RARE DECAYS OF HEAVY QUARKS — SEARCHING GROUND FOR NEW PHYSICS*

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The search for new physics beyond the standard model is proceeding nowadays intensively along experimental and theoretical lines. We review here the sector of charm radiative decays in this context. The calculation of $D \to V\gamma$, $D \to \ell^+\ell^-\gamma$ transitions reveals their unequivocal dominance by long-distance contributions. On the other hand, the beauty-conserving charm-changing electroweak transition $B_c \to B_u^*\gamma$ is shown to have unique properties which make it a promising avenue in the search for new physics. We describe a calculation of short- and long-distance contributions to this decay which finds them to be of comparable size. The branching ratio of this decay in the standard model is estimated to be $\simeq 10^{-8}$.

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1. Introduction

The standard model (SM) of strong and electroweak interactions [1], based on local gauge invariance with respect to the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, is presently in excellent shape. All experimental data are in agreement with SM and its relentless success is providing an ever increasing challenge to both experimentalists and theorists. The only missing fixture of the model is the SM Higgs boson, for which the existing experimental searches put a lower bound on its mass of 88.6 GeV/c^2 [2].

Despite its remarkable success, it is generally believed that the SM is in fact an effective theory at the energies presently accessible. This belief is fueled by the fact that the SM has about 20 arbitrary parameters and, moreover, there is no satisfactory explanation for many of its salient features. For instance, why does the gauge group of interactions have the structure

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expressed by its three factors? Why three generations of fermions? Why left-right asymmetry? How to explain the observed spectrum of quark and lepton masses and the pattern of mixing angles?

This situation has led theorists to propose many paths for the possible extension of the standard model. I do not plan to go into any detail here on the variety of possibilities, which was reviewed at many conferences [3,4], and I shall restrict myself to the mention of a few of the more widely-discussed proposals: supersymmetry [5], especially the "low-energy" Minimal Supersymmetric Standard Model [6] which is considered as a most likely possibility at the Fermi scale, grand-unified theories [7], right-left models]8], two Higgs doublet models [9], flavour changing neutral Higgs models [10], multiple Z^o bosons [11], and anomalous triple gauge boson couplings [12].

An important feature of the standard model is the flavour symmetry, as the gauge interactions do not distinguish among the three generations of leptons and quarks. In practice, this symmetry is broken, as it is evident from the pattern of masses and mixing angles of the SM fermions. The Higgs boson is an agent of flavour symmetry breaking in SM via its Yukawa couplings to fermions. However, this flavour symmetry breaking is realized in a particular way; the tree-level neutral couplings of the Higgs boson, as well as those of the photon and of the Z^o boson are all flavour diagonal. The observed neutral flavour-changing processes on the other hand are rather small, being made possible in SM by loop graphs only; as such their magnitude is determined by the values of quark masses and of the CKM matrix. In view of the smallness of flavour-changing neutral-current (FCNC) in the standard model, FCNC transitions are usually considered to be a fertile ground for the search of processes induced by new physics, which does not automatically suppress such processes. The charm sector plays a special role in this respect, since as a result of the effectiveness of the GIM mechanism in this sector, the short distance SM contributions to certain charm processes are very small. Accordingly, $D^o - \bar{D}^o$ mixing and rare charm decays have been singled out [4,13] as attractive candidates for the discovery of new physics effects.

In this lecture we consider the potential of the electroweak penguin transitions $c \to u\gamma$ in the search for new physics. To begin I shall review shortly the SM physics of the single quark transition $Q \to q\gamma$, then I shall present the status of short-distance and long-distance contributions in processes driven by $c \to u\gamma$ and $c \to u\ell^+\ell^-$ transitions and finally I shall describe our recent work [14] which singles out the $B_c \to B_u^*\gamma$ decay as a unique tool for the search of effects beyond the standard model.

2. The flavour changing $Q \to q\gamma$ transition

Flavour-changing photon transitions from a heavy quark Q to a light quark q are induced by loop diagrams and are a basic feature of the standard model [15], generally recognized as "electroweak penguins". Typical transitions are $s \to d\gamma$, $b \to s(d)\gamma$ for which up-quarks contribute in the loop, and $c \to u\gamma$, $t \to c(u)\gamma$ which are driven by down-quarks in the loop.

