ON THE PATTERN OF ASYMMETRIES IN THE POLE MODEL OF WEAK RADIATIVE HYPERON DECAYS

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We study the question whether the pole-model VMD approach to weak radiative hyperon decays can be made consistent with Hara's theorem and still yield the pattern of asymmetries characteristic of the quark model. It is found that an essential ingredient which governs the pattern of asymmetries is the assumed off-shell behaviour of the parity-conserving $1/2^- - 1/2^+ - \gamma$ amplitudes. It appears that this behaviour can be chosen in such a way that the pattern characteristic of the quark model is obtained, and yet Hara's theorem satisfied. As a byproduct, however, all parity-violating amplitudes in weak radiative and nonleptonic hyperon decays must then vanish in the SU(3) limit. This is in conflict with the observed size of weak meson–nucleon couplings.

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1. Introduction

Weak radiative hyperon decays (WRHD's) present a challenge to our theoretical understanding. Despite many years of theoretical studies, a satisfactory description of these processes is still lacking. For a review see Ref. [1] where current theoretical and experimental situation in the field is presented.

The puzzle posed by WRHD's manifests itself as a possible conflict between Hara's theorem [2] and experiment. Hara's theorem is formulated in the language of local field theory at hadron level, and is based on CP- and gauge- invariance. It states that the parity-violating amplitude of the $\Sigma^+ \to p\gamma$ decay should vanish in the limit of SU(3) flavour symmetry. For expected weak breaking of SU(3) symmetry the parity-violating amplitude in question and, consequently, the $\Sigma^+ \to p\gamma$ decay asymmetry should be small. Experiment [3] shows, however, that the asymmetry is large:

$\alpha(\Sigma^+ \to p\gamma) = -0.72 \pm 0.086 \pm 0.045$. Explanation of such a large value of this asymmetry is even more difficult when one demands a successful simultaneous description of the experimental values of the asymmetries of three related WRHD's, namely $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$.

Theoretical calculations may be divided into those performed totally at quark level (*e.g.* [5, 6]) and those ultimately carried out at hadron level (*e.g.* [4, 7]). Hadron-level calculations are based on the pole model, with Hara's theorem usually satisfied by construction. The only exception is the hadron-level vector-meson dominance (VMD) symmetry approach of Ref. [7] which admits a pole-model interpretation and yet violates the theorem. On the other hand, quark model calculation of Ref. [5] (and its phenomenological applications [6]), in spite of being explicitly CP- and gauge- invariant, directly violate the theorem. The problem is further confounded by the fact that experiment seems to agree with the predictions of the quark (or VMD) model, and not with those of the pole model satisfying Hara's theorem. Putting aside the approach of Ref. [7], for known pole and quark models there exists an important difference between their predictions concerning the pattern of the signs of asymmetries in the four WRHD's mentioned above. For the set of asymmetries ($\Sigma^+ \to p\gamma$, $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, $\Xi^0 \to \Sigma^0\gamma$) the pole model [4] predicts the pattern $(-,-,-,-)$, while the quark model [1,6] gives $(-,+,+,-)$. Experiment (and in particular the sign of the $\Xi^0 \to \Lambda\gamma$ asymmetry [8]) hints [1] that it is the latter alternative that is realized in Nature. Apart from the quark model, there are two other approaches that yield the pattern $(-,+,+,-)$. The first one is the hadron-level $SU(6)_W \times$ VMD approach of Ref. [7] which so far gives the best description of data [1]. The other is a diquark approach of Ref. [10].

The VMD prescription seems to violate Hara's theorem as well. Although a connection between the quark model and VMD result has been proposed [7], closer inspection [9] reveals that the origin of the violation of Hara's theorem is slightly different in the two models. In the quark model, the violation of Hara's theorem arises from bremsstrahlung diagrams in which photon is emitted from one of the pair of quarks exchanging the W-boson. The violation is connected with the intermediate quark entering its mass-shell in the $q_\gamma \to 0$ limit. The $SU(6)_W \times$VMD approach (related by symmetry to the standard pole model of nonleptonic hyperon decays) admits a pole-model interpretation. Then, the intermediate state is an excited $1/2^-$ state which is not degenerate with external ground state baryon. Hence, the intermediate excited baryon state cannot be on its mass shell.

