APACIC++, A PARTON CASCADE IN C++*

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The newly developped Monte-Carlo event generator APACIC++ suitable to describe multijet-events in high-energetic electron–positron annihilations is presented. A new ansatz to match the corresponding matrix elements for the production of jets via the strong and electroweak interactions to the subsequent parton shower modelling the inner-jet evolution is discussed in some detail. Results obtained with APACIC++ are compared to other QCD event generators and to some representative experimental data.

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1. Introduction

For decades, electron–positron collisions have been an extensively used testing ground for quantum field theory and particle physics. Especially e^+e^--annihilations into hadrons at high energies proved to be of continuos interest.In general, such events can be divided into three steps. First,

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a number of partons is produced at a scale of the order of the c.m. energy of the incoming electron–positron pair. Here, the standard method of perturbative quantum field theory of summing and squaring amplitudes related to corresponding Feynman-diagrams is applicable. In a second step, these primary partons loose virtual mass and energy by radiating additional partons giving rise to jets. Because of the possibly high and varying number of particles involved here one has to abandon the idea of summing the full amplitudes. Instead one considers only the limits of soft and small angle emissions resulting in a probabilistic description of jet-evolution as a chain of nearly independent single emissions in the perturbative regime of strong interactions. These radiations stop in a third step at some infrared scale of the order of a few Λ_{QCD} and hadronization sets in. Since this is essentially a non-perturbative, soft process it is usually modelled by some parameter-dependent phenomenological hadronization scheme, which does not alter the density- and energy-distribution of particles in phase-space drastically. However, the parameters entering the model are to a large extent scale-dependent. Therefore the jet-evolution via the parton shower has the additional purpose to connect the high-energy scale of jet-production with the low-energy scale of hadronization and thus guarantees the universality of the hadronization scheme once the parameter are fixed to fit the data.

Monte Carlo event generators are perfectly capable to model high--energetic e^+e^--annihilations into hadrons by means of the three steps as described above. In this respect, they are an indispensable tool to bridge the gap between theoretical considerations concering the dynamics of such events and their experimental observation and to provide testable signatures in a well-defined manner.

With rising energies, however, an increasing number of particles and of jets is produced, and the production, observation and theoretical description of such multijet-events is one of the cornerstones of current particle physics. Various reasons feed this interest and we would like to highlight only briefly some of them.

First of all, large parts of what is known as the Standard Model has been tested via multijet-events. QCD has been established as the correct gauge theory underlying the strong interactions by means of measurements [2] of the Casimir-operators [1]

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}\,, \quad C_A = N_c = 3 \tag{1.1}$$

of the fundamental and adjoint representation and by measuring the overall normalization

$$\mathrm{Tr}\left[T^a T^b\right] = \delta^{ab}\, T_F = \tfrac{1}{2} \tag{1.2}$$

of its generators. In addition, the electroweak sector of the Standard Model has been tested exhaustively by precision measurements of observables like for instance the widths of the gauge bosons and by establishing the non-Abelian structure of the gauge group via proving the existence of the triple gauge boson vertices.

Second, multijet-events open the door to new physics. For example, the Higgs-boson of the Standard Model has some clear signatures in e^+e^--collisions in so-called Higgsstrahlungs-processes [3] resulting in at least four final state fermions which may form jets. Of course, there is a large variety of other interesting signatures connected with cross-sections which depend sensitively on the number n_f of active flavours in the case of strong interacting particles [4].

Therefore it is of some interest, to have at hand some event generator capable to deal with such multijet-events. One of the major obstacles on that road is the question of how to match the parton shower responsible for jet-evolution and going down to the hadronization-scale with the matrix elements describing the high-energetic production of the jets, since only this guarantees the universality of the hadronization-scheme used and hence the predicitive power of the event generator. Within `APACIC++` we have implemented a new ansatz to that question enabling this code to deal with multijet-events due to the strong or electroweak interactions. So the outline of this article is as follows: In Section 2 we would like to introduce briefly some concepts and tools related to the perturbative treatment of jet-production via matrix elements and their implementation in `APACIC++` . The parton shower picture of jet-evolution and the way `APACIC++` handles it is discussed in Section 3. There, we briefly compare matrix elements and the parton shower and their regimes of validity. In Section 4 we will comment on two main approaches to the question of matching. Additionally, we will discuss in some length the ansatz used by `APACIC++` . We want to justify this ansatz in Section 5 by considering some results of `APACIC++` and comparing them to experimental data and the results obtained from other event generators. Finally, we would like to conclude in Section 6.

