RADIATIVE DECAYS OF HEAVY MESONS AND THE DETERMINATION OF THE STRONG *g*-COUPLING*

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The strong g-coupling characterizes the interaction of heavy mesons with pions in typical vertices $H^*H\pi$, $H^*H^*\pi$, where $(H^*;H)$ stands for vector and pseudoscalar $(B^*;B)$ or $(D^*;D)$ heavy mesons. Its estimation by different theoretical methods has led to a wide range of possible values. We describe a new approach to the determination of g, which exploits the rare radiative decays $B^* \to B\gamma\gamma$ and $D^* \to D\gamma\gamma$. It is shown that the branching ratio of $D^* \to D\gamma\gamma$ can be expressed as a function of a single unknown g and we calculate it to be in the measurable range between 1.6×10^{-6} and 3.3×10^{-5} for $0.25 < g < 1$.

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1. Introduction and experimental overview

We consider here the strong and electromagnetic decays of the heavy vector mesons D^* and B^* of spin-parity 1^-, with the special aim of gaining information on their strong couplings $g_{B^*B\pi}$, $g_{D^*D\pi}$. It is well known that the main decays of D^*'s proceed either as a strong transition $D^* \to D\pi$ with a final pion of about 40 MeV/c momentum, or as an electromagnetic transition $D^* \to D\gamma$ with a photon of momentum of nearly 140 MeV/c. On the other hand, since the mass difference $M_{B^*} - M_B$ is only 45.8 MeV, the decay $B^* \to B\pi$ cannot take place and the main decay of B^* is the radiative process $B^* \to B\gamma$.

The interaction between mesons containing a single heavy quark Q and pseudoscalar Goldstone bosons is presently best described by an effective theory [1–3] which contains flavour and spin symmetries in the heavy mesons sector and chiral $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ symmetry in the light one (for a recent

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review, see [4]). As a result of the heavy quark symmetry, the heavy meson chiral Lagrangian ($HM\chi L$) which implements this scheme contains just one coupling, denoted by g, to characterize the strength of several vertices, $D^*D^*\pi$, $D^*D\pi$, $B^*B^*\pi$ and $B^*B\pi$. While the $g_{B^*B\pi}$ vertex is not accessible in a direct decay, the $g_{D^*D\pi}$ coupling may be measured in principle from the decay width of the decay channel $D^* \to D\pi$. So far, only an upper limit is available from the ACCMOR Collaboration at CERN [5], $\Gamma(D^{*+}) <131$ KeV based on the high-resolution measurement of 127 (D^{*+}) events.

In view of the great interest in the strength of the $B^*B\pi$ and $D^*D\pi$ vertices, which are relevant for the analysis of various B and D decays, a large number of calculations has been performed to obtain the g-coupling. The results are quite divergent, as it will be described in the next chapter. Recently, Daphne Guetta and myself [6] have suggested a new approach to the determination of g. We considered the decays $B^* \to B\gamma\gamma$ and $D^* \to D\gamma\gamma$ within the framework of $HM\chi L$ and we have shown that the measurement of the branching ratios and spectra of these decays can provide information on the g-coupling. This method is especially profitable in the charm sector.

As a background for the model we have built, I review here succinctly the experimental status of the B^* and D^* decays and present a summary of the theoretical attempts to describe the main decays to a pion or a photon.

The main decay of B^*, the electromagnetic transition $B^* \to B\gamma$ has been observed both at the Cornell Electron Storage Ring (CESR) [7] and at LEP [8]. There is obviously no measurement of the width of this transition. It has been studied in a variety of theoretical models including quark models [9], the chiral bag model [10] followed by effective chiral Lagrangian approaches [11], potential models [12], QCD sum rules [13] and an analysis of experimental D^* branching ratios using $HM\chi L$ [14]. The predictions range from $\Gamma(B^{*o}(B^{*+}) \to B^0(B^+)\gamma) = 0.04(0.10)$ KeV [13] to 0.28(0.84) KeV [10,11], with most calculations concentrating in the higher range.