The diquark approach [10] contains a few free parameters, among them the masses of spin 0 and spin 1 diquarks. In the limit when these masses are equal to each other the approach yields the pattern $(-,+,+,-)$. Furthermore, all parity violating amplitudes are then proportional to the $m_s - m_d$

mass difference and, consequently, Hara's theorem is satisfied. The pattern $(-,+,+,-)$ for the diquark approach looks a little bit like an accident since it holds only when spin 0–spin 1 symmetry is satisfied. Still, the result of Ref. [10] poses the question if one can find other models which satisfy Hara's theorem and yet give the pattern $(-,+,+,-)$.

Specifically, the question that we put forward in this paper is: can the phenomenological success of VMD [1] be consistent with Hara's theorem? We will show that the answer to this question is "yes". However, consistency of the phenomenological success of the $SU(6)_W \times$ VMD approach with Hara's theorem implies that *dominant parts of all parity violating WRHD's amplitudes (as well as those of nonleptonic hyperon decays) must vanish in the SU(3) limit.* This markedly differs from the way in which Hara's theorem is satisfied in the standard pole model of Ref. [4]. That is, in Ref. [4] it is only the $\Sigma^+ \to p\gamma$ parity violating amplitude that vanishes in the SU(3) limit, while three remaining relevant WRHD parity-violating amplitudes remain constant and nonzero. The difference between the pole model of Ref. [4] and the pole model considered in this paper is connected to the off-shell behaviour of the $B^*B\gamma$ couplings. Throughout this paper, all our formulas will be consistent with Hara's theorem: we will not refer to Ref. [5] otherwise than in a discussion.

2. Photon–baryon couplings

Let us consider parity-violating, CP-conserving interaction of a photon with spin $1/2^+$ baryons. The most general conserved electromagnetic axial current of spin $1/2^+$ baryons may be written in this case as

$$j_5^\mu = g_{1,kl}(q^2)\overline{\psi}_k(q^2\gamma^\mu - q^\mu \not{q})\gamma_5\psi_l + g_{2,kl}(q^2)\overline{\psi}_k i\sigma^{\mu\nu}\gamma_5 q_\nu \psi_l \,, \qquad (1)$$

where $q = p_l - p_k$ and we use conventions of Ref. [11] for γ matrices. Note the factor of q^2 in the first term of Eq. (1). Indices l, k label initial and final baryons and may be different (*e.g.* $(l,k) = (\Sigma^+, p)$, *etc.*). Hermiticity and CP invariance of $j_5 \cdot A$ coupling require functions g_1, g_2 to be real (see *e.g.* Ref. [12]). Furthermore, g_1 is symmetric and g_2 antisymmetric in baryon indices

$$\begin{aligned} g_{1,kl} &= g_{1,lk} \,, \\ g_{2,kl} &= -g_{2,lk} \,. \end{aligned} \qquad (2)$$

For real photons ($q^2 = 0$, $q \cdot A = 0$) the coupling to a photon of the first term in Eq. (1) vanishes. Thus, the only contribution may come from the second term. Hara's theorem [2] states that in the SU(3) limit the function g_{2,Σ^+p} must vanish. The reason is simple: in the SU(3) limit wave functions of Σ^+

and p must be identical since they are obtained from each other by a simple replacement $s \leftrightarrow d$. Furthermore, photon is a U-spin singlet. Thus, function g_{2,Σ^+p} must be proportional to $g_{2,pp}$ (apart from the Cabibbo factor, nothing changes when we replace s by d in Σ^+). Because of its antisymmetry the function $g_{2,pp}$ is, however, zero. This proof does not specify, however, in what way the function g_{2,Σ^+p} vanishes. Furthermore, it says nothing about functions $g_{2,kl}$ for the remaining three WRHD's: $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$.

