17]:

$$\text{BR}(\text{p-Ps} \to 4\gamma) = \frac{\Gamma(\text{p-Ps} \to \gamma\gamma)}{\Gamma(\text{p-Ps} \to 4\gamma)} = 0.277(1)\left(\frac{\alpha}{\pi}\right)^2 \simeq 1.49 \cdot 10^{-6}\,. \quad (11)$$

The theoretical prediction, (10), agrees well with the experiment [18],

$$\Gamma_{\text{exp}}(\text{p-Ps}) = 7990.9(1.7)\ \mu\text{s}^{-1}. \quad (12)$$

3.2. Orthopositronium decays

The ground state of orthopositronium, 1^3S_1, can decay into an odd number of the photons only (if C is conserved). The three–photon (see Fig. 4) decay rate is given by

$$\Gamma(\text{o-Ps} \to \gamma\gamma\gamma) = \frac{2(\pi^2 - 9)m_e\alpha^6}{9\pi}\left[1 - 10.28661\frac{\alpha}{\pi} - \frac{\alpha^2}{3}\ln\frac{1}{\alpha} + B_o\left(\frac{\alpha}{\pi}\right)^2 - \frac{3\alpha^3}{2\pi}\ln^2\frac{1}{\alpha} + \mathcal{O}\left(\alpha^3 \ln\alpha\right)\right] \simeq \left(7.0382 + 0.39 \cdot 10^{-4}\ B_o\right)\ \mu\text{s}^{-1}. \quad (13)$$

Because of its three-body phase space and a large number of diagrams, a complete theoretical analysis of o-Ps decays is more difficult than in the case of p-Ps. The non-logarithmic two-loop effects, parameterized by B_o, have not been evaluated as yet, except for a subset of the so-called soft corrections. Those partial results depend on the scheme adopted for regularizing ultraviolet divergences and do not give a reliable estimate of the complete B_o. Further theoretical work is needed to find that potentially important correction.

Five-photon decay branching ratio is of order α^2 [17, 19],

$$\mathrm{BR}(\text{o-Ps} \to 5\gamma) = \frac{\Gamma(\text{o-Ps} \to 5\gamma)}{\Gamma(\text{o-Ps} \to \gamma\gamma\gamma)} = 0.19(1)\left(\frac{\alpha}{\pi}\right)^2 \simeq 1.0 \times 10^{-6}, \qquad (14)$$

and does not significantly influence the total width.

Table I lists the three latest experimental results for the o-Ps lifetime.

TABLE I

Recent experimental results for the o-Ps lifetime. "Method" in the second column refers to the medium in which o-Ps decays. The last column shows the value of the two-loop coefficient B_o, necessary to bring the theoretical prediction (13) into agreement with the given experimental value. The last line gives the theoretical prediction with $B_o = 0$.

Reference	Method	Γ(o-Ps) $[\mu s^{-1}]$	B_o
Ann Arbor [20]	Gas	7.0514(14)	338(36)
Ann Arbor [21]	Vacuum	7.0482(16)	256(41)
Tokyo [22]	Powder	7.0398(29)	41(74)
Theory without α^2		7.0382	0

The last column indicates the value of the two-loop coefficient B_o necessary to reconcile a given central experimental value with the theoretical prediction (13). We see that the most precise Ann Arbor experiments require an anomalously large value of B_o. This has been known as the "o-Ps lifetime puzzle." More recent Tokyo results, which are at present somewhat less accurate, are in good agreement with the QED prediction with a much smaller B_o. A new experiment is underway at Tokyo, and the data collected so far are in agreement [23] with the previous result [22]. This ongoing effort is expected to provide a measurement of Γ(o-Ps) with an accuracy of about 150 ppm, somewhat better than the presently available Ann Arbor results.

Concerning a possible large value of B_o, it should be mentioned that the perturbative coefficients are moderate in QED predictions for other observables studied with high accuracy, such as leptonic anomalous magnetic moments $(g-2)$, or various spectroscopic properties of positronium and muonium. If there are exceptions, they are caused by widely separated energy scales, such as the electron and muon masses in the muon $g-2$. In the Ps lifetime such effects arise as logarithms of α and have already been accounted for in the leading order.

It has also been argued that the smallness of the complete two-loop non-logarithmic effects in p-Ps, which have recently been calculated [13,14], indicates that analogous two-loop effects in o-Ps are unlikely to explain the large discrepancy with experiment. However, this issue remains controversial and can only be clarified by an explicit calculation of B_o, and renewed experimental studies of Γ(o-Ps).

4. Some implications for New Physics searches

It is interesting to compare the sensitivity of Ps decays and HFS measurements to various kinds of New Physics. Here we focus on the example of millicharged particles (for a review see [24]).