2. Jet-production in `APACIC++`

2.1. General features

Usually, the hadrons produced in e^+e^--annihilations are clustered in jets, objects separated in phase-space by some jet-measure. Popular jet-measures are the JADE- [5] and the DURHAM-scheme [6], defined by

$$
\begin{aligned}
(p_i+p_j)^2 = 2E_iE_j\,(1-\cos\theta_{ij}) \;&>\; y_{\rm cut}\,s_{ee}^{(0)} \quad \text{(JADE)} \\
2\min\{E_i^2,E_j^2\}\,(1-\cos\theta_{ij}) \;&>\; y_{\rm cut}\,s_{ee}^{(0)} \quad \text{(DURHAM)}
\end{aligned}
\tag{2.1}
$$

for two massless particles to belong to different jets. The parameter $y_{\rm cut}$ is a measure for the hardness of the jet. Within perturbation theory, the emergence of jets is described by the appropriate matrix elements for $e^+e^- \to n$ partons thus identifying jets with hard produced partons. Evaluating the corresponding cross sections in the standard way by squaring amplitudes and integrating over the phase space available one is, even at the tree-level, confronted with divergencies. Beyond the tree-level more divergencies occur due to additional loops or legs and have to be treated. Here, we would merely like to state, that mutual cancellations of the divergencies due to loops and legs connect topologies of varying numbers of legs and pose a major obstacle to any calculation beyond the tree-level. Some of the recent results can be found for instance in [7,8].

However, at the moment `APACIC++` deals with tree-level matrix-elements only. They can be kept finite quite easily by merely subjecting the initial partons to the condition that they form well-separated jets, *i.e.* by applying the restrictions of Eq. (2.1) to the integration over the final-state phase-space. It is not much of a surprise that the corresponding jet cross sections now are finite in Leading Order and become divergent for $y_{\rm cut} \to 0$.

Consequently a choice of this initial $y_{\rm cut}$ yields a parametrization of the reliability of LO matrix elements, and softer parton emissions are supposed to be better described by the appropriate Sudakov form factor, see Section 3. To summarize, this treatment is nothing else but the statement, that `APACIC++` considers jets to be entities whose production can be described in a reliable and controllable manner by traditional perturbation theory, *i.e.* by matrix elements.

Some of the efffects of higher order QCD-corrections are implemented in `APACIC++` by an overall factor $\kappa_s < 1$ for the scale of the strong coupling constant. Some similar treatment can be found for instance in [9,10]. This factor is a fit parameter for the scale of α_s used within the matrix elements in the form

$$
\alpha_s^{\rm M.E.} = \alpha_s(\kappa_s s)\,. \tag{2.2}
$$

`APACIC++` uses common the LO running of α_s and the quark masses (see for instance [1,11]), where the scale of the latter ones is not affected by κ_s.

2.2. Defining relative rates

Within APACIC++ there are matrix elements for the production of two and three QCD-jets via the exchange of a photon or a Z. Denoting the cross-sections by $\sigma_{q\bar{q}}$ and $\sigma_{q\bar{q}g}$, respectively, the corresponding rates are given by

$$\mathcal{R}_3 = \frac{\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}}\,, \quad \mathcal{R}_2 = 1 - \mathcal{R}_3\,, \tag{2.3}$$

based on the probabilistic picture of 3-jets being an exclusive subset of the inclusive production of hadrons [11]. When dealing with higher numbers of jets produced by QCD only, one is left with the task to extend this scheme in a sensible manner. Within APACIC++ we provide at least three schemes, namely a "direct" scheme, and two "rescaled" ones, which we denote by "rescaled1" and "rescaled2"

$$\begin{aligned} \mathcal{R}_n^{\mathrm{dir}} &= \frac{\sigma_n^{\mathrm{tot}}}{\sigma_{q\bar{q}}}\,, \\ \mathcal{R}_n^{\mathrm{res1}} &= \frac{\sigma_n^{\mathrm{tot}}}{\sigma_{q\bar{q}}} \cdot \prod_{m>n} (1 - \mathcal{R}_m^{\mathrm{re}}) \end{aligned}$$

or

$$\mathcal{R}_n^{\mathrm{res2}} = \mathcal{R}_n^{\mathrm{dir}} - \mathcal{R}_{n+1}^{\mathrm{dir}}\,, \tag{2.4}$$

where the last one uses the direct rates $\mathcal{R}_n^{\mathrm{dir}}$ and the corresponding rescaling applies for $n < n_{\max}$. The related rate $\mathcal{R}_{n_{\max}}$ remains unchanged.

It has to be stressed here, that these schemes are obviously by no means consistent in α_s, *i.e.* perturbation theory. Instead the evaluation of the rates and consequently the admixture of different jet-numbers within APACIC++ is to some extent just a phenomenological model with κ_s the parameter to be fitted to data.

Of course the situation above with QCD only changes drastically taking into account the production of jets via more than one electroweak gauge boson, for example when considering four-jet production via W-, Z- or Higgs-bosons beyond the corresponding thresholds. Currently this situation is handled in the following way. If the electroweak production of four or more jets is taken into account, the cross sections and the corresponding rates are divided into two sets. The first subset is defined by four or more fermions in the final state (electroweak subset), the second set is the conjugate subset (QCD subset). Interferences occuring between both of them, for example if an internal Z- or γ-line is replaced by a gluon, are awarded to the electroweak set. Then the rate of the first set is obtained by the sum of the corresponding cross sections and the rate of the second set still is defined via the cross section for the inclusive production of a quark–antiquark

pair. Within the electroweak subset the single rates are determined by the appropriate cross sections, within the QCD subset the determination of the relative rates is achieved in the fashion of Eq. (2.4).

