

IMPROVED INTERMITTENCY ANALYSIS OF SINGLE EVENT DATA*

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The intermittency analysis of single event data (particle moments) in multiparticle production is improved, taking into account corrections due to the reconstruction of history of a particle cascade. This approach is tested within the framework of the α -model.

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1. Introduction

The first data on possible intermittent behaviour in multiparticle production [1] came from the analysis of the single event of high multiplicity recorded by the JACEE collaboration [2]. It was soon realized, however, that the idea may be applied to events of any multiplicity provided that averaging of the distributions is performed [3]. This led to many successful experimental studies of intermittency [4], and allowed to express the effect in terms of the multiparticle correlation functions [5]. It should be realized, however, that the averaging procedure, apart from clear advantages, brings also a danger of overlooking some interesting effects if they are present only

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in a part of events produced in high-energy collisions. For example, the unique properties due to the presence of quark–gluon plasma in multiparticle production would manifest only in some events, see *e.g.* [6]. Taking into account the sample of events and averaging over them destroys such an information. Therefore, as already discussed in [8,9], there is a need for event-by-event analysis of multiparticle production processes. In this way the fluctuations of the measured physical quantities (*e.g.* factorial moments) from event to event can be observed and estimated, and any anomalous behaviour of them has a chance to manifest very clearly. Such studies should necessarily be restricted to high-multiplicity events because only there one may expect the statistical fluctuations to be under control.

Such an approach to the multiparticle data analysis has been already proposed in [7–9]. In [8] a new quantity: erraticity has been introduced to investigate the event-by-event fluctuations of factorial moments, and to search for their properties. Erraticity denotes the normalized moment of event-by-event distribution of a horizontally averaged factorial moment. It probes both types of fluctuations: horizontal ones connected with the spatial bin pattern and vertical ones *i.e.* event-by-event ones.

In [9] the event-by-event fluctuations of particle moments have been investigated directly for the one-dimensional α –model of random cascading. Monte Carlo simulations of the model allowed one to obtain the histograms of event-by-event distributions of horizontally averaged particle momenta and estimate the relation between the intermittency parameters obtained from such a histogram, and the intermittency parameters derived after usual procedure of averaging particle moments over all events. The results were promising: the average value of the intermittency exponent reproduced well the value obtained by averaging particle moments over events, however with the tendency to underestimate the theoretical value. Furthermore, the dispersion of the moment distribution was inversely proportional to the length of a generated cascade, and even for short cascades substantially smaller than the average value. The latter property was of a special importance: it allowed one to distinguish between groups of events emerging from cascades with different characteristics.

In this paper we would like to improve the analysis of single event data presented in [9]. Taking into account corrections due to the method of recovering the history of the multiparticle cascade [1,2], we expect to reduce the discrepancy between the theoretical value of intermittency exponent and its value estimated from the event-by-event histogram [9]. Our discussion will proceed as follows. In Section 2 we recall the definition of the intermittency exponents and the technique used to calculate them [1,2]. In Section 3 the definition of the α model will be briefly presented, and applied in Section 4 to calculate corrections for extracting intermittency exponents from single

event data. Section 5 is devoted to the comparison of theoretical results with numerical simulations. Finally in Section 6 we present our conclusions.

2. Intermittency exponents

Consider a multiparticle production cascade distributed into M bins. At the n^{th} stage of the cascade we measure the distribution of a particle density into M bins. Assume for simplicity that $M = 2^n$. We thus have 2^n numbers (quantities) denoting the content of each bin:

$$x_i^{(n)}, \quad i = 0, 1, \dots, 2^n - 1. \quad (1)$$

To perform the event-by-event analysis one is interested in the behaviour of particle moments with the stage of the cascade:

$$z_q^{(n)} = \frac{1}{2^n} \sum_{i=0}^{2^n-1} \left(x_i^{(n)} \right)^q. \quad (2)$$

The scaling behaviour of these moments is parametrized by intermittency exponents ϕ_q [1]:

$$z_q^{(n)} \sim 2^{n \cdot \phi_q}. \quad (3)$$

The task is to estimate the value of an intermittency exponent. There are two different ways of doing it. The first one is to calculate the average moment $z_q^{(n)}$ for the whole ensemble of individual events, and from this to reconstruct the intermittency exponent. The second one is to calculate the exponent ϕ_q for each event separately, and then to recover the average ϕ_q . The latter approach has the advantage of being able to distinguish between two independent cascading processes each with different ϕ_q . This could be done by looking at the distribution of individual ϕ_q 's. In the former method both of these possibly independent processes would be artificially forced to be described by a single 'effective' ϕ_q .