In the pole model of Ref. [4] WRHD's proceed in two stages: a virtual decay of the initial ground-state baryon B_i into a photon and an excited spin $1/2^-$ B^* baryon followed by a weak interaction transforming the latter into a final ground-state baryon B_f (a reverse order of interactions is of course also taken into account). To describe these processes one has to know in particular the $B^*B\gamma$ couplings.

In Ref. [4] these couplings are given in the form of a parity-conserving interaction of the photon with a current whose form (after setting $q^2 = q \cdot A = 0$) is fully analogous to Eq. (1)

$$j^{\mu}_{(2)}(B^*B) = f_{2,kl}(q^2)\overline{\psi}_k i\sigma^{\mu\nu}\gamma_5 q_\nu \psi_l \,, \tag{3}$$

where a pair of indices k, l denotes a pair of baryons B, B^* under consideration, *i.e.* $(k,l) \equiv (B^*_k, B_l)$ or (B_k, B^*_l). Following Ref. [12] one can check that hermiticity and CP invariance of $j_{(2)} \cdot A$ coupling require function f_2 to be purely imaginary and symmetric

$$f_{2,kl} = f_{2,lk} \,. \tag{4}$$

In Ref. [4] the corresponding function is stated to be real and antisymmetric. This difference is inessential because one can always absorb our purely imaginary phase of f_2 into the definition of the spinor of the intermediate excited state. The relation valid for both our convention and that of Ref. [4] is $f_2^\dagger = -f_2$.

There is one problem with Eq. (3) that was not discussed in Ref. [4] at all: the form of the right-hand side of Eq. (3) is <u>not</u> the most general form for the situation under consideration. In fact, Eq. (3) is fully correct only when particles B^*, B are *on their mass shells*. In the pole model, however, the intermediate excited states are certainly not on their mass shells. Thus, the use of Eq. (3) is not fully justified.

To substantiate our claim we shall consider the current

$$j^{\mu}_{(1)}(B^*B) = f_{1,kl}(q^2)(-i)(p_k + p_l)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{\psi}_k\gamma_\rho\psi_l \tag{5}$$

which is quadratic in external momenta. As before, $(k,l) = (B^*_k, B_l)$ or (B_k, B^*_l). Hermiticity and CP invariance of the coupling of $j_{(1)}$ to a photon

require f_1 to be purely imaginary and antisymmetric ($f_1^\dagger = f_1$ in phase-convention-independent form). We observe that a form totally analogous to Eq. (5) might also be used as an axial current relevant for describing the parity violating coupling of a photon to ground-state baryons

$$\widetilde{j_5^\mu} = \tilde{g}_{kl}(q^2)(-i)(p_k + p_l)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{\psi}_k\gamma_\rho\psi_l \tag{6}$$

with initial and final spin $1/2^+$ baryons k, l. Hermiticity and CP invariance of $\widetilde{j_5} \cdot A$ interaction require $\tilde{g}$ to be real and symmetric

$$\tilde{g}_{kl} = \tilde{g}_{lk}\,. \tag{7}$$

Using the identity

$$\gamma^\alpha\gamma^\beta\gamma^\mu = g^{\alpha\beta}\gamma^\mu - g^{\alpha\mu}\gamma^\beta + g^{\beta\mu}\gamma^\alpha - i\gamma_5\varepsilon^{\alpha\beta\mu\nu}\gamma_\nu \tag{8}$$

it is straightforward to show that

$$\begin{aligned} &-i(p_k + p_l)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{\psi}_k\gamma_\rho\psi_l \\ &= \overline{\psi}_k(q^2\gamma^\mu - q^\mu \not{q})\gamma_5\psi_l + \overline{\psi}_k(\not{p}_k i\sigma^{\mu\nu}\gamma_5 q_\nu - i\sigma^{\mu\nu}\gamma_5 q_\nu \not{p}_l)\psi_l\,. \end{aligned} \tag{9}$$

Thus, for particles k, l on their mass shell the current $\widetilde{j_5}$ of Eq. (6) reduces to the current j_5 of Eq. (1) with $g_{1,kl} = \tilde{g}_{kl}$ and $g_{2,kl} = (m_k - m_l)\tilde{g}_{kl}$. Interaction with real transverse photons of the first term on the rhs of Eq. (9) vanishes. As to the second term, please note that the obtained function $g_{2,kl}$ is antisymmetric and that it vanishes for equal masses of baryons k, l. Although for the parity-conserving current $j^\mu_{(1)}(B^*B)$ the identity of Eq. (9) also holds, in the pole model of WRHD's one cannot in general replace $\not{p}_k$ and $\not{p}_l$ by the corresponding baryon masses: the intermediate baryons B^* are not on their mass shell. We shall see later what are the consequences of this lack of sufficient generality of the current of Eq. (3).