2.3. Multijet-matrix elements available

To allow for the formation of higher jet-configurations we have added three matrix element generators.

1. In its present state, `AMEGIC++` [12] describes the production of up to five massive jets via the strong or electroweak interaction in Leading Order.

   Recently, the production of up to five jets via the strong interaction has been successfully tested. Results obtained by [10] in this channel have been reproduced for both massless and massive quarks and over the full ranges of the two jet-measures considered, namely the JADE- and the DURHAM-scheme. Additionally, the production of four jets by the exchange of two electroweak gauge bosons (W, Z or γ) has been tested by reproducing some of the results of [14].

2. `DEBRECEN` [13] accounts for the QCD-production of up to 5 jets in Leading Order and up to 4 jets in Next-to Leading Order.

3. `EXCALIBUR` [14] describes processes with 4 quark-jets, generated via strong or electroweak interactions in Leading Order.

One of them, `AMEGIC++` , has not been published yet. In its final version it is meant to allow for the production of up to six massive jets in all Standard Model channels including the full electroweak and Higgs-sectors. `AMEGIC++` uses the helicity method of [10] originally proposed in [15].

For all of the matrix element generators `APACIC++` provides interfaces. In addition `APACIC++` and `AMEGIC++` , respectively, allow for the inclusion of QED-Coulomb corrections to the production of heavy particles near the threshold [16] and for some initial state radiation of photons off the electrons [17].

3. Final state parton shower

3.1. Space-time picture and LLA

We would like to dwell on the jet-evolution of the initial partons produced by the appropriate matrix elements. Here, the common approach of evaluating and squaring amplitudes fails due to the high number of particles involved. To deal with this, one restricts oneself to the kinematical enhanced

regions of small angles and low energies of the emitted particles. This allows for the probabilistic construction of the jet-evolution in terms of subsequent independent branchings of one parton into two. The kinematical enhancement is a common feature of all field theories with massless bosons, where the regions of soft and collinear emissions give rise to the corresponding divergencies. To illustrate this point, we would like to consider the amplitude squared of a $(N+1)$-particle matrix element obtained via one additional radiation from a N-particle matrix element [11],

$$|\mathcal{M}_{N+1}|^2 \propto \frac{\alpha_s}{t_a} C\, P_{ba}(z)\, |\mathcal{M}_N|^2 \,, \tag{3.1}$$

where C is some appropriate colour factor, t_a the virtual mass of the particle a decaying into b and c, and $P_{ba}(z)$ is the corresponding splitting function depending on the energy fraction z particle b carries away. People familiar with the splitting functions will appreciate the fact, that within the framework of QCD event-generators, the notorious divergencies related to the limits $z \to 0$ and $z \to 1$ are regularized kinematically in quite a natural way by imposing some minimal virtual mass for any outgoing parton. As can be deduced directly from Eq. (3.1) each decay process $a \to bc$ may be described by means of two variables only, namely t_a and z. Note, that there are different possibilities to interpret those two parameters and they refer to different schemes of organizing the parton shower to be reviewed later on. To specify the process kinematics of the decay $a \to bc$ completely, an additional azimuthal angle ϕ of the decay plane around the direction a is needed. As a first guess ϕ is distributed isotropically, but rather weak spin correlations of two subsequent branching processes lead to some non-trivial plane correlation, which is included in `APACIC++` , too [18].

However, the cross section related to the process of Eq. (3.1) can now be written as

$$d\sigma_{N+1} \propto \alpha_s\, dz \frac{dt_a}{t_a}\, \hat{P}_{ba}(z)\, d\sigma_N \,. \tag{3.2}$$

Iterating this equation it is easy to see that a strong ordering of the virtual masses related to subsequent emissions yields the largest enhancement of the form

$$d\sigma_N \propto d\sigma_0\, \alpha_s^n \int\limits_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int\limits_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \cdots \int\limits_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} = d\sigma_0\, \frac{\alpha_s^n}{n!} \left(\log \frac{Q^2}{Q_0^2}\right)^n \,, \tag{3.3}$$

with Q^2 the hard scale of the first parton taking part in the jet evolution and Q_0^2 the infrared scale characterizing usually the onset of hadronization.

This can be plugged into a form suitable for the implementation within a code by considering first the well-known DGLAP-Equation [19]

$$t\,\frac{\partial}{\partial t}\,q(x,t) = \int dz\,\frac{\alpha_s}{2\pi}\,P(z)\,\left[\frac{1}{z}\,q\left(\frac{x}{z},t\right) - q(x,t)\right] \tag{3.4}$$

describing the evolution of a parton density $q(z,t)$ inside a hadron. Introducing the Sudakov-form factor [20]

$$\Delta(t,t_0) \equiv \exp\left\{-\int\limits_{t_0}^{t}\frac{dt'}{t'}\int\limits_{\varepsilon}^{1-\varepsilon} dz\,\frac{\alpha_s}{2\pi}\,P(z)\right\}, \tag{3.5}$$

one is able to construct an evolution equation similar to the DGLAP-equation and to rewrite it as an integral equation,