In the following we would like to address the question of reliably reconstructing the correct value of ϕ_q from single event data. Numerical simulations in [9] showed that there is an inherent discrepancy between the theoretical value and the distributions of event-by-event ϕ_q (see Tables I, II). The aim of this letter is to analyze this result and introduce a correction which improves the estimation.

A convenient way of calculating ϕ_q is to make a linear fit to the points $(n, \log z_q^{(n)})$ (all logarithms are taken to be calculated in base 2, *i.e.* $\log x \equiv \ln x / \ln 2$):

$$\log z_q^{(n)} = n \cdot \phi_q + b. \quad (4)$$

TABLE I

Standard and corrected intermittency exponents (determined from the position of the maximum of the histograms) and their dispersions (errors) for $a = 0.8$, $b = 1.1$ and $n = 5, \dots, 10$ cascade steps. Theoretical values for intermittency exponents are $\varphi_{2,\text{theor}} = 2.85 \times 10^{-2}$ and $\varphi_{3,\text{theor}} = 8.13 \times 10^{-2}$.

	$\varphi_i = 10^{-2} \varphi_i$					
i	5	6	7	8	9	10
φ_2	1.90 ± 0.89	2.00 ± 0.82	2.26 ± 0.75	2.49 ± 0.66	2.52 ± 0.59	2.46 ± 0.52
$\varphi_{2,\text{corr}}$	2.66 ± 0.75	2.79 ± 0.66	2.66 ± 0.56	2.85 ± 0.46	2.85 ± 0.43	2.85 ± 0.36
φ_3	5.66 ± 2.46	5.66 ± 2.46	6.48 ± 2.05	6.64 ± 1.72	6.81 ± 1.72	6.72 ± 1.49
$\varphi_{3,\text{corr}}$	7.79 ± 2.13	7.63 ± 1.81	7.62 ± 1.64	8.00 ± 1.40	8.12 ± 1.15	8.12 ± 0.98

TABLE II

Intermittency exponents (determined from the position of the maximum of the histograms) and their dispersions (errors) for $a = 0.5$, $b = 1.5$ and $n = 5, \dots, 10$ cascade steps. Theoretical values for intermittency exponents are $\varphi_{2,\text{theor}} = 3.22 \times 10^{-1}$ and $\varphi_{3,\text{theor}} = 8.07 \times 10^{-1}$.

	$\varphi_i = 10^{-1} \varphi_i$					
i	5	6	7	8	9	10
φ_2	2.00 ± 1.00	2.09 ± 0.92	2.43 ± 0.74	2.61 ± 0.70	2.44 ± 0.74	2.26 ± 0.70
$\varphi_{2,\text{corr}}$	3.13 ± 0.79	3.13 ± 0.74	2.96 ± 0.61	2.87 ± 0.57	3.05 ± 0.52	3.00 ± 0.48
φ_3	4.52 ± 2.23	5.31 ± 2.23	5.83 ± 2.02	5.90 ± 1.83	6.03 ± 1.83	5.70 ± 1.70
$\varphi_{3,\text{corr}}$	7.53 ± 1.90	7.93 ± 1.83	7.66 ± 1.57	7.53 ± 1.31	7.66 ± 1.31	7.66 ± 1.18

This procedure has the advantage of cancelling out the major part of the correction coming from the fact that we are effectively reconstructing the exponents from $\langle \log z_q^{(n)} \rangle$ while the true value is defined in terms of $\log \langle z_q^{(n)} \rangle$.