The millicharged particles we are interested in here are fermions, similar in properties to the electron, but with an unknown small mass m_x and electric charge ηe (where $-e$ is electron's charge and $\eta \ll 1$). Such particles were searched for in decays of o-Ps [25], where a 90% confidence level upper bound $\eta < 8.6 \times 10^{-6}$ was found for $m_x \ll m_e$ (this bound is not very sensitive to m_x, except for m_x close to m_e, where it becomes weaker). That bound was found by comparing a theoretical prediction for the decay rate of o-Ps into a pair $X\bar{X}$ of millicharged particles,

$$\Gamma(\text{o-Ps} \to X\bar{X}) = \eta^2 \frac{m_e \alpha^6}{6} \sqrt{1 - \frac{m_x^2}{m_e^2}} \left(1 + \frac{m_x^2}{2m_e^2}\right), \tag{15}$$

with the measured bound on the maximum decay rate of o-Ps into invisible particles, $\Gamma(\text{o-Ps} \to \text{invisible}) < 2.8 \times 10^{-6} \Gamma(\text{o-Ps} \to \gamma\gamma\gamma)$.

How does this compare with the sensitivity of HFS to possible extra loops in the photonic vacuum polarization (*cf.* Fig. 2(c))? We find that a pair of millicharged particles would have a negative contribution to Ps HFS, given by the formula

$$\Delta\nu(\text{millicharged}) = \frac{\eta^2 m_e \alpha^5}{12\pi} \times \left\{ \frac{1}{3} + 2\left(1 + \frac{m_x^2}{2m_e^2}\right)\left[\sqrt{\left(\frac{m_x^2}{m_e^2} - 1\right)}\,\text{arccot}\sqrt{\left(\frac{m_x^2}{m_e^2} - 1\right)} - 1\right]\right\}$$
$$\simeq -\frac{\eta^2 m_e^3 \alpha^5}{15\pi m_x^2} \qquad (\text{for } m_x \gg m_e). \tag{16}$$

We note that the o-Ps decay search for $X\bar{X}$ has the advantage that the Standard Model background (o-Ps$\rightarrow \gamma\gamma\gamma$) is suppressed by an additional factor of α^2. Therefore, with presently achievable experimental accuracy, HFS cannot compete with o-Ps if m_x is smaller than m_e. For this reason we have displayed the simple limiting behavior of the HFS contribution for large m_x.

Another potentially interesting observable which might be sensitive to $X\bar{X}$ effects is the electron anomalous magnetic moment ($a_e = (g_e - 2)/2$). If $m_x \gg m_e$, the contribution of an $X\bar{X}$ loop is

$$\Delta a_e(\text{millicharged}) = \frac{\eta^2 \alpha^2 m_e^2}{45\pi^2 m_x^2}. \tag{17}$$

We can now express the millicharged pair contribution to the HFS by its effect on a_e:

$$\Delta\nu(\text{millicharged}) = -3\pi m_e \alpha^3 \Delta a_e(\text{millicharged})$$
$$= -\frac{36\pi}{7\alpha}\Delta\nu^{(0)}\Delta a_e(\text{millicharged}), \tag{18}$$

where $\Delta\nu^{(0)}$ denotes the lowest order Ps HFS, given in (6). In the lowest order in α we have $a_e^{(0)} = \alpha/2\pi$, so that

$$\frac{\Delta\nu(\text{millicharged})}{\Delta\nu^{(0)}} = -\frac{18}{7}\frac{\Delta a_e(\text{millicharged})}{a_e^{(0)}}. \tag{19}$$

Since the present agreement of theory and experiment for a_e is at the level of 3 parts in 10^8 [26], we see that the largest compatible effect of millicharged particles in HFS could be of the order of 10^{-7}. This is insufficient to reconcile the theory with experiment since, as we saw in Section 2, the observed HFS value is smaller than the theoretical prediction by about 3 MHz, or $1.4 \times 10^{-5}\Delta\nu^{(0)}$. It might, however, be worth noting that the differences of central theoretical and experimental values of a_e and Ps HFS have opposite signs, and an $X\bar{X}$ pair would decrease both discrepancies.

Positronium decays were also used to search for light pseudoscalar bosons (see *e.g.* [27]). It turns out that such searches give much more stringent constraints on the coupling of pseudoscalars to electrons than do present HFS measurements (by a few orders of magnitude). If pseudoscalars are heavier than o-Ps, the relative sensitivity of a_e and Ps HFS to such particles was discussed in [28].