$$\begin{aligned} q(x,t) \;&=\; \Delta(t,t_0)\,q(x,t_0) \\ &\quad+\int\limits_{t_0}^{t}\frac{dt'}{t'}\,\frac{\Delta(t,t_0)}{\Delta(t',t_0)}\int\limits_{\varepsilon}^{1-\varepsilon}\frac{dz}{z}\,\frac{\alpha_s(p_\perp^2)}{2\pi}\,P(z)\,q\left(\frac{x}{z},t'\right), \end{aligned} \tag{3.6}$$

allowing for a probilisitc interpretation of the Sudakov form factor $\Delta(t,t_0)$ as the probability, that no branching occurs between t and t_0. This interpretation is further motivated by the observation, that

$$\Delta(t_0,t_0) = 1 \text{ and } \quad \Delta(t,t') = \frac{\Delta(t,t_0)}{\Delta(t',t_0)}\,. \tag{3.7}$$

In Eq. (3.6) we have included explicitly the scale of the strong coupling constant in terms of the transversal momentum, which is in LLA given by

$$p_\perp^2 = z(1-z)t\,. \tag{3.8}$$

As mentioned above, there is some minimal virtual mass for each parton, t_0, regularizing the divergencies via limiting the z-integration

$$\varepsilon(t) \le z \le 1-\varepsilon(t)\,, \quad \varepsilon(t) = \frac{1}{2} - \frac{1}{2}\,\sqrt{1-4\,\frac{t_0}{t}}\,. \tag{3.9}$$

We have seen, that forcing a strong ordering of the virtual masses results in a resummation of the leading logarithms (leading logarithmic approximation, LLA). The divergencies related to the singular behaviour of the splitting function at the edges of the z-space are cut in this approach. Their resummation is achieved in the modified leading logarithmic approximation (MLLA). Its basic ideas will be discussed below.

3.2. Coherence effects and MLLA

To illustrate the idea of coherence, we want to refer first to the Chudakov-effect of QED [21]. Here, an e^+e^--pair is produced off an initial virtual photon and emits an additional photon, see Fig. 1. Assuming the photon

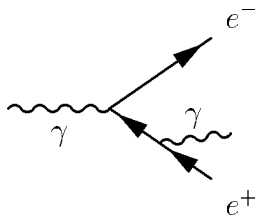

Fig. 1. Emission of a photon by an e^+e^--pair in QED

to stem from the positron, the formation time t_f of this photon can be estimated from the uncertainty principle as

$$t_f \approx \frac{1}{k\theta_{\gamma e}^2} \approx \frac{\lambda_\perp}{\theta_{\gamma e}}, \tag{3.10}$$

where k is the photon momentum, $\lambda_\perp$ the component of its wavelength vector transversal to the positron and $\theta_{\gamma e}$ is the positron–photon angle. To allow this photon to resolve the electron–positron pair and hence to be produced at all, the distance the pair separates during the photon formation should be larger than the transversal wavelength. Therefore we end up with

$$\rho_\perp \approx t_f \theta_{ee} \approx \frac{\theta_{ee}}{\theta_{\gamma e}} \lambda_\perp \geq \lambda_\perp \quad \longrightarrow \quad \theta_{\gamma e} \leq \theta_{ee} \tag{3.11}$$

yielding an angular ordering. Stated the other way around, the emission of a photon is suppressed at angles larger than θ_{ee} since it experiences only the effect of the overall charge of the pair. A similar reasoning applies for QCD and thus motivates the angular ordering of subsequent emissions within jets to model coherence effects like the one of the example above [11,22].

Within `APACIC++` , angular ordering of the jet-evolution is accomplished in two ways. The first approach is to subject subsequent emissions to a hard veto on rising angles of subsequent branchings [23]. The second method utilizes the fact, that it is equally possible to construct a Sudakov form factor for angular ordering as

$$\Delta_i(\zeta E^2) = \exp\left[-\int_{4t_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\varepsilon(\zeta)}^{1-\varepsilon(\zeta)} dz \frac{\alpha_s(z^2(1-z)^2 t')}{2\pi} \hat{P}_{ji}(z) \right] . \tag{3.12}$$

by taking into account not only Leading Logarithmic contributions, but also double leading logarithmic terms. Note, that neglecting some redefinitions of integration variables and adjusting the regions of integration this form is exactly the one of the LLA Sudakov form factor yielding the same interpretation as above.

The new evolution variable is given with the parton's energy E by

$$t' = \zeta E^2 \ , \quad \zeta = \frac{p_b \cdot p_c}{E_b E_c} \approx 1 - \cos\theta_{bc} \ , \quad \varepsilon(\zeta) = \sqrt{t_0/(E^2\zeta)} \tag{3.13}$$

and the transversal momentum is now

$$p_\perp^2 = z^2(1-z)^2\,\zeta E^2 \ . \tag{3.14}$$

An additional remark is in order here. Since the construction of the MLLA Sudakov form factor relies on the assumption of small branching angles as can be deduced from Eq. (3.13), this ordering scheme might not be applicable to the first branchings within a jet, which can very well include regions of $\cos\theta_{bc}$ and ζ yielding a virtual mass $\sqrt{t} = \sqrt{\zeta}E$ of particle a larger than its energy. To cure this problem, within `APACIC++` the first decay is always performed using the LLA form factor. Other codes employ the fact, that $\cos\theta_{bc}$ and ζ are not boost invariant and perform the evolution of the jets in suitable reference frames.