However there is still one caveat to (4). Since we cannot in general separate out the various stages of the cascade, one reconstructs the previous stages from the last one by summing the $x_i^{(n)}$'s in adjacent bins using the technique described in [1] (and applied there to JACEE event [2]). Namely one approximates the true value of $x_i^{(n-k)}$ by:

$$x_i^{(n-k)} \longrightarrow y_i^{(n-k)} = \frac{1}{2^k} \sum_{j=0}^{2^k-1} x_{2^k \times i + j}^{(n)}. \quad (5)$$

Therefore in (4) one really uses the reconstructed moments:

$$z_{q,\text{reconstructed}}^{(k)} = \frac{1}{2^k} \sum_{j=0}^{2^k-1} \left(y_i^{(k)} \right)^q. \quad (6)$$

We will now use the α model of random cascading [1] to calculate explicitly the difference between the true and reconstructed moments and the resulting shift of the ϕ_q distribution from the theoretical value.

3. The α model of random cascading

In the α model of random cascading [1] the root of the cascade — $x_0^{(0)}$ is taken to be a with probability p_a and b otherwise (with probability $p_b = 1 - p_a$). One generates the next stages of the cascade recursively. The two bins $x_{2i}^{(n+1)}$ and $x_{2i+1}^{(n+1)}$ are obtained from $x_i^{(n)}$ by:

$$x_{2i}^{(n+1)} \longrightarrow a \cdot x_i^{(n)} \quad \text{with probability } p_a, \quad (7)$$

$$x_{2i+1}^{(n+1)} \longrightarrow b \cdot x_i^{(n)} \quad \text{with probability } p_b, \quad (8)$$

and same for $x_{2i+1}^{(n+1)}$. The parameters a and b are taken to satisfy:

$$ap_a + bp_b = 1. \quad (9)$$

Particle moments fulfil the relation:

$$z_q^{(n)} = 2^{(n+1) \cdot \phi_q}, \quad (10)$$

where intermittency exponents ϕ_q are equal to:

$$\phi_q = \log(a^q p_a + b^q p_b). \quad (11)$$

4. Reconstructed moments

The reconstructed moments in the α model are related to the true ones by:

$$z_{q,\text{reconstructed}}^{(n-k)} = \frac{1}{2^n} \sum_{i=0}^{2^{n-k}-1} \left\langle \left(\sum_{j=0}^{2^k-1} x_{2^k i+j}^{(n)} \right)^q \right\rangle \equiv z_q^{(n-k)} \cdot p_q(k). \quad (12)$$

where the average $\langle \dots \rangle$ is taken over the random choices made *only* above the $(n-k)$ -th stage of the cascade. The factor $p_q(k)$ can be calculated

exactly (see below) and we propose to use it to compensate for the errors introduced by the reconstruction procedure. In particular the reconstructed moments entering (4) will be shifted by:

$$\log z_{q,\text{reconstructed}}^{(n-k)} \longrightarrow \log z_{q,\text{reconstructed}}^{(n-k)} - \log(p_q(k)). \quad (13)$$

We will now determine the explicit form of the correction $p_q(k)$. By the definition of the α model, the correction $p_q(k)$ can be calculated just by evaluating:

$$p_q(k) = \left\langle \left(\frac{1}{2^k} \sum_{i=0}^{2^k-1} x_i^{(k)} \right)^q \right\rangle \quad (14)$$

in the α model modified by taking the starting bin $x_0^{(0)} = 1$.

First it is easy to see that for $q = 1$ there is no correction $p_1(k) = 1$. This is due to (9). Also all corrections vanish for $k = 0$:

$$p_q(0) = 1. \quad (15)$$

The appearance of a correction for $q > 1$ comes from the fact that the ‘number’ of particles in this model has a nonzero dispersion.

Consider first the case of $q = 2$. We will now split the bins (x_i ’s) appearing in (14) into a left half ($i < 2^{k-1}$) and a right half ($i \geq 2^{k-1}$):

$$\begin{aligned} p_2(k) = \left\langle \left(\frac{1}{2^k} \sum_i l_i + r_i \right)^2 \right\rangle &= \frac{1}{4} \left\langle \left(\frac{1}{2^{k-1}} \sum_i l_i \right)^2 + \left(\frac{1}{2^{k-1}} \sum_i r_i \right)^2 \right. \\ &\quad \left. + 2 \left(\frac{1}{2^{k-1}} \sum_i l_i \right) \left(\frac{1}{2^{k-1}} \sum_i r_i \right) \right\rangle. \end{aligned} \quad (16)$$