The D^* meson was discovered more than twenty years ago [15] and its decay branching ratios have been studied extensively. The current PDG averages [16] are $\mathrm{Br}(D^{*+} \to D^+\pi^o) : \mathrm{Br}(D^{*+} \to D^o\pi^+) : \mathrm{Br}(D^{*+} \to D^+\gamma) = (30.6 \pm 2.5)\% : (68.3 \pm 1.4)\% : (1.1 \pm 2.1 \pm 0.7)\%$ and $Br(D^{*o} \to D^o\pi^o) : \mathrm{Br}(D^{*o} \to D^o\gamma) = (61.9 \pm 2.9)\% : (38.1 \pm 2.9)\%$. A recent [17] CLEO experiment on D^{*+} decays gives the more accurate branching ratios $\mathrm{Br}(D^{*+} \to D^+\pi^o) : \mathrm{Br}(D^{*+} \to D^o\pi^+) : \mathrm{Br}(D^{*+} \to D^+\gamma) = (30.7 \pm 0.7)\% : (67.6 \pm 0.9)\% : (1.7 \pm 0.6)\%$.

These D^* electromagnetic and strong decays have been calculated with the same models used for B^* and most papers of Refs. [9]-[14] have considered also D^* decays. Additional calculations referring to D^* only include quark models [18], the chiral bag model [19] and use of sum rules [20]. Here again the theoretical calculations span an order of magnitude range for the prediction of the widths, from $\Gamma(D^{*o}) \simeq (3-10)$KeV [13] to (60-120)KeV [9,11]. Many of these theoretical results are fitted to obtain correct relative widths. The real test will come when the absolute width will be measured. According to several of the models, typical values are $\Gamma(D^{*+} \to \text{ all}) \simeq 80$KeV, $\Gamma(D^{*o} \to \text{ all}) \simeq 60$ KeV [9,18,19], not far from the present upper limit [5].

2. The heavy meson chiral Lagrangian and estimation of g

The treatment of the physical processes involving soft pions we described in the previous section is performed within the framework of an effective theory, the "Heavy Meson Chiral Lagrangian" ($HM\chi L$) which embodies two principal symmetries of Quantum Chromodynamics. At one end, there is the SU(2N$_f$) heavy flavour-spin symmetry characteristic of the infinite heavy quark mass limit. In this limit, the interactions with the light hadrons are independent of the mass of the heavy quarks; moreover, the independence of the interaction on the spin s_Q of the heavy quark allows to define degenerate doublets of heavy states with spin-parity $s_Q^P = (s_\ell \pm \frac{1}{2})^P$ where s_ℓ is the spin of the light quark, which in our case is $q = u, d, s$.

The doublet members are the pseudoscalar and vector mesons corresponding to $s_\ell = \frac{1}{2}$ and we make the assumption that both the c and the b quark are sufficiently heavy. Symmetry-breaking corrections to this scheme are taken into account by terms obtained in a $\frac{1}{M_Q}$ expansion [4].

At the other end of the energy scale is the chiral limit of QCD realized for $M_\ell \to 0 (\ell = u, d, s)$; the QCD Lagrangian is then invariant under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformations. This symmetry is spontaneously broken to the vector subgroup $\mathrm{SU}(3)_V$ and the resulting Goldstone bosons are the eight pseudoscalar mesons π, K and η. The quark mass terms breaking the chiral symmetry help the Goldstone bosons to acquire their mass.

The effective Lagrangian which contains these symmetries is expressed [1-4] in terms of the hadronic fields for heavy and light mesons. The heavy vector (B^*, D^*) and pseudoscalar (B, D) mesons are represented by a 4×4 Dirac matrix H, with one spinor index for the heavy quark and the second one for the light degree of freedom,

$$H = \frac{1+\not{v}}{2}\left[P^*_\mu v^\mu - P\gamma_5\right], \quad \bar{H} = \gamma_o H^\dagger \gamma_o \,. \tag{1}$$

P^*_μ and P are the respective annihilation operators of vector (1^-) and pseudoscalar (0^-) heavy mesons with four-velocity v_μ.

The Goldstone bosons are represented with the aid of a unitary 3×3 matrix $\sum = \exp(2iM/f)$ with the M being the usual 3×3 hermitian traceless matrix describing the octet of pseudoscalar bosons.

The most general Lagrangian describing the interaction of heavy mesons with Nambu–Goldstone bosons, which is invariant under Lorentz transformations, parity, heavy-quark spin flavour symmetry and chiral symmetry is given in the framework we described by [1–4]

$$\begin{aligned}\mathcal{L} = \quad & i\,\mathrm{Tr}\,\left\{\bar{H}_a v^\mu D_{\mu ba} H_b\right\} + \frac{f^2}{8}\partial^\mu \Sigma_{ab}\partial_\mu \Sigma^\dagger_{ba} \\ & ig\,\mathrm{Tr}\,\left\{\bar{H}_a \gamma_\mu \gamma_5 A^\mu_{ab} H_b\right\}\end{aligned} \tag{2}$$

where D_μ is a covariant derivative. $D_\mu = \partial_\mu + V_\mu$, and f is the pion decay constant, $f = 132$ MeV.