For further details on the construction of the Sudakov form factors in the two ordering schemes, virtualities (LLA) and angles (MLLA) we refer to [24] and to the concise textbook [11].

3.3. Matrix elements vs. parton shower

To compare matrix elements and the parton shower and discuss their regions of reliability, it is sufficient, to stress once again, that the construction of the Sudakov form factor and consequently the organization of the shower relies on the expansion around the soft and collinear limit including proper resummation of the large logarithms attached to each region. Therefore the parton shower performs better than matrix elements in this region. However, vice versa in the region of hard and large-angle emissions we should assume the matrix elements to account for a much better description, since they include interference effects, which become important when leaving the soft and collinear region.

4. Matching of matrix elements and the parton shower

4.1. Basic ideas

We now turn to the question of how to match the matrix elements and the parton shower. Actually, this question can be stated in another way, namely of how to supply the particles produced in the hard process with virtual masses to allow them to radiate additional partons. Since the matrix elements describe the production of on-shell particles only, this question already suggests an interpretation of the virtual masses as order parameters inside the parton shower and as small perturbation not altering anything else. Before we discuss in some detail the answer to the question above as given within `APACIC++` we would like to describe briefly the matching algorithms employed by other event-generators in the framework of e^+e^--annihilations.

In general, two approaches exist. The first possibility is to utilize the matrix elements merely to correct the kinematics of the shower evolution of the two initial partons with the help of a veto-algorithm [25]. This is the approach chosen by `PYTHIA` [9], where at the present state the first radiation according is corrected in a way to reproduce exactly the three-jet matrix element. Of course, this accounts for some of the features of four-jet production as well. Alternatively, one might try to divide the phase space for the emission of partons additional to the two initial ones in two regions, the hard one dominated by the matrix elements and describing the production of further jets, and the soft one modelling the inner jet-evolution [26, 27]. The two regions have to be separated, this is to be achieved by defining and fitting accordingly a fixed matching scale Q^2 determining in some sense the virtualities of the out-going partons. Consequently, below this scale the parton shower governs the emissions, above the matrix elements are responsible. This is the approach chosen within `HERWIG` [28] in the framework of a MLLA parton shower.

`APACIC++` rather follows the second approach of splitting the phase space into two regions. But instead of fixing a scale the matching is achieved in a different way. First of all, we define a $y_{\mathrm{cut}} = y_{\mathrm{crit}}$ characterizing jets at the parton level. Then, since for any hard jet production characterized by $y_{\mathrm{cut}} > y_{\mathrm{crit}}$ the matrix elements do a better job, they are responsible for all such emissions. Reversely, the parton shower performs better in the soft region characterized by relative low y_{cut}. Therefore the parton shower governs all branchings with $y_{\mathrm{cut}} < y_{\mathrm{crit}}$. Thus within `APACIC++` the matching strategy is to use the matrix elements for jet-production and the parton shower for their evolution. The virtual masses of the outgoing partons are always provided by the Sudakov form factor and subjected to the condition that the parton shower does not produce any additional jet as specified by y_{crit} [29].

4.2. Matching procedure

Invoking the example of four jet-production we will now explain in some detail the single steps of the matching procedure used by `APACIC++` . The relevant graphs are depicted in Fig. 2.

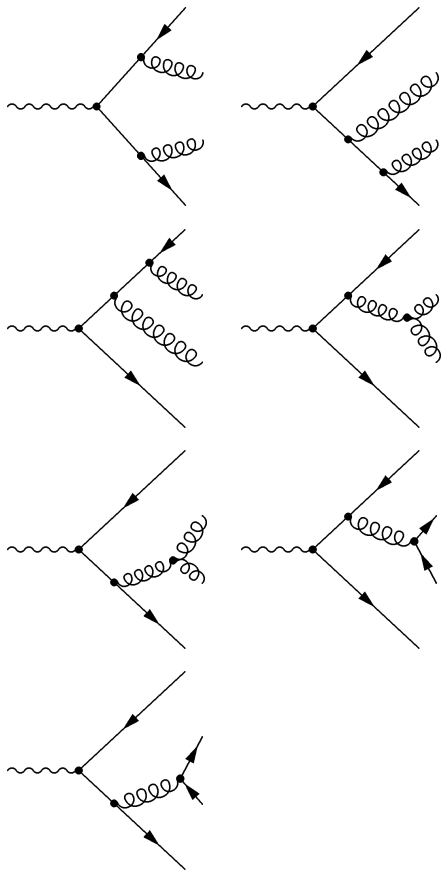

Fig. 2. Feynman graphs contributing to $e^+e^- \to$ four jets at LO

1. Step: Choice of number of jets and flavours

Presuming that we have chosen a sensible $y_{\rm crit}$ for the evaluation of the matrix elements we are able to choose the number of jets according to the rates given above, Eq. (2.4). Assume that we are left with four jet-production, than two possible final states are

$$e^+e^- \to q\bar{q}q'\bar{q}' \quad \text{and} \quad e^+e^- \to q\bar{q}gg \tag{4.1}$$

which do not mix and can therefore be chosen according to their relative cross sections. Therefore we will consider in the following the latter combination $q\bar{q}gg$ only.