Using the fact that the left and right bins are independent one gets the recurrence relation:

$$p_2(k) = \frac{1}{2} \underbrace{(p_a a^2 + p_b b^2)}_{d_2} p_2(k-1) + \frac{1}{2}. \quad (17)$$

This can be solved together with the initial data (15), to yield a closed form solution:

$$p_2(k) = \left(\frac{d_2}{2} \right)^k \frac{1 - d_2}{2 - d_2} + \frac{1}{2 - d_2}. \quad (18)$$

In general one can obtain the recurrence relation for general q in exactly the same way:

$$p_q(k) = \frac{1}{2^q} \sum_{i=0}^q \binom{q}{i} d_i d_{q-i} \cdot p_i(k-1) p_{q-i}(k-1), \quad (19)$$

where

$$d_i = p_a a^i + p_b b^i. \quad (20)$$

A similar recurrence relation has been obtained in a different context in [10].

5. Discussion

We have performed numerical simulations of the α -model in order to test the improved single data analysis in practice. In Fig. 1 and Fig. 2 the histograms of the corrected (with the shift (19) taken into account) and standard (without the correction (19)) values of intermittency exponents φ_2, φ_3 are plotted for 90000 generated cascades of 5 and 10 steps. The peaks with the correction included are significantly closer to the theoretical value. The dispersion of the distribution estimated directly from the observed peak, for the “corrected” histogram is smaller than the dispersion of the “standard” one. It decreases with the number of cascade steps. The numerical values of “corrected” and “standard” dispersion as a function of the cascade length are presented in Tables I, II for 2 different sets of cascade parameters. The corrected dispersion is relatively small, and it allows to distinguish between the cascades with different parameters (Figs 1, 2).

The influence of the correction (19) on the value of the intermittency exponents obtained from averaging over the ensemble of events (‘center of mass’ of the histogram) was also investigated. The results are presented in Tables III, IV for 2 different sets of cascade parameters. The estimation of intermittency exponents for the corrected case is much better than for the standard one.

In the preceding, the formula for the correction (see *e.g.* (18)) depends on the values of the parameters a, b of the α -model. In practice, however, one would like to implement some sort of model independent correction. A possible way of doing this is to use the fact that the corrections $\log p_2(i)$ and $\log p_3(i)$ seem to change most dramatically in the first few steps of the reconstruction procedure (near the ‘end’ of the cascade). After that they seem to stabilize at some constant value. This would suggest using just the reconstructed moments near the beginning of the cascade in the fit (4). In practice, however, this might perhaps suffer from low statistics and large fluctuations.