Using $\Sigma(x) = \xi^2(x)$ one introduces a vector $V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ and an axial current $A_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$. The first two terms of (2) are the kinetic energy terms and the third one is an interaction term, defining the strong-interaction coupling g. Explicit interaction terms are obtained by expanding the axial current in (2) and keeping the first term $A^\mu = (i/f)\partial_\mu M + \ldots$. Expressing the interaction in terms of (D^*, D) fields (the same holds for B^*, B fields) one has

$$\mathcal{L}^{\mathrm{int}}_{\mathrm{eff}} = \left[-\frac{2g}{f} D^*_\mu \partial^\mu M D^\dagger + \mathrm{h.c.}\right] + \frac{2gi}{f}\epsilon_{\mu\nu\sigma\tau} D^{*\mu}\partial^\sigma M D^{*\dagger\nu} v^\tau \tag{3}$$

and we see that, *e.g.*, $D^*D\pi$ and $D^*D^*\pi$ vertices are characterized with the same strength g. The g-coupling is directly related to the hadronic coupling $g_{D^*D\pi}$ which is defined by the on-shell matrix element

$$\langle D^o(p)\pi^+(q)|D^{*+}(p+q)\rangle = g_{D^*D\pi}\epsilon_\mu q^\mu \tag{4}$$

where ϵ^μ is the polarization vector of D^{*+}. Likewise,

$$\langle D^{*o}(p,\epsilon_1)\pi^+(q)|D^{*+}(p+q,\epsilon_2)\rangle = g_{D^*D^*\pi}\epsilon_{\mu\nu\sigma\tau}\epsilon_1^\mu \epsilon_2^\nu p^\sigma q^\tau\,. \tag{5}$$

From (3)–(5) one finds the link

$$g_{D^*D\pi} = g_{D^*D^*\pi} = \frac{2M_D}{f} g\,. \tag{6}$$

Moreover, isospin symmetry requires

$$g_{D^*D\pi} \equiv g_{D^{*+}D^o\pi^+} = -\sqrt{2}g_{D^{*+}D^+\pi^o} = \sqrt{2}g_{D^{*o}D^o\pi^o} = -g_{D^{*o}D^+\pi^-}\,. \tag{7}$$

The interest in the value of g is not limited to the theoretical interest in the strength of the axial interaction defined in Eq. (2). The knowledge of g is of great phenomenological value, since its strength is required in the analyses of many electroweak processes [4]. Among these, for example, are heavy-to-light semileptonic exclusive decays like $B \to \pi\ell\nu$, $D_s \to K\ell\nu$ which are promising processes for the extraction of CKM matrix elements like $|V_{ub}|$, and their analysis with VMD requires the knowledge of $g_{B^*B\pi}$. Other processes requiring such knowledge are $B \to D^*\pi\ell\nu$ decays, chiral corrections to $B \to D$ processes, decay constants of heavy mesons, radiative processes like $D^* \to D\gamma$, $B^* \to B\gamma$ and more. Thus, no wonder that a large number of theoretical papers has been devoted during the last years to the computation of g. In the rest of this Section we present a succint overview of the main attempts in this direction.

The theoretical attempts may be grouped into several classes; however, even within a single class of models, the variation of g turns out to be quite large. The non-relativistic quark model leads to [3] the largest value $g = 1$, while slightly modified quark models [18, 21] bring this value down to about $g = 0.8$. On the other hand, in a calculation [22] in which the effect of the relativistic motion of the light antiquark is taken into account by the use of Salpeter equation one arrives at $g = \frac{1}{3}$. Somewhat higher values were obtained in recent quark-model calculations: with a relativistic quark model based on the light front formalism Jaus obtains [9] $g = 0.56$; in a relativisitic quark model with direct quark-meson interactions [23] one arrives at $g = 0.46$; and a quark model with Dirac equation [24] finds $g = 0.61$. The QCD sum rules have also been used [25] extensively to the calculation of g. The outcome of these calculations is generally in the direction of small g values, between 0.15 and 0.35, which would imply that the decay width of D^* is rather small, *i.e.* below 45 KeV. Finally, we mention a recent lattice QCD determination [26] of $g = 042(4)(8)$ and the analysis of Stewart [14] of the experimental data on $D^* \to D\pi, D\gamma$ which incorporates symmetry breaking terms in the Lagrangian and deduces $g = 0.27^{+0.09}_{-0.04}$.