2. Step: Choice of a specific parton history

Within the framework of Monte Carlo methods aiming merely at the correct average it is perfectly justified to choose now one of the five remaining topologies to provide the partons with virtual masses and to account for the correct colour statistics. Various possibilities exist for this choice, "a winner takes it all"-strategy with regard to the relative probabilities of the individual topologies encountered as well as an equal probability distribution between the five diagrams. Within `APACIC++` , however, we choose the diagram according to the relative probabilities.

In principle there are various possibilities to define relative probabilities of Feynman-diagram like topologies. Within `APACIC++` we have implemented two. First, if available, the probabilities $\mathcal{P}_i$ of each of the five topologies can be defined as the squares of the corresponding subamplitudes $\mathcal{M}_i$, namely

$$\mathcal{P}_i = |\mathcal{M}_i|^2 \ . \tag{4.2}$$

The second possibility applies for example for `DEBRECEN` where the individual subamplitudes are not supplied. Then, the relative probabilities are reconstructed using the parton shower in the fashion of [25]. Consider the topologies depicted in Fig. 3. Its relative probability $\mathcal{P}$ can be defined as

$$\mathcal{P} = \mathcal{P}_{1\to 34}\mathcal{P}_{4\to 56} = \frac{1}{t_1} P_{qg}(z_{34}) \ \frac{1}{t_4} P_{gg}(z_{56}) \,, \tag{4.3}$$

with the $P(z)$ the well-known splitting functions, the t_i the squares of the corresponding four momenta and the z_i the usual energy fractions.

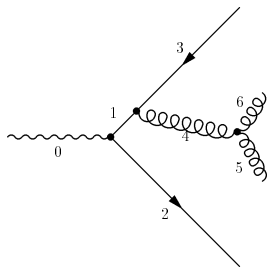

Fig. 3. Typical graph for $e^+e^- \to$ four jets at LO

3. Step: Providing virtual masses Having chosen a specific topology it is easy to supply the outgoing partons with virtual mass invoking the parton shower picture as determined by the Sudakov form factor. The starting virtuality for each evolution downwards is given by the kinematics of the topology. for example in Fig. 3, the virtual mass of parton 4 is given by summing and squaring the known four-momenta of partons 5 and 6 and this virtuality t_4 is employed to determine t_5 and t_6. Both of them are subject to the condition, that no further jets are produced by any subsequent branching of them. The same procedure yields virtual masses to any $q\bar{q}$ pair in two jet production, where the starting scale is given by the invariant mass of the intermediate vector boson.

4. Step: Correcting the kinematics Since we want to guarantee four-momentum conservation, the only task left is to account for the slight changes in the kinematics due to the fact, that the outgoing partons now have acquired a virtual mass. Considering subkinematics $a \to bc$ the corrected four momenta $p_i^{\rm cor}$ are given by

$$p_{b,c}^{\rm cor} = p_{b,c}^{(0)} \pm \left(r_c p_c^{(0)} - r_b p_b^{(0)} \right) , \tag{4.4}$$

where for the various r_i one has to encounter the following two cases:

1. Case 1: b is an internal line, c is outgoing.

$$\begin{aligned} r_b &= \frac{t_a + (t_c - t_b) - \lambda}{2t_a} , \\ r_c &= \frac{t_b(t_b - t_c + \lambda) - t_a(t_a - t_c - \lambda)}{2t_a(t_b - t_a)} . \end{aligned} \tag{4.5}$$

2. Case 2: b and c are outgoing.

$$r_{b,c} = \frac{t_a \pm (t_c - t_b) - \lambda}{2t_a} . \tag{4.6}$$

Obviously, not only the virtual masses are provided. Additional changes alter slightly the z and tend to narrow the angels θ_{bc}. However, a careful study, like for instance of the various four-jet correlation angles [2] showed that these are only minor changes.

5. Results

We have performed a comparison of a variety of observables at a c.m. energy of 91 GeV at the level of matrix elements, parton showers and hadrons using `PYTHIA` [9], `HERWIG` [28] and our event generator `APACIC++`. for the latter we used matrix elements for the production of up to five jets via QCD provided by `AMEGIC++` . For `APACIC++` we employed the string hadronization [30] in the form of [31] by linking the corresponding routines of JETSET to our code. We did not take into account any initial state radiation.

We would like to divide the presentation and discussion of results into two parts, one part flashing over some representative event-shape observables and the like, proving clearly, that `APACIC++` is perfectly capable to reproduce the experiment. In the other part we will restrict ourselves to the parton level only and show, that our matching formalism has some clear benefits describing the topological structure of four jet-events.

