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TOPOLOGICAL DEFECTS FROM AN INHOMOGENEOUS QUENCH: SECOND AND FIRST ORDER TRANSITIONS*

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Kibble-Zurek scenario of topological defects formation is extended to inhomogeneous first and second order transitions. In both cases there is characteristic threshold velocity of critical front propagation below which no topological defects are produced. Instead oriented condensate is grown behind moving temperature or pressure front.

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1. Introduction

It was pointed out by Kibble [1,2] in the context of cosmology that topological defects can be formed during a rapid symmetry breaking phase transition. The Kibble mechanism can be best illustrated for first order phase transitions. Such a transitions proceeds by bubble nucleation. When the temperature drops below T_c the system at first remains in the supercooled symmetric phase with vanishing order parameter. For sufficient supercooling this phase decays thanks to nucleation of bubbles of the symmetry broken phase. Each bubble is nucleated independently so the orientation of the order parameter in each bubble is chosen at random. For simplicity let us restrict to a planar system with a planar order parameter. It may happen that 3 bubbles are nucleated such that their order parameters are more or less radial with respect to the bubbles' center of mass. The at first isolated bubbles are expanding and they eventually touch one another and coalesce.

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The order parameter smoothly interpolates between orientations in each bubble. The resulting hedgehog configuration is a topologically stable vortex. In this way the random statistical fluctuations, which are responsible for nucleation of bubbles and their orientation, are frozen in and preserved in the symmetry broken phase as topological defects. Such defects resulting from transitions on the GUT scale are believed by some cosmologist to be seeds for structure formation [3] in the early Universe. Independently, the proposal passed tests in liquid crystals experiments.

The situation in second order transitions is slightly different as it was clarified by Żurek [4–8]. There are no bubbles to provide well defined sites for the topological triangulation. However, rapid phase transition imprints on the order parameter a characteristic length scale $\hat{\xi}$ which depends on the rate of the quench. An appropriate inverse power of the scale determines the density of topological defects: for kinks or domain walls it is $n \sim \hat{\xi}^{-1}$, for vortex lines $n \sim \hat{\xi}^{-2}$, for monopoles or skyrmions $n \sim \hat{\xi}^{-3}$. The scale is a result of critical slowing down at $T_{\rm c}$. If ε is minus mass-squared in the $\varepsilon\varphi^2$ term of the Ginzburg-Landau model (φ is the order parameter), it vanishes at $T_{\rm c}$. The relaxation time scales as τ_0/ε. As we approach $T_{\rm c}$ from above the relaxation time diverges, at certain moment of time the time left to the transition becomes comparable to the actual relaxation time, that is the moment when the system goes out of equilibrium. For $\varepsilon(t) = -t/\tau$, this happens when $|t| = \tau\tau_0/t$. At this instant the correlation length, which is given by $\xi = \xi_0/\sqrt(\varepsilon)$, is $\hat{\xi} = \xi_0(\tau/\tau_0)^{1/4}$. As the system goes out of equilibrium the scale is preserved in the subsequent evolution and later determines density of topological defects. This prediction was verified in a number of numerical experiments [9–13] and several experiments in superfluid ^{3}He and ^{4}He [14–18]. The experimental results are still a source of much controversy mainly because of an indirect or delayed way of counting topological defects.

Homogeneous quenches are a convenient idealization and may be a good approximation in some cases. However, in reality, the change of thermodynamic parameters is unlikely to be ideally uniform:

1) Experiments carried out in ^{3}He [14, 15], where a small volume of superfluid is re-heated to normal state, and subsequently rapidly cools to the temperature of the surrounding superfluid, are a good example of an inhomogeneous quench: The normal region shrinks from the outside. Yet, topological defects are created, thus suggesting that the phases of distinct domains within the re-heated region are selected independently.

2) Another example are relativistic heavy ion collisions where, according to Bjorken scenario (Bjorken 1983 [19]), a finite volume of quark-gluon

plasma can be created. The plasma expands in the direction of collision and cools from the outside in the perpendicular direction. The phase transition in this case can be first or second order (or a smooth crossover) depending on the parameters of the collision.

3) Any generic experiment based on pressure and/or temperature quench is to some degree inhomogeneous because of finite velocity of sound and/or finite heat conductance.