5. Summary

We have reviewed the basic properties of the positronium spectrum and decay modes. We have seen that there are discrepancies between the theoretical predictions and experimental determinations of the hyperfine splitting (by 3.3σ) and o-Ps lifetime (by about $6-9\sigma$ for Ann Arbor experiments). It is unlikely that New Physics effects are responsible for such large differences. It is also very unlikely that the uncalculated higher-order effects in QED could alone account for such discrepancies. Most likely, both problems could be clarified by new experiments. While efforts are underway to obtain a more accurate value of the o-Ps lifetime, it would also be very useful to re-measure the hyperfine splitting.

I am grateful to S. Asai for helpful discussions on o-Ps lifetime measurements, to S. Davidson and W. Marciano for careful reading of the manuscript and helpful remarks, and to K. Melnikov and A. Yelkhovsky for collaboration on positronium physics. I thank Professor Andrzej Białas and Professor Michał Praszałowicz for inviting me to the Cracow School of Theoretical Physics. This research was supported by DOE under grant DE-AC02-98CH10886.

REFERENCES

[1] K. Pachucki, S. G. Karshenboim, *Phys. Rev. Lett.* **80**, 2101 (1998), hep-ph/9709387.

[2] A. Czarnecki, K. Melnikov, A. Yelkhovsky, *Phys. Rev.* **A59**, 4316 (1999), hep-ph/9901394.

[3] J. Pirenne, *Arch. Sci. Phys. Nat.* **29**, 265 (1947).

[4] V. B. Berestetski, L. D. Landau, *JETP* **19**, 673,1130 (1949).

[5] R. A. Ferrel, *Phys. Rev.* **84**, 858 (1951).

[6] R. Karplus, A. Klein, *Phys. Rev.* **87**, 848 (1952).

[7] A. Czarnecki, K. Melnikov, A. Yelkhovsky, *Phys. Rev. Lett.* **82**, 311 (1999), hep-ph/9809341.

[8] W. E. Caswell, G. P. Lepage, *Phys. Lett.* **B167**, 437 (1986).

[9] K. Pachucki, *Phys. Rev.* **A56**, 297 (1997).

[10] G. S. Adkins, J. Sapirstein, *Phys. Rev.* **A58**, 3552 (1998).

[11] M. W. Ritter, P. O. Egan, V. W. Hughes, K. A. Woodle, *Phys. Rev.* **A30**, 1331 (1984).

[12] S. G. Karshenboim, *Sov. Phys. JETP* **76**, 541 (1993), Zh. Eksp. Teor. Fiz. **103**, 1105 (1993).

[13] A. Czarnecki, K. Melnikov, A. Yelkhovsky, *Phys. Rev. Lett.* **83**, 1135 (1999), hep-ph/9904478.

[14] A. Czarnecki, K. Melnikov, A. Yelkhovsky, α^2 *corrections to parapositronium decay: a detailed description* (1999), hep-ph/9910488.

[15] A. Billoire, R. Lacaze, A. Morel, H. Navelet, *Phys. Lett.* **78B**, 140 (1978).

[16] T. Muta, T. Niuya, *Prog. Theor. Phys.* **68**, 1735 (1982).

[17] G. S. Adkins, F. R. Brown, *Phys. Rev.* **A28**, 1164 (1983).

[18] A. H. Al-Ramadhan, D. W. Gidley, *Phys. Rev. Lett.* **72**, 1632 (1994).

[19] G. P. Lepage, P. B. Mackenzie, K. H. Streng, P. M. Zerwas, *Phys. Rev.* **A28**, 3090 (1983).

[20] C. I. Westbrook, D. W. Gidley, R. S. Conti, A. Rich, *Phys. Rev.* **A40**, 5489 (1989).

[21] J. S. Nico, D. W. Gidley, A. Rich, P. W. Zitzewitz, *Phys. Rev. Lett.* **65**, 1344 (1990).

[22] S. Asai, S. Orito, N. Shinohara, *Phys. Lett.* **B357**, 475 (1995).

[23] S. Asai, private communication.

[24] S. Davidson, B. Campbell, D. Bailey, *Phys. Rev.* **D43**, 2314 (1991).

[25] T. Mitsui, R. Fujimoto, Y. Ishisaki, Y. Ueda, Y. Yamazaki, S. Asai, S. Orito, *Phys. Rev. Lett.* **70**, 2265 (1993).

[26] A. Czarnecki, W. J. Marciano, *Nucl. Phys. B (Proc. Suppl.)* **76**, 245 (1999), hep-ph/9810512.

[27] S. Asai, S. Orito, K. Yoshimura, T. Haga, *Phys. Rev. Lett.* **66**, 2440 (1991).

[28] L. M. Krauss, M. Zeller, *Phys. Rev.* **D34**, 3385 (1986).