3. Parity–violating amplitudes in pole model

The pole model is built from two basic building blocks. The first describes weak interaction, the second - electromagnetic emission of a photon. Parity violation comes from weak interactions which transform ground-state baryons into excited spin $1/2^-$ baryons and *vice versa*.

The parity-violating weak transitions are described by

$$a_{kl}\overline{\psi}_k\psi_l\,, \tag{10}$$

where the pair of indices k, l describes a pair of baryons (B,B^*), *i.e.* $(k,l) = (B_k^*, B_l)$ or (B_k, B_l^*). Hermiticity and CP invariance require a to be purely imaginary and antisymmetric

$$a_{kl} = -a_{lk}\,. \tag{11}$$

(Again we differ in conventions with Ref. [4], where a is real and symmetric. A convention-independent condition is $a^\dagger = a$.)

The electromagnetic emission is described by coupling the photon to the sum $j(B^*B)$ of currents of Eqs. (3),(5)

$$\begin{aligned} j^\mu(B^*B) \;=\; & f_{1,kl}(q^2)(-i)(p_k+p_l)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{\psi}_k\gamma_\rho\psi_l \\ & + f_{2,kl}(q^2)\overline{\psi}_k i\sigma^{\mu\nu}\gamma_5 q_\nu \psi_l\,. \end{aligned} \tag{12}$$

The calculation of Ref. [4] corresponds to $f_1 = 0$, $f_2 \neq 0$ and leads to the pattern $(-,-,-,-)$ (see *e.g.* Ref. [1]). Since this case was studied elsewhere [1, 4], we will consider it only in a discussion, a little later. The really novel feature is the first term (f_1) on the right hand side of Eq. (12). We turn now to the evaluation of its effects. We will show that this term generates asymmetry pattern $(-,+,+,-)$.

There are two pole-model diagrams (Fig. 1(a),(b)) contributing to the decay $B_i \to B_f\gamma$. The amplitude corresponding to these diagrams is built from our basic blocks in a simple way. Weak interaction (symbolized by blobs in Fig. 1) is described by Eq. (10) while the electromagnetic current by Eq. (12). In addition, there must be a pole factor $1/(p^2 - m_*^2)$ corresponding to the propagation of the off-shell excited baryon B^*.

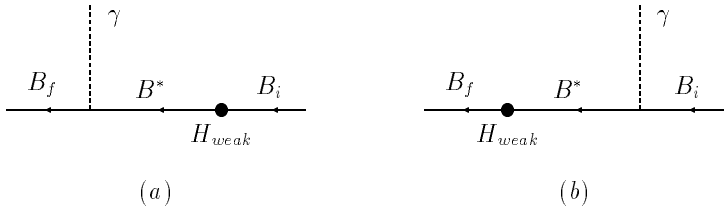

Fig. 1. Baryon-pole diagram for parity-violating WRHD amplitudes.