The mass parameter $\varepsilon(t,\vec{r})$, varying in both time and space, must be considered in defect formation. As a consequence, locations entering the broken symmetry phase first could communicate their choice of the new vacuum as the phase ordered region spreads in the wake of the phase transition front. When this process dominates, symmetry breaking in various, even distant, locations is no longer causally independent. The domain where the phase transition occurred first may impose its choice on the rest of the volume, thus suppressing or even halting production of topological defects. This happens if velocity of the critical front is less than certain characteristic velocity.

2. Second order transition

The characteristic velocity in an overdamped transition can be estimated as follows: The freeze-out healing length is set at $\hat{t}$ as $\hat{\xi} = \xi_0 \ (\tau_Q/\tau_0)^{1/4}$. At the same instant the relaxation time is $\hat{\tau} = (\tau_Q \tau_0)^{1/2}$. These two scales can be combined (Z 1985) to give a velocity scale

$$\hat{v} = \hat{\xi}/\hat{\tau} = v_0 \ (\tau_0/\tau_Q)^{1/4} \, , \tag{2.1}$$

where $v_0 = \xi_0/\tau_0$.

The density of defects N as a function of critical front velocity is expected to change qualitatively at $\hat{v}$. Above $\hat{v}$ the homogeneous estimates should hold. Below $\hat{v}$ the density should be suppressed. Kibble and Volovik [20] suggested that $N \sim v/\hat{v}$ for small $v < \hat{v}$. Dziarmaga, Laguna and Zurek [21] argued that N is exponentially suppressed below $\hat{v}$. There is a lot of qualitative difference between the two proposals. The former option suggests that however you make a quench you will always get some defects, the latter implies that if your inhomogeneous quench is sufficiently slow you will get no defects at all. In what follows we will quantify what “sufficiently slow” means.

2.1. Decay of the false vacuum

As a simple warm up exercise, let us consider decay of a false symmetric vacuum to a true symmetry broken ground state in a one-dimensional dissipative φ^4 model

$$\partial_t^2\varphi + \eta\,\partial_t\varphi \;-\; \partial_x^2\varphi \;+ \frac{1}{2}\left(\varphi^3 - \varepsilon\varphi\right) = 0\;, \tag{2.2}$$

where $\varphi(t,x)$ is a real order parameter and ε measures the degree of symmetry breaking i.e. $m^2 = -\varepsilon$. Without loosing generality, we look for a solution $\varphi(t,x)$ which interpolates between $\varphi(t,-\infty) = -\sqrt{\varepsilon}$ and $\varphi(t,+\infty) = 0$. Such a solution cannot be static. It is a stationary half-kink

$$\varphi(t,x) = -\sqrt{\varepsilon}\left(1 + \exp\left[\frac{\sqrt{\varepsilon}}{2}\frac{(x - v_t t)}{\sqrt{1 - v_t^2}}\right]\right)^{-1} \tag{2.3}$$

moving with characteristic velocity

$$v_t = \left[1 + \left(\frac{2\eta}{3\sqrt{\varepsilon}}\right)^2\right]^{-1/2} \overset{\eta\to\infty}{\approx} \frac{3\sqrt{\varepsilon}}{2\eta}\,. \tag{2.4}$$

It is worth noting that the decay velocity v_t increases with ε.

2.2. Shock wave

Our shock wave inhomogeneous quench model consists of a sharp pressure front propagating with velocity v; that is,

$$\partial_t^2\varphi + \eta\,\partial_t\varphi \;-\; \partial_x^2\varphi \;+ \frac{1}{2}\left(\varphi^3 - \varepsilon(t,x)\,\varphi\right) = \mathcal{O}(t,x)\,, \tag{2.5}$$

where

$$\varepsilon(t,x) \;= \mathrm{Sign}(t - x/v) \tag{2.6}$$

is the relative temperature and $\mathcal{O}(t,x)$ is a Gaussian white noise of temperature Θ.

There are two qualitatively different regimes:

1) $v > v_t$, the phase front propagates faster than the false vacuum can decay. The half-kink (2.3) lags behind the front (2.6); a supercooled symmetric phase grows with velocity $v - v_t$. The supercooled phase cannot last for long; it is unstable, and the noise makes it decay into the true vacuum.