The amplitude for such transitions, with the quarks Q, q on the mass-shell is given by [15]

$$A_\mu^{(Q\to q\gamma)} = \frac{eG_F}{4\pi^2\sqrt{2}} \sum_j V_{jQ}^* V_{jq} \bar{u}(q) \left[F_{1,j}(k^2) k_\mu \not{k} - k^2 \gamma_\mu \right.$$
$$\left. \times \frac{1-\gamma_5}{2} + F_{2,j}(k^2) i\sigma_{\mu\nu} k^\nu M_Q \frac{1+\gamma_5}{2} + m_q \frac{1-\gamma_5}{2} \right] u(Q)\,. \qquad (1)$$

F_1, F_2 are the charge-radius and magnetic form factors respectively and V_{ab} are CKM matrices; F_1, F_2 were first calculated in the electroweak SM by Inami and Lim [16]. The F_1 term does not contribute to decays with real photons, however, it is relevant in leptonic decays like $B \to X(s)\ell\bar{\ell}$, $K \to \pi\ell\bar{\ell}$, $D \to V\ell\bar{\ell}$. In order to compare the calculations of these processes with experiment, one must complement the electroweak SM calculation by the inclusion of QCD corrections [17]. In this section we shall mention the $s \to d\gamma$ and $b \to s\gamma$ transitions and in the next section we turn to the charm sector in more detail.

The contribution of $s \to d\gamma$ to various radiative K-decays [18,19] and hyperon decays [20–25] has been studied extensively in the last twenty years. As it turns out [26] radiative processes which are in the $\sim (10^{-4} - 10^{-7})$ range of branching ratios like $K^+ \to \pi^+\pi^o\gamma$, $K^+ \to \pi^+ e^+ e^-$, $\Sigma^+ \to p\gamma$, $\Xi^- \to \Sigma^-\gamma$ have both short-distance and long-distance contributions and the latter are dominant; this prevents a direct and trustworthy check of the SM or of deviations from it in these decays. In order to investigate the short distance $s \to d$"γ" transition one must turn to very rare decays [27], like $K^+ \to \pi^+\nu\bar{\nu}$, $K_L^o \to \pi^o e^+ e^-$, $K_L^o \to \pi^o\nu\bar{\nu}$. In these, the short-distance contribution is prominent and the QCD corrections to the decay amplitudes have been estimated [28]. The most frequent of these is $K^+ \to \pi^+\nu\bar{\nu}$, which is expected [28] in SM with a branching ratio $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu}) = (9.1 \pm 3.8) \times 10^{-11}$. Recently [29], one event has been detected in this channel, which gives $\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = \left(4.2^{+9.7}_{-3.5}\right) \times 10^{-10}$. The other two decays are expected with branching ratios of the order of 10^{-11} and one must wait for the planned experiments in order to find out whether the $s \to d$"γ" and the box diagrams involved of SM give an accurate picture for these transitions. In the domain of hyperon radiative decays a similar

situation prevails [25]; however, there might be an exception as it appears [22] that the yet unobserved $\Omega^- \to \Xi^-\gamma$ decay is affected in a measurable manner [24,26] by the SM single quark $s \to d\gamma$ transition.

Although the $s \to d\gamma$ was the first to be investigated with the aim of relating it to the observed radiative decays of kaons and hyperons, it is the $b \to s\gamma$ transition [30] which has been the center of attention during the last dozen years. Since it was pointed out [31] that the enhancement provided by QCD corrections to $b \to s\gamma$ (in which the top quark in the loop gives the main contribution) would bring the inclusive $B \to X_s\gamma$ and exclusive $B \to K^*\gamma$ decays into the realm of observability, a considerable amount of theoretical activity has proceeded alongside the experimental observation. The CLEO collaboration was the first to measure the inclusive rate [32] $\mathrm{Br}(B \to X_s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$ as well the exclusive (charged and neutral) decay [33] $\mathrm{Br}(B \to K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. The theoretical effort has been directed on the one hand towards a best determination of the QCD corrections to the inclusive process in SM and on the other hand to establishing the limitations imposed by the observed rate on various "beyond the standard model" theories. For typical Refs. on the latter effort see [34]. The latest theoretical calculations within the SM[35] give $\mathrm{Br}(B \to X_s\gamma) = (3.32 \pm 0.30) \times 10^{-4}$ which should be compared with two recent experimental results: the CLEO update giving [36] $\mathrm{Br}(b \to s\gamma) = [3.15 \pm 0.35\ (\mathrm{stat}) \pm 0.32\ (\mathrm{syst}) \pm 0.26\ (\mathrm{mod})] \times 10^{-4}$ which is derived from an analysis of 3.3×10^6 $B\bar{B}$ pairs and the ALEPH result [37] of $[3.11 \pm 0.80\ (\mathrm{stat}) \pm 0.72\ (\mathrm{stat})] \times 10^{-4}$. Obviously, the agreement with the SM is impressive.