3. A model for two-photon decays $D^* \to D\gamma\gamma$, $B^* \to B\gamma\gamma$

Recently we have considered the two-photon processes $B^*(D^*) \to B(D)\gamma\gamma$ [6], not discussed previously in the literature, by using the $HM\chi L$ and we found that the $D^* \to D\gamma\gamma$ could provide a measurement of the much sought after g-coupling.

The calculation of radiative processes requires the addition of the electomagnetic interaction to the Lagrangian of Eq. (2). This is performed by the usual procedure of minimal coupling which leads to the replacement of derivative operators by covariant derivatives containing the photon field. However, this does not suffice to account for the observed magnetic dipole transitions $B^* \to B\gamma$, $D^* \to D\gamma$; to account for these, a contact gauge invariant electromagnetic term proportional to $F_{\mu\nu}$ must be added, which has the form in the heavy mass limit [11, 14]

$$\mathcal{L}^{(\mu)} = \frac{e\mu}{4} \mathrm{Tr}\,(\bar{H}_a \sigma_{\mu\nu} F^{\mu\nu} H_b \delta_{ab}), \tag{8}$$

where μ is the strength of this anomalous magnetic dipole interaction and has mass dimension $[1/M]$.

In the following, we present our calculation for the charm sector; then, we shall comment on the features arising in the beauty sector. From (8), additional electromagnetic vertices obtain representing $D^*D^*\gamma$ and $D^*D\gamma$ interactions of same strength. These are

$$\langle \gamma(k,\epsilon) D(v_1) | D^*(v_2,\epsilon_2)\rangle = -ieM_{D^*}\mu\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu k^\nu v_2^\alpha \epsilon_2^\beta \tag{9}$$

$$\langle \gamma(k,\epsilon) D^*(v_1,\epsilon_1) | D^*(v_2,\epsilon_2)\rangle = e\mu M_{D^*}(\epsilon_1 \cdot k\epsilon \cdot \epsilon_2 - \epsilon_2 \cdot k\epsilon \cdot \epsilon_1)\,. \tag{10}$$

The calculation is performed to leading order in chiral perturbation theory and to this order there are no counterterms [14,27]. In addition to the Feynman diagrams obtained from Eq. (2) with minimal electromagnetic interaction and from Eq. (8), we must include to the same order the pion axial anomaly, whose strength is known. All these terms are of the same order in an $1/N_c$ expansion.

We start with the description of the decay of the neutral D^*. The decay amplitude A for $D^{*o} \to D^o\gamma\gamma$ may be written as

$$A = A_{\text{anomaly}} + A_{\text{tree}} + \sum_{i=1}^{6} A^{(i)}_{\text{loops}}\,. \tag{11}$$

We shall describe now the eight contributions to the amplitude, indicating the couplings entering into each of them, without giving here the detailed expressions which can be found in Ref. [6].