2) $v < v_t$, the phase front is slow enough for a half-kink to move in step with the front, $\varphi(t,x) = H_v(x - vt)$. The symmetric vacuum decays into one definite non-symmetric vacuum, the choice is determined by the boundary condition at $x \to -\infty$. No topological defects are produced in this regime. The stationary solution $H_v(x - vt)$ is stable against small perturbations [21].

These expectations are borne out by the numerical study of kink formation in [21]. Numerical results are presented in Fig. 1.

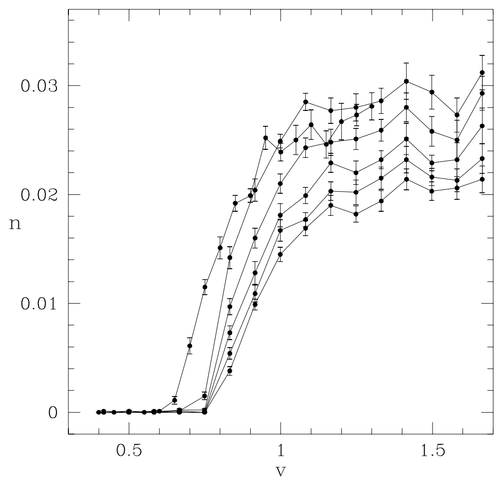

Fig. 1. Density of kinks n as a function of velocity v for the shock wave (2.6) with $\eta = 1$ (overdamped system). In this overdamped regime, the predicted threshold velocity is $v_t = 0.83$. The plots from top to bottom correspond to $\Theta = 10^{-1}$, 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}. At low Θ, we get a clear cut-off velocity at $v \approx 0.8$, which is consistent with the prediction.

2.3. Linear front

Let us consider now a system in which the inhomogeneous quench takes place via linear transition

$$\varepsilon(t,x) = (t - x/v)/\tau_Q \,. \tag{2.7}$$

In the absence of noise, the propagating front is followed by a stationary half-kink. This half-kink moves somewhat behind the front, its location is determined by the place where the threshold velocity (2.4) is equal to the front velocity, $v_t[\varepsilon(t,x)] = v$. The distance between the front and the half-kink increases as v^3. This distance gives the size of the supercooled region. When the supercooled region is narrow then it is stable against small perturbations so that no defects are produced. If

$$v \;>\; v_t \equiv \left(1+\frac{\eta^{3/2}\tau_Q^{1/2}}{11.7}\right)^{-1/2} \stackrel{\eta\to\infty}{\approx} \frac{3.42}{\eta}\left(\frac{\eta}{\tau_Q}\right)^{1/4} \equiv \; 4.07\,\hat{v}\,, \tag{2.8}$$

then the region is broad enough to be unstable [21] and the production of defects is no longer suppressed.

This prediction is confirmed by the numerical study of linear quenches in Ref. [21], compare Fig. 2. However, it is seen that the threshold velocity apparently gradually decreases with increasing noise temperature Θ. This decrease of the threshold for kink formation is due to the thermal nucleation of kinks. Quantitative estimates for this effect are given in [22].

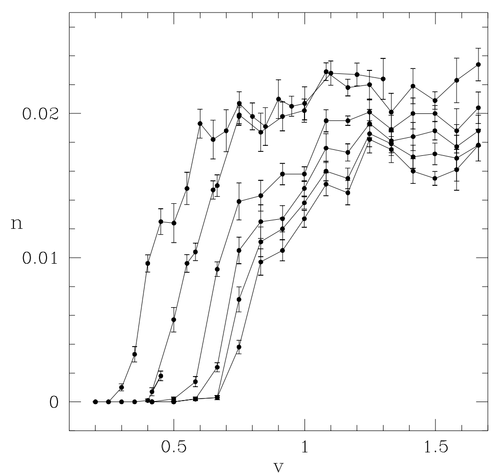

Fig. 2. Density of kinks n as a function of velocity v for the linear inhomogeneous quench (2.7) with $\tau_Q = 64$ and $\eta = 1$. The predicted threshold is $v_t = 0.77$. This cut-off is achieved for low Θ. The plots from top to bottom correspond to $\Theta = 10^{-1}$, 10^{-2}, 10^{-4}, 10^{-6}, 10^{-8}, 10^{-10}.