There are two remarks to be made here. Firstly, the conclusion on the excellent agreement with SM assumes that LD contributions are small, which is indeed the result of many calculations (approximately 5-10%) [24]. Secondly, we await for experimental results on the complementary process $b \to s\ell^-\ell^+$ (including $B \to K^*\ell^+\ell^-$, $B \to K\ell^+\ell^-$) which should be compared with SM theoretical expectations of a branching ratio in the 10^{-6} range.

Before turning to the charm sector, we conclude that the study of the $Q \to q\gamma$, $Q \to q\ell^+\ell^-$ transitions in SM is waiting for the measurement of very rare decays in the strangeness domain, while in the beauty sector, where experiments are available, the standard model does very well so far.

3. Short distance $c \to u\gamma$ and $c \to u\ell^+\ell^-$

The $c \to u\gamma$ transition is induced by the electroweak penguin with the down quarks running in the loop. In the absence of QCD corrections this transition is extremely small as a result of the small masses of the quarks in the loop and the smallness of the CKM factors. The electroweak SM calculation [38] gives for this strongly GIM suppressed transition a branching

ratio of $\sim 10^{-17}$ only. Including the QCD corrections at the leading log approximation [38], the C_7 Wilson coefficient of the $\sigma_{\mu\nu}$ operator gets the admixture of C_1, C_2 Wilson coefficients and the amplitude is increased by two orders of magnitude, giving a branching ratio of about 10^{-12}. The calculation of the complete two-loop QCD corrections [39] leads, after using unitarity of CKM, to the folowing effective Lagrangian

$$\begin{aligned} L_{\rm SD}^{c\to u\gamma} &= -\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}V_{cs}V_{us}^* C_7(\mu)\bar{u}\sigma^{\mu\nu}[m_c(1+\gamma_s)+m_u(1-\gamma_s)]cF_{\mu\nu}\ , \\ C_7(m_c) &= 0.0068 - 0.020i\,. \end{aligned} \tag{2}$$

From this expression, another increase of two orders of magnitude in the SD amplitude obtains, giving rise to ${}^{\rm SD}\Gamma(c\to u\gamma)/\Gamma(D^o)\sim 2.5\times 10^{-8}$. This implies that in exclusive modes, like $D\to V\gamma$, the SD contribution to the branching ratio would be about $(3-5)\times 10^{-9}$. In order to ascertain the possibility of detecting the SD transition, one must now consider the size of the LD contribution.

The SD amplitude for $c\to u\ell^+\ell^-$ can be obtained from the general electroweak amplitude [16], and the explicit expression for the effective Lagrangian after certain simplifications is [40]

$$\begin{aligned} L_{\rm SD}^{c\to u\ell^+\ell^-} &= -\frac{G_F}{\sqrt{2}}\frac{e^2 A}{8\pi^2\sin^2\theta_W}\bar{u}\gamma^\mu(1-\gamma_5)c\bar{\ell}\gamma_\mu\ell\ , \\ A &= -0.065\,. \end{aligned} \tag{3}$$

This electroweak transition is not strongly suppressed, in contrast to $c\to u\gamma$ and although the QCD corrections have not been evaluated explicitly, they are not expected to change the value of A appreciably [40]. From (3) one finds ${}^{\rm SD}\Gamma(c\to u\ell^+\ell^-)/\Gamma(D^o)\sim 3\times 10^{-9}$. Hence, like in the $c\to u\gamma$ case, one has to ascertain the LD contribution before one may use these leptonic decays for checking the standard model.

4. *D*-mesons radiative decays — the long distance aspect

Several treatments have addressed recently the problem of estimating LD contributions to radiative D decays. These approaches include a pole model [38], a quark model [41] , the use of QCD sum rules [42] and effective Lagrangians [43]. Already from these works one learns that the $D\to V\gamma$ decays are expected to have branching ratios of the order of $10^{-4}-10^{-6}$, much larger than from the SD part.