5.1. Comparison with event-shapes

Comparing results of `PYTHIA` and `APACIC++` with each other and experimental data provided by the DELPHI collaboration we found an encouraging agreement for most of the observables. For `PYTHIA` we employed as an additional channel we denote by `JETSET` the matrix elements provided there with subsequent hadronization without intermediate parton shower. For a representative extract of various event shape observables see Fig. 4. There, we depict the sphericity distribution, the $1-$thrust distribution as well as the inner and outer transversal momentum distribution. Additionally we depict the aplanarity and the rapidity with respect to the thrust axis. Obviously the results of `APACIC++` are in pretty good agreement with data indicating that our approach to match matrix elements and parton showers describes reasonably the interplay of various numbers of jets as well as the overall features of e^+e^- events.

We would like to introduce briefly the observables we display to the reader unfamiliar with them. For this purpose we consider first the tensor constructed out of the three-momenta p of final-state particles,

$$S^{\alpha\beta} = \frac{\sum_i (p_i^\alpha p_i^\beta)}{\sum_i p_i^2}, \quad \alpha, \beta = 1, 2, 3\,, \tag{5.1}$$

with eigenvalues $\lambda_{1,2,3}$. The combination

$$S = \frac{3}{2}(\lambda_1 + \lambda_2) \tag{5.2}$$

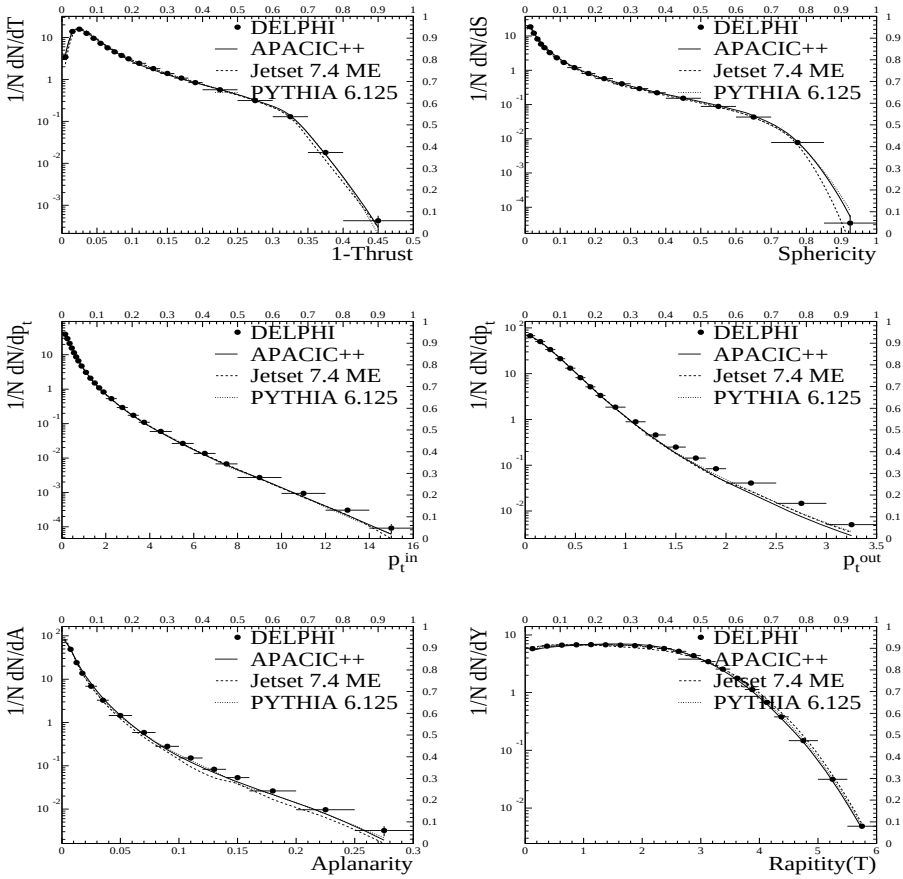

Fig. 4. Comparison of experimental data and event generators for a variety of event shape observables at the hadron level at the Z-pole. We employed the Lund–String hadronization of `PYTHIA` for `APACIC++`.
The plots stem from [32] utilizing data from [33].

of the two smalles eigenvalues defines the sphericity. In contrast, aplanarity is given by

$$A = \frac{3}{2}\,\lambda_1\,. \tag{5.3}$$

Thrust is defined by the maximal value of

$$T = \max_{\vec{n}} \left(\frac{\sum\limits_i |\vec{p}_i \vec{n}|}{\sum\limits_i |\vec{p}_i|} \right)\,, \tag{5.4}$$

where $\vec{n}$ is a free vector to be chosen accordingly. The vector $\vec{n}$ yielding the maximal value of T is the thrust-axis.

$p_\perp^{\rm in}$ and $p_\perp^{\rm out}$ are the components of the transversal momenta being inside the event-plane or perpendicular. The rapidity here is taken with respect to the thrust-axis.