Using the first term ($j_{(1)}$) of the current of Eq. (12) the following expression corresponds then to Fig. 1(a):

$$f_{1,fk^*}(-i)(p_f+p_{k^*})_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{u}_f\gamma_\rho u_{k^*}\cdot\frac{1}{p_i^2-m_{k^*}^2}\cdot a_{k^*i}\overline{u}_{k^*}u_i\,, \tag{13}$$

where k^* labels intermediate excited states (summation over admissible k^* is implied). The contribution corresponding to Fig. 1(b) is

$$a_{fk^*}\overline{u}_f u_{k^*}\cdot\frac{1}{p_f^2-m_{k^*}^2}\cdot f_{1,k^*i}(-i)(p_{k^*}+p_i)_\lambda q_\nu\varepsilon^{\lambda\mu\nu\rho}\overline{u}_{k^*}\gamma_\rho u_i \tag{14}$$

with appropriate m_{k^*}, different from that in Eq. (13). However, since we are mainly concerned with the limit $m_s - m_d \to 0$, for our purposes it is

sufficient to consider $1/2^+ - 1/2^-$ mass splitting to be much larger than $m_s - m_d$. Thus, we may put the same m_{k^*} everywhere. Upon summing the above two contributions and replacing the factor $u_{k^*}\overline{u}_{k^*}$ by $\not{p}_{k^*} + m_{k^*}$, we act with $\not{p}_{k^*}$ on u_i ($\overline{u}_f$) for the contributions of Fig 1(a), 1(b), respectively. This yields m_i (m_f). Using $p_i^2 = m_i^2$ and $p_f^2 = m_f^2$ we obtain the total pole-model contribution from f_1 terms

$$-i(p_i + p_f)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{u}_f\gamma_\rho u_i \cdot \left\{\frac{f_{1,fk^*}a_{k^*i}}{m_i - m_{k^*}} + \frac{a_{fk^*}f_{1,k^*i}}{m_f - m_{k^*}}\right\}. \qquad (15)$$

Now, for real photons and external baryons on their mass shell the factor in front of the braces in Eq. (15) can be reduced using Eq. (9). In this way, Eq. (15) is brought into our final form and the parity-violating WRHD amplitude is obtained from

$$(m_f - m_i)\left\{\frac{f_{1,fk^*}a_{k^*i}}{m_i - m_{k^*}} + \frac{a_{fk^*}f_{1,k^*i}}{m_f - m_{k^*}}\right\} \cdot \overline{u}_f i\sigma^{\mu\nu}q_\nu\gamma_5 u_i A_\mu \,. \qquad (16)$$

As Eq. (16) shows, <u>all</u> parity-violating WRHD amplitudes vanish now in the limit $m_i \to m_f$. Furthermore, this vanishing does not come about as a result of the cancellation between the contributions from the s- and u-channel poles as in Ref. [4]. In fact, for $f = i$ the denominators of the two terms in braces are identical and the same can be shown to hold for the numerators since: 1) $f_{1,fk^*} = f_{1,ik^*} = -f_{1,k^*i}$ and 2) $a_{fk^*} = a_{ik^*} = -a_{k^*i}$ leads to $f_{1,fk^*}a_{k^*i} = (-f_{1,k^*i})(-a_{fk^*})$. One can also easily see that under $i \leftrightarrow f$ interchange the expression in braces in Eq. (16) is symmetric, *i.e.* $\{\ldots\}_{if} = +\{\ldots\}_{fi}$, and therefore the whole expression $(m_f - m_i)\{\ldots\}$ is antisymmetric , in agreement with the second of Eqs. (2).

Let us now try to use the current $j_{(1)}$ while putting intermediate baryons B^* on their mass shell. For real transverse photons the current $j_{(1)}$ of Eq. (5) may be then reexpressed using the simplified version of Eq. (9)

$$-i(p_k + p_l)_\lambda q_\nu \varepsilon^{\lambda\mu\nu\rho}\overline{u}_k\gamma_\rho u_l = (m_k - m_l)\overline{u}_k i\sigma^{\mu\nu}q_\nu\gamma_5 u_l \qquad (17)$$

for $(k,l) = (B_k^*, B_l)$ or (B_k, B_l^*). The electromagnetic currents in Eqs. (13), (14) are then replaced by

$$f_{1,k^*i}(m_{k^*} - m_i)\overline{u}_{k^*}i\sigma^{\mu\nu}q_\nu\gamma_5 u_i \qquad (18)$$

for Fig. 1(a) (Eq. (13)) and

$$f_{1,fk^*}(m_f - m_{k^*})\overline{u}_f i\sigma^{\mu\nu}q_\nu\gamma_5 u_{k^*} \qquad (19)$$