In a more comprehensive and systematic treatment for these decays [44] we used an effective hybrid Lagrangian combining heavy quark symmetries

and chiral symmetry [45] to calculate nine decay modes of the $D \to V\gamma$ type. The effective nonleptonic Lagrangian used is given by

$$L_{\rm LD} \;=\; -\frac{G_F}{\sqrt{2}} V_{uq_i} V^*_{cq_i} \left[a_1 (\bar{u} q_i)^\mu (\bar{q}_j c)_\mu + a_2 (\bar{u} c)_\mu (\bar{q}_j q_i)^\mu \right] \tag{4}$$

and for the QCD-induced constants a_1, a_2 we take $a_1 = 1.26$, $a_2 = -0.55$ as determined [46] from nonleptonic D decays. In order to evaluate the matrix elements of (4) we use the factorization approximation for the $\langle V V_o | (\bar{q}_i q_j)^\mu (\bar{q}_k c)_\mu | D \rangle$ amplitudes.

The general gauge invariant amplitude for the decay $D(p) \to V(p_V) + \gamma(k)$ is

$$\begin{aligned} A(D \to V + \gamma) = \frac{eG_F}{\sqrt{2}} V_{uq_j} \cdot V^*_{cq_j} \Big\{ & \epsilon_{\mu\nu\alpha\beta} k^\mu \varepsilon^{*\nu}_{(\gamma)} p^\alpha \varepsilon^{*\beta}_{(V)} A_{\rm PC} \\ & + i \left[(\varepsilon^*_{(V)} \cdot k)(\varepsilon^*_{(\gamma)} \cdot p_{(V)}) - (p_{(V)} \cdot k)(\varepsilon^*_{(V)} \varepsilon^*_{(\gamma)}) \right] \Big\} A_{\rm PV} \,. \end{aligned} \tag{5}$$

In Ref. [44] all diagrams contributing to $A_{\rm PC}$, $A_{\rm PV}$ are classified and their explicit expressions are presented. In Table I below we give the predicted widths [44] as well as the existing experimental upper limits [47]. Since the amplitudes contain several terms, with unknown relative phases, we can present only their expected range. The first two decays in the Table are Cabibbo-allowed, the next five are Cabibbo-forbidden and the last two are doubly forbidden. To give an indication, the photon energy in the first two decays is 717 and 834 MeV respectively. As it is obvious from Table I, in all these decays the LD contribution masks totally the SD one — preventing the detection of deviations from it by orders of magnitude.

TABLE I

$D \to V\gamma$ Transition	Br Ratio $\times 10^5$ [44]	Exp. limits [47]
$D^o \to \bar{K}^{*o}$	6-36	$< 7.6 \times 10^{-4}$
$D_s^+ \to \rho^+$	20-80	
$D^o \to \rho^o$	0.1-1	$< 2.4 \times 10^{-4}$
$D^o \to \omega$	0.1-0.9	$< 2.4 \times 10^{-4}$
$D^o \to \varphi$	0.4-1.9	$< 1.9 \times 10^{-4}$
$D^+ \to \rho^+$	0.4-6.3	
$D_s^+ \to K^{*+}$	1.2-5.1	
$D^+ \to K^{*+}$	0.03-0.44	
$D^o \to K^{*o}$	0.03-0.2	

Turning now to decays of type $D \to V\ell^+\ell^-$, these were also calculated recently [40] using generally the same theoretical framework [45] as for $D \to V\gamma$ transitions. Since the SD transition (Eq. (3)) is considerably larger here than in the $c \to u\gamma$ case before the application of the QCD corrections, one could expect that the gap between SD and LD contributions is narrower for the leptonic decays in the SM. Such a situation could open the window to new physics.

The authors of Ref. [40] have considered the same hadronic transitions as in Table I. The SD contribution due to $c \to u\ell^+\ell^-$ is present in the five Cabibbo suppressed decays $D^0 \to (\rho^o, \omega^o, \varphi^o)\ell^+\ell^-$, $D^+ \to \rho^+\gamma$, $D_s^+ \to K^{*+}\gamma$ while in the other four decays signals for new physics might come from more exotic contributions. The calculation is performed [40] again using factorization for matrix elements of (4) which leads to three classes of diagrams: the annihilation contribution, the V_o-spectator part and the V-spectator part, where V is the final state particle and V_o an intermediate vector meson (ρ, ω, φ). There are thus two kinds of LD contributions: the *resonant* mechanism, where in addition to V also V_o is produced in the final state and converts to a photon through vector meson dominance, and a *nonresonant* mechanism with the photon emitted directly from the initial D state, as prescribed by the structure of the hybrid lagrangian [45]. The latter should contain in our approach also possible contributions from intermediate $c\bar{c}$ states. The predicted branching ratios for $D \to V\mu^+\mu^-$, including SD + LD contributions, and the exisiting experimental upper limits are given in Table II. The range in column two is due to coupling parameter uncertainties.