A_{anomaly} represents $D^{*o} \to D^{o\prime\prime}\pi^{\prime\prime} \to D^o\gamma\gamma$ via a virtual neutral pion. Since the physical decay $D^{*o} \to D^o\pi^o$ is allowed, we limit ourselves to a region for $s = (k_1 + k_2)^2$ which goes up to 20 MeV away from the pion mass. Given the strength of the pion axial anomaly of $(\alpha/\pi f)$, where f is the pion decay constant and $\alpha = e^2/4\pi$, A_{anomaly} is proportional to $\alpha g_{D^*D\pi}$. The tree level graph is due to the transition $D^{*o} \to {}''D^{*o}{}''\gamma \to D^o\gamma\gamma$, containing two insertions of the anomalous magnetic operator. Hence, A_{tree} is proportional to $\alpha\mu^2$. $A^{(1)}_{\text{loop}}$ describes the transition $D^{*o} \to (D^{*+}\pi^-) \to D^o\gamma\gamma$ with the two photons radiated from the virtual charged pion. Additional graphs, required by gauge invariance have one photon radiated from the loop and the second emitted from the $D^*D^*\pi$, $D^*D\pi$ vertices or both photons emitted from these vertices. The other loop diagrams, $A^{(2)}_{\rm loop} - A^{(6)}_{\rm loop}$ come from diagrams where both the strong coupling and the magnetic one are involved. $A^{(2)}_{\rm loop}$ is given by $D^{*o} \to "D^{*o}"\gamma \to (D^{*+}\pi^-)\gamma \to D^o\gamma\gamma$, being thus proportional to $\alpha g_{D^*D^*\pi} g_{D^*D\pi}\mu$. In $A^{(3)}_{\rm loop}$ a $D^{*o}D^o\gamma$ vertex replaces the $D^{*o}D^{*o}\gamma$ one in the initial step, the diagram being proportional to $\alpha g^2_{D^*D\pi}\mu$. $A^{(4)}_{\rm loop}$ describes the transition $D^{*o} \to (D^+\pi^-) \to D^{*o}\gamma \to D^o\gamma\gamma$, where the first photon is emitted by the charged pion from the $(D^+\pi^-)$ loop, while the second one comes from the $D^{*o} \to D^o\gamma$ transition. The expression is then proportional to $\alpha g^2_{D^*D\pi}\mu$. Exchanging D with a D^* in the loop one gets $A^{(5)}_{\rm loop}$, which is thus proportional to $\alpha g^2_{D^*D^*\pi}\mu$. Finally, we have a diagram given by $D^{*o} \to (D^+\pi^-)$ and then the virtual pion emits one photon while the virtual D^+ emits the other one becoming D^{*+}. The virtual $(D^{*+}\pi^-)$ recombine to a D^o. This is described by $A^{(6)}_{\rm loop}$ and is proportional to $\alpha g^2_{D^*D\pi}\mu^{(+)}$, where $\mu^{(+)}$ is the strength of the $D^{*+} \to D^+\gamma$ transition.

Calculating from (11) the decay width, one obtains [6] an expression containing 13 terms, which depend on various products $g^\alpha \mu^\beta {\mu^{(+)}}^\gamma$, where α, β have values 0 to 4 and γ has values 0-2. At this point, the crucial step is to use the existing experimental information on the relative branching ratios $\Gamma(D^{*o} \to D^o\pi^o) : \Gamma(D^{*o} \to D^o\gamma) = (61.9 \pm 2.9)\% : (38.1 \pm 2.9)\%$ and $\Gamma(D^{*+} \to D^o\pi^+) : \Gamma(D^{*+} \to D^+\gamma) = (67.6 \pm 0.9)\% : (1.7 \pm 0.6)\%$ [16,17]. This allows us to establish $\mu \simeq 6.6g/M_{D^*}$ and $\mu_+ \simeq 1.7g/M_{D^*}$. As a result, we were able to express $\Gamma(D^{*o} \to D^o\gamma\gamma)$ [6] *as a function of g only*:

$$\begin{aligned}\Gamma(D^{*o} \to D^o\gamma\gamma) = \Big[&2.52 \times 10^{-11}g^2 + 5.66 \times 10^{-11}g^3 \\ &+4.76 \times 10^{-9}g^4 + 3.64 \times 10^{-10}g^5 + 1.53 \times 10^{-9}g^6\Big]\ \text{GeV}\,. \end{aligned} \tag{12}$$

In obtaining (12) we assumed that the coupling constants are relatively positive, as indicated by theoretical analysis [14]. However, if we assume opposite sign for various pairs of couplings, we found that the changes are rather small, the reason being that the main contribution is given by quadratic terms.

Let us define now the branching ratio for this decay

$$\mathrm{BR}\,(D^{*o} \to D^o\gamma\gamma) = \frac{\Gamma(D^{*o} \to D^o\gamma\gamma)}{\Gamma(D^{*o} \to D^o\gamma) + \Gamma(D^{*o} \to D^o\pi^o)}\,. \tag{13}$$

In the denominator, we can use for $\Gamma(D^{*o}\to D^o\pi^o)$ the expression obtainable from Eq. (4)

$$\Gamma(D^{*o}\to D^o\pi^o)=\frac{1}{12\pi}\frac{g^2}{f^2}|\vec{p}_\pi|^3=1.25\times 10^{-4}g^2\ \text{GeV} \tag{14}$$

while for $\Gamma(D^{*o}\to D^o\gamma)$ we use the experimental fact [16] $\Gamma(D^{*o}\to D^o\pi^o)$: $\Gamma(D^{*o}\to D^o\gamma)=61.9:38.1$, to reexpress $\Gamma(D^{*o}\to D^o\gamma)$ in terms of g^2 as well. Thus one obtains [6]

$$\Gamma(D^o\to\ \text{all})=(2.02\pm 0.12)\times 10^{-4}g^2\ \text{GeV} \tag{15}$$

and as a consequence

$$\text{BR}\,(D^{*o}\to D^o\gamma\gamma)=\frac{(0.025+0.057g+4.76g^2+0.36g^3+1.53g^4)\times 10^{-9}g^2}{2.02\times 10^{-4}g^2}. \tag{16}$$

Obviously, a measurement of this ratio will constitute a measurement of g.