for Fig. 1(b) (Eq. (14)). Please note that now the factors $f_{1,kl^*}(m_k - m_{l*})$ multiplying spinorial expressions in Eqs. (18), (19) have symmetry properties of the f_2 factors, *i.e.* they are symmetric under $k \leftrightarrow l^*$ interchange,

as in Eq. (4). We might write $\tilde{f}_{2,fk^*} \equiv f_{1,fk^*}(m_f - m_{k^*})$ and $\tilde{f}_{2,k^*i} \equiv f_{1,k^*i}(m_{k^*} - m_i)$ with $\tilde{f}_2$ symmetric, and thus fully analogous to f_2 in Eqs. (3), (12). Consequently, results of Ref. [4] should follow. Indeed, applying the procedure described above for the true current $j_{(1)}$ we obtain now the counterpart of Eq. (16) for the current of Eqs. (18), (19)

$$\left\{ \frac{f_{1,fk^*}(m_f - m_{k^*})a_{k^*i}}{m_i - m_{k^*}} + \frac{a_{fk^*}f_{1,k^*i}(m_{k^*} - m_i)}{m_f - m_{k^*}} \right\} \cdot \overline{u}_f i\sigma^{\mu\nu} q_\nu \gamma_5 u_i \,. \qquad (20)$$

Now, for $f = i$ the denominators of the two terms in Eq. (20) are identical but the numerators differ in sign since: $f_{1,fk^*} = f_{1,ik^*} = -f_{1,k^*i}$, $a_{fk^*} = a_{ik^*} = -a_{k^*i}$ and $(m_f - m_{k^*}) = -(m_{k^*} - m_i)$. Thus, for $f = i$ the two terms in Eq. (20) cancel. This is precisely the case considered in Ref. [4] where only the current $j_{(2)}$ was considered and the cancellation between the two diagrams of Fig. 1 was invoked as a way in which Hara's theorem is satisfied. In Ref. [4] such a cancellation does not occur, however, for the remaining three relevant WRHD's, namely $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$.

4. Discussion

Phenomenologically, the most successful model seems to be the VMD model of Ref. [7] (and its update in Ref. [1]). In the VMD approach the crucial assumption (apart from the VMD prescription) is the assumed $SU(6)_W$ symmetry relating WRHD's to the well measured experimentally nonleptonic hyperon decays (NLHD's). Thus, the size and the pattern of parity violating WRHD amplitudes are determined by symmetry from NLHD's.

The symmetry structure of the parity-violating WRHD and NLHD amplitudes of Refs. [7] may be understood in terms of the pole model. In view of:

(1) considerations of the preceding section in which two different possible patterns of WRHD asymmetries were obtained in the pole model, and

(2) the symmetry connection between WRHD's and NLHD's that forms the basis of the successful approach of Refs. [7],

it is pertinent to discuss nonleptonic hyperon decays in the pole model along the lines of the preceding section and to study the relation between the symmetry structures of WRHD's and NLHD's. This is what we will turn to now.

For the sake of further discussion let us assume that masses of octet pseudoscalar mesons are negligible, $m_P^2 \approx 0$. Thus, we shall discuss the parity-violating CP-conserving amplitudes for the $B_i \to P^0 B_f$ couplings

with P^0 a $CP = -1$ pseudoscalar meson (π^0 or η_8) and $B_{i,f}$ — ground-state baryons. Consider the following coupling

$$b_{fi}^{(0)}\overline{u}_f u_i P^0 + b_{fi}^{(1)}\overline{u}_f \not{q} u_i P^0 + b_{fi}^{(2)}\overline{u}_f(-i\sigma_{\mu\nu}(p_f + p_i)^\mu q^\nu)u_i P^0 \,, \tag{21}$$

where (by CP-invariance and hermiticity) all $b^{(n)}$ are imaginary, with $b_{fi}^{(0)}$, $b_{fi}^{(2)}$ antisymmetric and $b_{fi}^{(1)}$ symmetric under $i \leftrightarrow f$ interchange. For baryons B_f, B_i on mass shell the coupling of Eq. (21) may be rewritten ($q^2 = m_P^2$) as

$$\left\{ b_{fi}^{(0)} + (m_i - m_f) b_{fi}^{(1)} + [(m_i - m_f)^2 - m_P^2] b_{fi}^{(2)} \right\} \overline{u}_f u_i P^0 \,, \tag{22}$$

where we may put $m_P^2 = 0$. The *a priori* possible term $b_{fi}^{(1')}\overline{u}_f(\not{p}_f + \not{p}_i)u_i P^0$, linear in external momenta, may be absorbed into the $b_{fi}^{(0)}$ term.