TABLE II

$D \to V\mu^+\mu^-$	Calculation [4] of Br(LD+SD)	Exp. limits [48]
$D^o \to \bar{K}^{*o}$	$(1.6 - 1.9) \times 10^{-6}$	$< 1.18 \times 10^{-3}$
$D_s^+ \to \rho^+$	$(3.0 - 3.3) \times 10^{-5}$	
$D^o \to \rho^o$	$(3.5 - 4.7) \times 10^{-7}$	$< 2.3 \times 10^{-4}$
$D^o \to \omega^o$	$(3.3 - 4.5) \times 10^{-7}$	$< 8.3 \times 10^{-4}$
$D^o \to \varphi^o$	$(6.5 - 9.0) \times 10^{-8}$	$< 4.1 \times 10^{-4}$
$D^+ \to \rho^+$	$(1.5 - 1.8) \times 10^{-6}$	$< 5.6 \times 10^{-4}$
$D_s^+ \to K^{*+}$	$(5.0 - 7.0) \times 10^{-7}$	$< 1.4 \times 10^{-3}$
$D^+ \to K^{*+}$	$(3.1 - 3.7) \times 10^{-8}$	$< 8.5 \times 10^{-4}$
$D^o \to K^{*o}$	$(4.4 - 5.1) \times 10^{-9}$	

The short-distance contributions alone are $\sim 10^{-9}$ for $D^o \to \rho^o(\omega^o)\mu^+\mu^-$, 5×10^{-9} for $D^+ \to \rho^+\mu^+\mu^-$ and 1.6×10^{-9} for $D_s^+ \to K^{*+}\mu^+\mu^-$, hence between 2 and 3 orders of magnitude lower than the total Br. The situation is therefore more favourable than in the $D \to V\gamma$ case. Branching ratios well above 10^{-6} for $D^o \to (\rho^o, \omega^o)\mu^+\mu^-$ or in the 10^{-5} range for $D^+ \to \rho^+\mu^+\mu^-$ would be indicative of new physics. It is satisfactory to note that present experimental bounds are not far above.

Lastly, we mention the $D^{+,o} \to \pi^{+,o}\ell^+\ell^-$ decays, whose short distance contribution is again related to $c \to u\ell^+\ell^-$. In this case, the LD contribution reaches [49] a branching ratio of the order of 10^{-6} in the φ-resonance region and a few times 10^{-7} in the nonresonant region, a situation similar to what was encountered in $D \to V\ell^1\ell^-$ decays.

5. $B_c \to B_u^*\gamma$ — a unique opportunity

The situation described in the previous sections indicates that the probability of observing new physics in $D \to V\gamma$, $D \to V\ell^+\ell^-$ or $D \to P\ell^+\ell^-$ is rather modest. It would require a mechanism which increases the SD amplitude of $c \to u\gamma$ or $c \to u\ell^+\ell^-$ by at least one or two orders of magnitude, a rather unlikely though not impossible proposition.

Fajfer, Prelovsek and Singer [14] have turned to the domain of very rare decays and have proposed the idea of exploring the $c \to u\gamma$ transition when c is embedded in a beauty particle. In other words, they consider a "beauty-conserving" and "charm-changing" decay, which is driven by the $c \to u\gamma$ transition. As it has been shown by these authors explicitly, such a transition has about equal SD and LD contributions, making it an ideal testing ground for deviations from SM [14,50].

The B_c-meson, a compact bound state of two heavy quarks of different flavour, c and $\bar{b}$, has been discovered recently at Fermilab [51] and its lifetime has been determined as $\tau(B_c) = 0.46^{+0.18}_{-0.16} \pm 0.03$ps. The transition $c \to u + \gamma$ would lead to the decay $B_c \to B_u^* + \gamma$, in which the $\bar{b}$-quark is merely a spectator. In order to estimate the SD and the LD contributions to the decay one uses the effective Lagrangians of (2) and (4). In (2), the appropriate scale for $C_7(\mu)$ is indeed $\mu = m_c$ also for the decay $B_c \to B_u^* + \gamma$, and not m_b, in view of the spectator role of the $\bar{b}$-quark. The general form of the decay amplitude is as given in Eq. (5) and we turn now to the calculation of $A_{\rm PC}$ and $A_{\rm PV}$, which have both SD and LD contributions.