Turning to the $B^{*o}\to B^o\gamma\gamma$ decay, one has a rather different situation. Firstly, the branching ratio is now defined as

$$\text{BR}\,(B^{*o}\to B^o\gamma\gamma)=\frac{\Gamma(B^{*o}\to B^o\gamma\gamma)}{\Gamma(B^{*o}\to B^o\gamma)} \tag{17}$$

since there is no strong decay of B^{*o}. The quantity in Eq. (17) turns out to be a function of g, μ and μ^+ and an analysis similar to the one we performed for D^{*o} decay is not possible. Still, a certain amount of information is obtainable by a judicious analysis in the above parameter space [6]. However, we shall not address this topic here.

4. Discussion and summary

Firstly, some remarks about the framework of the calculation of [6]. The analysis of $D^{*o}\to D^o\gamma\gamma$ has been performed to leading order in chiral perturbation theory and mostly to leading order in a $1/M$ expansion. Corrections to the leading order of (2) have been studied extensively in recent years [4,14,28]. A comprehensive treatment of such corrections is beyond the scope of the calculation presented in Ref. [6]. However, one notes that several features belonging to the next order are included in their [6] treatment, like the use of physical masses for the degenerate doublet of heavy mesons in the decay calculations and in propagators. For the latter, as well as for vertices and normalizations the convention of Ref. [4] is used. Thus, the propagator of heavy vector mesons is given by $-i(g^{\mu\nu}-v^\mu v^\nu)/2[(v\cdot k)-\Delta/4]$ and of pseudoscalar mesons by $i/2[(v.k)+3\Delta/4]$, where $\Delta=M_{D^*}(M_{B^*})-M_D(M_B)$ and v,k are the velocity and the residual momentum.

Additional technical points, which should be mentioned are:

(i) the loop calculations for $D^{*o} \to D^o\gamma\gamma$ include also contributions from intermediate states containing $K-$mesons, like $D^{*o} \to (K^- D_s^{*+}) \to D^o\gamma\gamma$ with the photons emitted from the virtual K^-'s;

(ii) contributions from diagrams containing three heavy meson propagators were neglected, as these are very small indeed; diagrams with two heavy meson propagators were included; however their contribution is quite small;

(iii) the contribution of the η^o-anomaly has been estimated and found to be small;

(iv) the off-the-mass-shell q^2-dependence of the anomaly has been neglected.

The calculation described here was performed for neutral D^{*o} decay (likewise for B^*). Obviously there are also the $D^{*+} \to D^+\gamma\gamma$, $B^{*+} \to B^+\gamma\gamma$ and $D_s^{*+} \to D_s^+\gamma\gamma$ decays. For these decays, one has to consider also the bremsstrahlung radiation emitted by the initial or final charged particles. We have estimated these decays and we found that the bremsstrahlung part is orders of magnitude larger than the direct one; hence a different type of analysis is required [29] and we do not address this here.

To summarize, we have shown [6] that the measurement of the $D^{*o} \to D^o\gamma\gamma$ branching ratio constitutes an ideal tool for obtaining the magnitude of the strong g-coupling, since it can be expressed in terms of g only (Eq. 16). This has been achieved by combining a theoretical calculation of $D^{*o} \to D^o\gamma$ using the Heavy Meson Chiral Lagrangian with the experimental information relating the electromagnetic ($D^* \to D\gamma$) and strong ($D^* \to D\pi$) partial decay channels. The various theoretical estimates put g in the range $0.25 < g < 1$. Hence, from Eq. (16) one obtains

g	Br$[(D^{*o} \to D^o\gamma\gamma)/D^{*o} \to$ all$]$
0.25	1.7×10^{-6}
0.38	3.9×10^{-6}
0.5	6.9×10^{-6}
0.7	1.4×10^{-5}
1	3.3×10^{-5}

These figures indicate that the suggested measurement is indeed feasible in the not too distant future.

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