3. First order transition

We assume the transition is strongly first order and that it goes by bubble nucleation. To be more specific we consider a toy model in 3 dimensions

$$\partial_t\varphi = \nabla^2\varphi - a\varphi + b\varphi^3 - c\varphi^5 + \mathcal{O}\,, \tag{3.1}$$

where φ is real order parameter. The effective potential is of the φ^6 type. Provided that $b^2 > 4ac$, it has symmetric minimum at $\varphi = 0$ and two symmetry broken minima at $\varphi = \pm\varphi_m \equiv \pm\sqrt{(b+\sqrt{b^2-4ac})/2c}$. We assume that b, c are constant and that symmetry breaking transition is driven by a decreasing below its critical value $a_c = 3b^2/16c$. At $a = a_c$ all three minima are degenerate.

3.1. Decay of the false vacuum

Suppose that $a < a_c$. Let us consider decay of the false symmetric vacuum to the true symmetry broken phase in a one dimensional version of the model (3.1). We look for a solution which interpolates between $\varphi = \varphi_m$ for $x \to -\infty$ and $\varphi = 0$ for $x \to +\infty$. The solution is found as a stationary half-kink $H(x - v_t t)$ moving with velocity

$$v_t = \frac{-b + 2\sqrt{b^2 - 4ac}}{\sqrt{3c}} \tag{3.2}$$

which has an envelope function

$$H(x) = \frac{\varphi_m}{\sqrt{1 + \frac{\exp \alpha x}{2c}}}\,, \tag{3.3}$$

where $\alpha = \sqrt{4c/3}\varphi_m^2$. This way the false $\varphi = 0$ vacuum decays into the true $\varphi = \varphi_m$ vacuum in the absence of noise. The decay velocity v_t is zero for $a = a_c$, it increases with increasing supercooling or with decreasing a.

3.2. Shock wave

In the shock wave model a sharp pressure front propagates with velocity v

$$a = a_c - \Delta a \,\mathrm{Sign}(t - x/v)\,. \tag{3.4}$$

Similarly as for second order transitions there are two regimes:

1) $v > v_t$, the pressure front propagates faster than the false vacuum can decay. The half-kink lags behind the front. The supercooled phase in between them grows linearly with time. The phase is unstable, it decays by bubble nucleation just as for a homogeneous transition. Homogeneous estimates of defect density apply in this case.

2) $v < v_t$, the half-kink is faster. It moves in step with the front with its tail penetrating into the symmetric phase. There is no supercooled phase where bubbles could be nucleated. The symmetric phase goes smoothly into one of the symmetry broken phases.

3.3. Linear front

Let the inhomogeneous quench proceed by a linear front moving with velocity v

$$a = a_{\mathrm{c}} - (t - x/v)/\tau_Q \,. \tag{3.5}$$

The half-kink follows the critical front keeping certain distance behind it. The distance D is such that the half-kink velocity v_t, which depends on the local value of a, is equal to the front velocity v, $v_t(a) = v$. With increasing v the half-kink settles at increasing values of local a. Close to the critical front the radius of the critical bubble is infinite and at the same time the nucleation rate is infinitely small. As we go away from the front in the direction of the half-kink the critical radius shrinks. At a certain distance L from the front the energy of the critical bubble becomes comparable to the temperature Θ. At this point bubble nucleation becomes possible. If $L < D$ bubbles can be nucleated in the supercooled region between the front and the half-kink. If $L > D$ then there is no bubble nucleation and no defects can be born in the supercooled area.