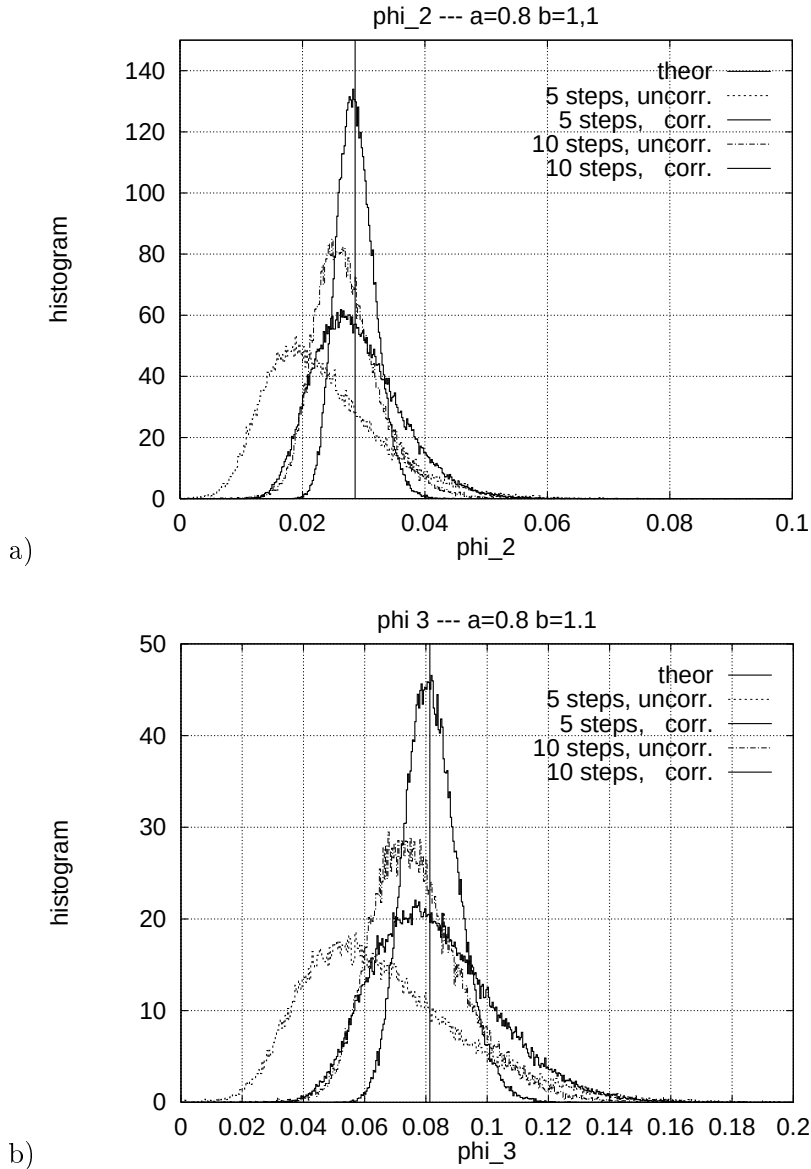


Fig. 1. a-b Histograms of the intermittency exponents a) ϕ_2 and b) ϕ_3 simulated for the set of parameters $a = 0.8$, $b = 1.1$ in 90000 events for 5 and 10 cascade steps. The wider curves correspond to 5 stages of the cascade. ‘Solid’ curves represent the histograms with the correction $p_q(k)$ taken into account.