6. A model for $B_c \to B_u^* \gamma$

The SD contribution calculated from (2) can be expressed in terms of two form factors $F_1(0)$, $F_2(0)$:

$$\varepsilon_\mu^* \langle B_u^*(p', \varepsilon') | \bar{u} i \sigma^{\mu\nu} q_\nu c | B_c(p) \rangle_{q^2=0} = i \epsilon^{\mu\nu\alpha\beta} \varepsilon_\mu^* \varepsilon_\nu^{'*} p'_\alpha p_\beta F_1(0) \ , \tag{6}$$

$$\varepsilon_\mu^* \langle B_u^*(p', \varepsilon') | \bar{u} i \sigma^{\mu\nu} q_\nu \gamma_5 c | B_c(p) \rangle_{q^2=0} = \Big[(M_{B_c}^2 - M_{B_u^*}^2) \varepsilon^* \cdot \varepsilon^{'*} - 2(\varepsilon^{'*} \cdot q)(p \cdot \varepsilon^*) \Big] F_2(0). \tag{7}$$

The LD contributions may be separated into two classes related to the two terms of (5). The class (I) is related to the a_2 term and represents processes $c \to u \bar{q}_i q_i$ followed by $\bar{q}_i q_i \to \gamma$, with $\bar{b}$ as spectator. The $\bar{q}_i q_i \to \gamma$ transitions are expressed by $\bar{q}_i q_i$ hadronization into vector meson, thus we have a vector meson dominance (VMD) approximation. The class II of diagrams is related to the a_1 term and corresponds to the quark process $c\bar{b} \to u\bar{b}$ with the photon attached to quark lines. Only the lowest (pole) states are included in the calculation [14,50].

VMD amplitudes of class I are proportional to $\varepsilon_\mu^* \langle B_u^* | \bar{u} \gamma^\mu (1 - \gamma_5) c | B_c \rangle$ taken at $q^2 = 0$. This involves one vector and four axial-vector form factors. However, requirements of finiteness at $q^2 = 0$ [46] and gauge invariance imply [14] the vanishing of two axial form factors and a relation between the other two and accordingly the VMD contribution is expressible in terms of two form factors only, $V(0)$ and $A_1(0)$. The amplitudes thus obtained in [14] are

$$A_{\rm PV} = -\frac{G_F}{\sqrt{2}} e \left(V_{cs} V_{ud}^* \left[\frac{C_7(m_c)}{2\pi^2} (m_c - m_u) F_2(0) + 2 a_2(m_c) C_{\rm VMD}^1 \frac{A_1(0)}{M_{B_c} - M_{B_u^*}} \right] \right) , \tag{8}$$

$$A_{\rm PC} = -\frac{G_F}{\sqrt{2}} e \left(V_{cs} V_{ud}^* \left[\frac{C_7(m_c)}{4\pi^2} (m_c + m_u) F_1(0) + 2 a_2(m_c) C_{\rm VMD}^1 \frac{V(0)}{M_{B_c} + M_{B_u^*}} \right] + V_{cb} V_{ub}^* a_1(m_b) \times \left[\frac{\mu_{B_c} g_{B_c^*} g_{B_u^*}}{M_{B_c^*}^2 - M_{B_u^*}^2} + \frac{\mu_{B_u} M_{B_c}^2 f_{B_c} f_{B_u}}{M_{B_c}^2 - M_{B_u}^2} \right] \right) . \tag{9}$$

In these expressions the first term is from SD, the second is the LD VMD contribution and the third term is the LD pole contribution. Also,

$$C^1_{\rm VMD} = \frac{g^2_\rho(0)}{2M^2_\rho} - \frac{g^2_\omega(0)}{6M^2_\omega} - \frac{g^2_\varphi(0)}{3M^2_\varphi} = (-1.2 \pm 1.2) \times 10^{-3}{\rm GeV}^2 \qquad (10)$$

and $\langle V(q,\epsilon)|V_\mu|0\rangle = g_V(q^2)\epsilon^*_\mu$. μ_i, f_i and g_i are couplings related to the axial and vector currents and are defined in [14].