5.2. Topological structure of four jet-events

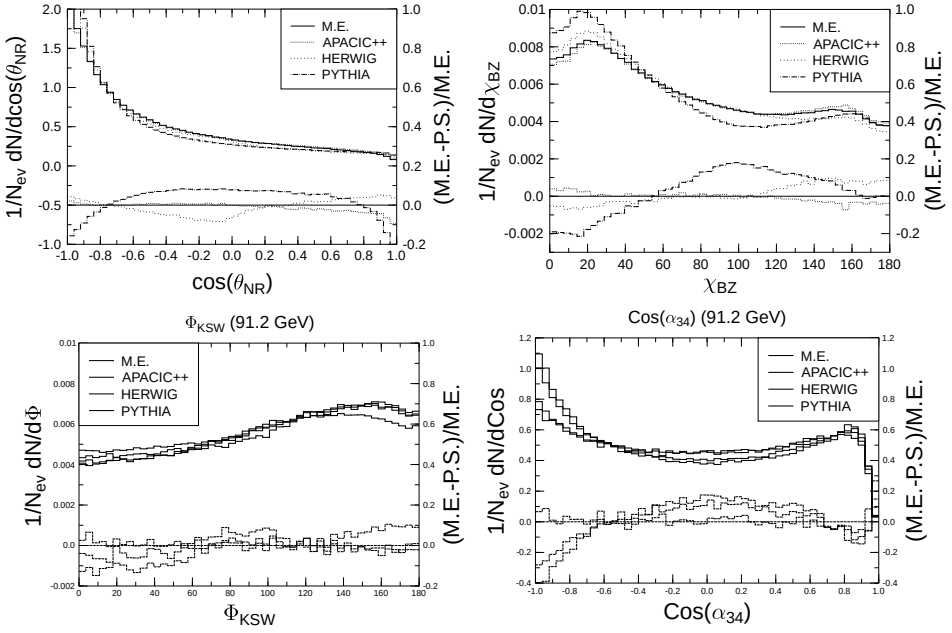

Fig. 5. Distributions for the modified Nachtmann–Reiter-, the Bengtson–Zerwas-, the Körner–Schierholz–Willrodt-angle and for α_{34} as given in Eq. (5.5) obtained by the various event generators. For the definition of jets the Durham-scheme with $y_{\rm cut} = 0.002$ was employed for all final states as well as for the matching of the matrix elements and the parton shower. The upper lines show the corresponding differential rates with respect to the numbers on the left axis whereas the errors relative to the matrix element expression, namely (M.E.-P.S.)/M.E are given by the appropriate lower lines with respect to the numbers on the right axis.

However, the validity of our matching procedure can be verified in more depth considering the topological structure of multijet-events as exemplified by four-jet events. Ordering the jets by their energies, $E_1 \geq E_2 \geq E_3 \geq E_4$, typical observables describing these processes are the modified Nachtmann–Reiter-, the Bengtson–Zerwas- and the Körner–Schierholz–Willrodt-angle as

well as the angle α_{34} between the two least energetic jets [2, 11],

$$\begin{aligned} \theta^*_{\rm NR} &= \angle(\vec{p_1}-\vec{p_2},\vec{p_3}-\vec{p_4})\,, \\ \chi_{\rm BZ} &= \angle(\vec{p_1}\times\vec{p_2},\vec{p_3}\times\vec{p_4})\,, \\ \Phi_{\rm KSW} &= \angle(\vec{p_1}\times\vec{p_3},\vec{p_2}\times\vec{p_4})\,. \end{aligned} \tag{5.5}$$

In Fig. 5 we display the angular distributions of the partons after the shower generated by the various event generators in comparison to the distributions resulting from the corresponding matrix elements.

6. Conclusions

Obviously APACIC++ is perfectly capable to describe in a precise and reliable manner the four jet topologies. Therefore one is tempted to conlude, that the parton shower and the matrix elements are matched appropriately. The few sizeable deviations of the topologies at the parton shower level from the matrix elements are collimated in the region of nearly collinear jets. This is not too surprising, however, since the jet evolution softens the initial partons to jets and widens them to jet-cones which in turn may easily overlap. Of course this alters the results slightly. However, in principle, this exactly reflects the picture employed of hard produced partons widening to jets. We therefore conclude, that the matching succeeded.

In contrast, the two other event generators considered at the present state do not include an accurate matching procedure for four-jet events. Therefore their failure in describing such topologies consistently at the parton level merely reflects the fact, that the angular structures of four-jet events are due to correlations not embedded in the parton shower like for instance interferences of single diagrams.

On the other hand, it should be noted, that all of the event generators displayed here reproduce the overall features of e^+e^--annihialtions into hadrons in a fairly satisfying manner, even though the intrinsic parameters of our code `APACIC++` have not been fully tuned.

Summarizing we would like to state, that we have proposed a obviously working general approach to match parton showers and arbitrary matrix elements in the framework of QCD event generators. We have implemented this ansatz into the newly developped event generator `APACIC++` , which linked with `AMEGIC++` will offer new possibilities to describe precision data concerning multijet-events at LEP II and beyond.

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Both codes, `APACIC++` and `AMEGIC++` can be obtained upon request from F. Krauss E-mail: krauss@physics.technion.ac.il and R. Kuhn E-mail: kuhn@theory.phy.tu-dresden.de.

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