Now we estimate the critical velocity such that $L = D$. The half-kink is located at such an a that $v_t(a) = v$. $L = D$ providing that for this a the energy of the critical bubble $E(a)$ is equal to temperature Θ. The critical bubble is a metastable spherically symmetric static solution of Eq. (3.1) with, say, φ_m vacuum inside and 0 vacuum outside its wall. Its energy can be easily estimated when the width of its wall is negligible as compared to its radius $R_{\mathrm{c}}(a)$. An approximate solution is given by $H[r - R_{\mathrm{c}}(a)]$, where the critical radius is

$$R_{\mathrm{c}}(a) = \frac{\sqrt{12c}}{-b + 2\sqrt{b^2 - 4ac}} \,. \tag{3.6}$$

The energy of the critical bubble $E(a)$ has a negative volume contribution, $(4\pi R_{\mathrm{c}}^3/3)V(\varphi_m)$, and a positive surface tension term,

$$(4\pi R_{\mathrm{c}}^2) \int dx [H'(x)]^2 .$$

When the solution of $v_t(a) = v$ is put into $E(a)$ and then the equation $E(a) = \Theta$ is solved, one obtains a critical velocity

$$v_{\mathrm{cr}} = \left(\frac{\pi b(3b^2 - 6bc + 16c^2)}{4c^3 \Theta} \right)^{1/3} \tag{3.7}$$

when $L = D$. For $v > v_{\mathrm{cr}}$ bubbles can nucleate in between the half-kink and the front and thus the necessary condition for topological defects production is satisfied.

The formula for $v_{\rm cr}$, Eq. (3.7), is still a crude lower estimate for the critical velocity. In fact it is not sufficient to nucleate some bubbles. Individual bubbles would coalesce with the half-kink without any chance to trap any nontrivial winding number. The bubbles should be nucleated in large numbers or have enough time to grow so that they can mutually coalesce before merging with the half-kink. Still, the argument which leads to $v_{\rm cr}$ demonstrates that there is a threshold velocity for defect formation.

4. Higher dimensions

The theory can be generalized to higher dimensions and to complex order parameter in a straightforward manner. Its major result is that a subthreshold inhomogeneous quench does not produce any variation of the order parameter in the direction normal to the front. This excludes any possibility of production of vortex loops or closed membranes entirely contained in the bulk, as well as of any pointlike defects. Some extended defects can grow into the bulk provided their seeds were created at this edge of the system where the symmetry was broken first. In first approximation such, say, vortices grow into the bulk, following the passing front, keeping their direction normal to the front. In the end we do not get any chaotic tangle of strings and string loops but parallel "combed" vortices. There are two important perturbations to this "combed" picture:

1) Thermal fluctuations make the strings look more random but without backtracking and with string tension tending to smooth the small scale fluctuations. The ends of the strings and antistrings at the critical front are wandering around. Eventually an end of a string and of an antistring may meet so that the strings join into a half-loop with its both ends attached to the initial edge of the system. String tension shrinks the half-loop to the edge where it unwinds.

2) A much more efficient factor to remove vortices from the bulk are their mutual interactions. Global parallel string and antistring attract one another so that their ends at the critical front do not seek each other at random but tend to fuse in a deterministic way. This mechanism makes the number of strings in the bulk decay quickly with increasing distance between the front and the initial edge.

The factors (1) and (2) lead to a picture in which the critical front initially draws some parallel strings and antistrings from the edge, then the strings recombine by joining ends and shrinking back to the edge. In the end only the net surplus of strings (or antistrings) is left in the bulk.

These ideas are supported by experiments:

1) Disclinations produced during a quench from disordered to nematic phase in liquid crystals. This is a weakly first order transition. In early attempts to make cosmological experiments in liquid crystals the disclinations were observed to grow approximately combed, join ends and shrink to the initial edge. Later on it was realized that these quenches were not homogeneous enough [23].

2) Czochralski method of growing monocrystals, which is widely used to grow silicon monocrystals necessary for microchips. In this method, discovered in the thirties, a surface of liquid material is touched with a monocrystal template. As the template is slowly lifted up it drags a column of crystal out of the container. The top part of the column is cold while its bottom part is at the melting temperature - the transition is inhomogeneous. If the template is lifted slowly enough, then no defects of the crystal lattice are produced which might spoil the monocrystal.

To conclude: in an inhomogeneous quench there is a threshold velocity v_t of the critical front. Above the threshold defects are produced like in a homogeneous quench. Below the threshold one gets no defects; instead a clean monocrystal or a "disoriented chiral condensate" is grown with a vacuum which may be uniform over significant distances, but which differs from the true vacuum.

This talk is partially based on work done in collaboration with Pablo Laguna and Wojciech Żurek [21].

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