In order to determine the form factors $A_1(0)$, $V(0)$, $F_1(0)$, $F_2(0)$ and the various μ_i, f_i, g_i the authors of Ref. [14] have chosen the nonrelativistic constituent Isgur-Score-Grinstein-Wise (ISGW) model [52]. This model is considered to be reliable for a state composed of two heavy quarks; in addition, the velocity of B^*_u in the rest frame of B_c is to a good measure nonrelativistic. In the ISGW model the quarks of mass M move under the influence of the effective potential $V(r) = -4\alpha_s/(3r) + c + br$ with $c = -0.81$ GeV, $b = 0.18$ GeV^2 [53]. The authors [14] use variational solutions of the Schrödinger equation, $\psi(\vec{r}) = \pi^{-\frac{3}{4}}\beta^{\frac{3}{2}}e^{\frac{-\beta^2 r^2}{2}}$ for S state with β as variational parameter. Using accepted values for current quark masses, CKM matrix elements and constituent quark masses, one calculates the SD and the LD contributions separately, as well as the total branching ratio of $B_c \to B^*_v\gamma$. It is found [14]:

TABLE III

	$\mathrm{Br}^{(\mathrm{SD})}$	$\mathrm{Br}^{(\mathrm{LD})}$	$\mathrm{Br}^{(\mathrm{tot})}$
$B_c \to B^*_u\gamma$	4.7×10^{-9}	$\left(7.5^{+7.9}_{-4.3}\right) \times 10^{-9}$	$\left(8.5^{+5.8}_{-2.5}\right) \times 10^{-9}$

As evidenced by the results of Table III, the SD and LD contributions are comparable, which in principle allows one to probe the $c \to u\gamma$ transition in $B_c \to B^*_u\gamma$ decay. Experimental detection of $B_c \to B^*_u\gamma$ at a branching ratio well above 10^{-8} would clearly indicate a signal for new physics. It is worth mentioning here that at LHC one expects [50] to produce well above $10^8 B_c$ mesons.

Finally, we mention a recent calculation of Aliev and Savci [54] which confirms our conclusions [14,50]. They calculate the SD contribution to $B_c \to B^*_u\gamma$ by the use of QCD sum rules and find a value for $F_i(0)$ which leads to an SD branching ratio for $B_c \to B^*_u\gamma$ of $\sim 1.6 \times 10^{-8}$, slightly higher than presented above, but with the same general conclusions.

7. Summary

We have reviewed the possibility of using various processes to detect deviations from the standard model in the charm sector, using the $c \to u\gamma$, $c \to u\ell^+\ell^-$ transitions. The $D \to V\gamma$ decays are shown to be dominated by long distance contributions which usually prevents one from observing deviations from the standard model short distance ones. The situation is somewhat better in $D \to V\ell^+\ell^-$ decays, where the gap between SD and LD is smaller. Here, branching ratios well above 10^{-6} for $D^o \to \rho^o\mu^+\mu^-$ or $D^o \to \omega^o\mu^+\mu^-$ or in the 10^{-5} range for $D^+ \to \rho^+\mu^+\mu^-$ would indicate new physics. Of particular interest is the novel decay $B_c \to B_u^*\gamma$ suggested in Ref. [14]. In this decay both the SD and LD contributions to the branching ratio are in the 10^{-8} range. The SD contribution is at its natural value. The LD one is strongly suppressed, as follows: the Class I VMD contribution is very small as a result of the smallness of $C^1_{\rm VMD}$ (Eq. 10), which represents a cancellation of vector mesons contributions at a level better than 10% as a result of GIM and $SU(3)_F$ symmetry; on the other hand, the class II pole contributions is also strongly supressed in view of the appearance of the factor $V_{cb}V_{ub}^*$ in the $c\bar{b} \to u\bar{b}$ pole diagrams. (In D decays we had the much bigger $V_{cs}V_{us}^*$ factor, which made the LD pole contributions dominant). This fortuituous occurrence of SD, LD contributions equality establishes the $B_c \to B_u^*\gamma$ decay mode as an ideal testing ground for physics beyond the standard model. To conclude, we stress that this decay has a clear signature: the detection requires the observation of a B_u decay in coincidence with two photons – a high energy one (985 MeV) and a low energy photon (45 MeV) in the respective centers of mass of B_c and B_u^*.

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