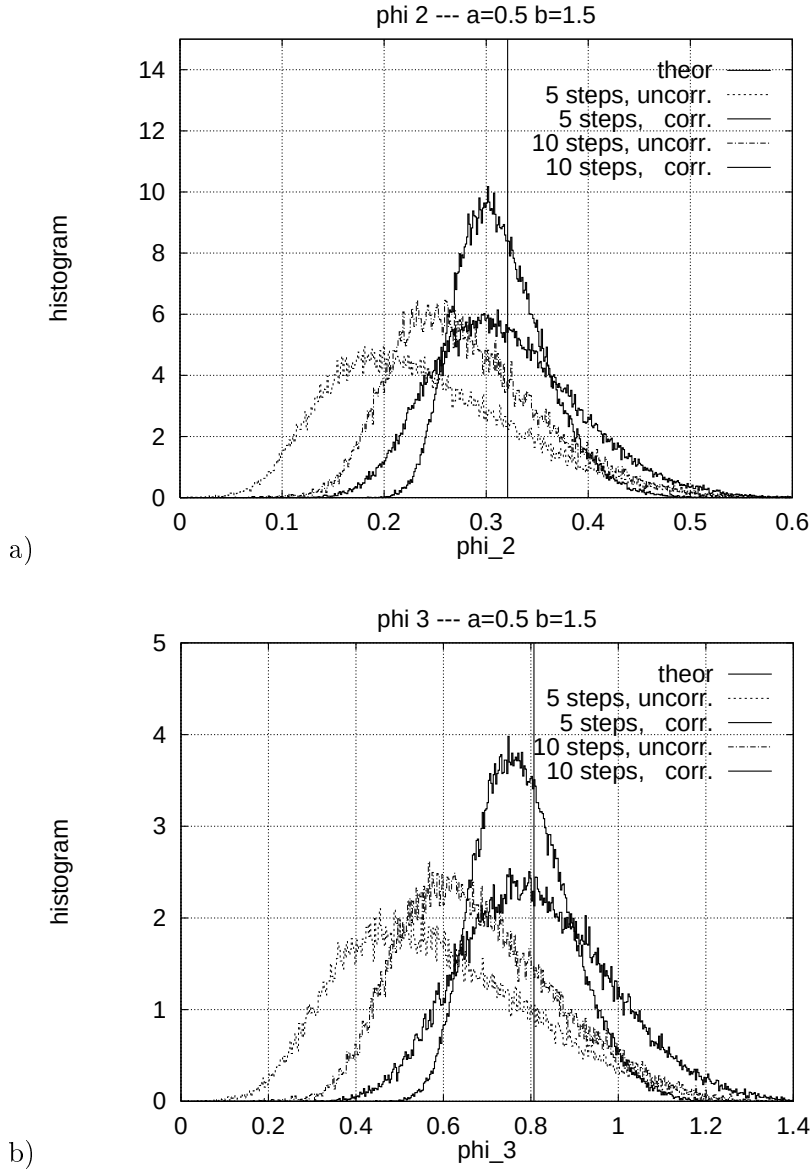


Fig. 2. a-b Histograms of the intermittency exponents a) ϕ_2 and b) ϕ_3 simulated for the set of parameters $a = 0.5$, $b = 1.5$ in 90000 events for 5 and 10 cascade steps. The wider curves correspond to 5 stages of the cascade. ‘Solid’ curves represent the histograms with the correction $p_q(k)$ taken into account.

TABLE III

Standard and corrected intermittency exponents and their dispersions (errors) for $a = 0.8$, $b = 1.1$ and $n = 5, \dots, 10$ cascade steps obtained after averaging over the sample of 90000 events. Theoretical values for intermittency exponents are $\varphi_{2,\text{theor}} = 2.85 \times 10^{-2}$ and $\varphi_{3,\text{theor}} = 8.13 \times 10^{-2}$.

	$\varphi_i = 10^{-2} \varphi_i$					
i	5	6	7	8	9	10
φ_2	2.16 ± 4.34	2.45 ± 1.51	2.57 ± 0.74	2.63 ± 0.66	2.68 ± 0.60	2.72 ± 0.54
$\varphi_{2,\text{corr}}$	2.90 ± 0.70	2.89 ± 0.58	2.88 ± 0.49	2.88 ± 0.42	2.87 ± 0.36	2.87 ± 0.31
φ_3	6.54 ± 4.80	7.03 ± 2.62	7.33 ± 2.11	7.53 ± 1.90	7.68 ± 1.73	7.77 ± 1.58
$\varphi_{3,\text{corr}}$	8.32 ± 2.00	8.29 ± 1.67	8.25 ± 1.40	8.23 ± 1.18	8.22 ± 1.02	8.20 ± 0.89

TABLE IV

Intermittency exponents and their dispersions (errors) for $a = 0.5$, $b = 1.5$ and $n = 5, \dots, 10$ cascade steps obtained after averaging over the sample of 90000 events. Theoretical values for intermittency exponents are $\varphi_{2,\text{theor}} = 3.22 \times 10^{-1}$ and $\varphi_{3,\text{theor}} = 8.07 \times 10^{-1}$.

	$\varphi_i = 10^{-1} \varphi_i$					
i	5	6	7	8	9	10
φ_2	2.33 ± 1.2	2.50 ± 0.95	2.62 ± 0.82	2.69 ± 0.76	2.77 ± 0.74	2.81 ± 0.72
$\varphi_{2,\text{corr}}$	3.20 ± 0.70	3.17 ± 0.63	3.15 ± 0.57	3.15 ± 0.52	3.14 ± 0.47	3.14 ± 0.43
φ_3	5.78 ± 2.36	6.13 ± 2.06	6.38 ± 1.90	6.55 ± 1.81	6.71 ± 1.78	6.81 ± 1.75
$\varphi_{3,\text{corr}}$	8.16 ± 1.71	8.05 ± 1.51	7.96 ± 1.36	7.90 ± 1.24	7.86 ± 1.13	7.84 ± 1.06

An alternative procedure would be to first determine the parameters a and b using the standard (uncorrected) method, and then substitute those parameters into (19) and use the improved analysis to obtain a better approximation of the exponents. One could repeat this until the result no longer changed.

6. Conclusions

Our conclusions can be summarized as follows:

- the value of intermittency exponent estimated from the maximum of “corrected” histogram moves closer to the theoretical value,
- the dispersion of the distribution estimated directly from the observed peak for the “corrected” histogram is smaller than the dispersion of the

“standard” one,

- (c) the corrected value of intermittency exponent obtained after averaging over the sample of events estimates the theoretical value better than in the standard case,
- (d) a possible procedure of improving the analysis without the knowledge of α -model parameters is proposed.

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