In the pole model of NLHD's the couplings of Eq. (22) arise from the parity-violating weak transition of Eq. (10) followed by parity-conserving π^0 (or η_8) emission from the excited spin $1/2^-$ baryon (a reverse order of interactions is also taken into account). Consider parity-conserving P^0 emission couplings described by

$$f_{kl}^{(0)}\overline{u}_k u_l P^0 + f_{kl}^{(1)}\overline{u}_k \not{q} u_l P^0 + f_{kl}^{(2)}\overline{u}_k(-i\sigma_{\mu\nu}(p_k + p_l)^\mu q^\nu)u_l P^0 \tag{23}$$

with $(k, l) = (B_k^*, B_l)$ or (B_k, B_l^*). Hermiticity and CP-invariance require all $f_{kl}^{(n)}$ to be real with $f_{kl}^{(0)}$, $f_{kl}^{(2)}$ symmetric and $f_{kl}^{(1)}$ asymmetric under $k \leftrightarrow l$ interchange. Since excited intermediate spin $1/2^-$ baryon is not on its mass shell we are not allowed to replace Eq. (23) by a momenta-independent form analogous to Eq. (22). (In Eq. (23) we have neglected an a priori possible term $f_{kl}^{(1')}\overline{u}_k(\not{p}_k + \not{p}_l)u_l P^0$; calculation shows that its effect is fully analogous to that of the $f^{(0)}$ term.) Working out the pole model contributions from various terms of Eq. (23) we obtain (as in the previous section)

(1) from the $f^{(0)}$ term

$$\left\{ \frac{f_{fk^*}^{(0)} a_{k^*i}}{m_i - m_{k^*}} + \frac{a_{fk^*} f_{k^*i}^{(0)}}{m_f - m_{k^*}} \right\} \overline{u}_f u_i P^0 \tag{24}$$

with the factor in braces antisymmetric under $i \leftrightarrow f$ interchange (this is the term usually considered in papers on nonleptonic hyperon decays),

(2) from the $f^{(1)}$ term

$$(m_i - m_f) \left\{ \frac{f_{fk^*}^{(1)} a_{k^*i}}{m_i - m_{k^*}} + \frac{a_{fk^*} f_{k^*i}^{(1)}}{m_f - m_{k^*}} \right\} \overline{u}_f u_i P^0 \tag{25}$$

with the factor in braces symmetric under $i \leftrightarrow f$ interchange,

(3) from the $f^{(2)}$ term

$$[(m_i - m_f)^2 - m_P^2] \left\{ \frac{f^{(2)}_{fk^*} a_{k^* i}}{m_i - m_{k^*}} + \frac{a_{fk^*} f^{(2)}_{k^* i}}{m_f - m_{k^*}} \right\} \overline{u}_f u_i P^0 \qquad (26)$$

with the factor in braces antisymmetric under $i \leftrightarrow f$ interchange. Thus, the pole model yields specific predictions for $b^{(0)}_{fi}$, $b^{(1)}_{fi}$, and $b^{(2)}_{fi}$ of Eq. (22), which are given by factors in braces in Eqs. (24), (25), (26).

Assuming now that one of the two patterns of parity-violating NLHD amplitudes (corresponding to the symmetry or antisymmetry of the factor in braces) is dominant, there appears the question which pattern is actually realized in Nature.

Calculations of Desplanques, Donoghue and Holstein (Ref. [13]) and those of Ref. [7] correspond to the pattern obtained from terms $f^{(0)}$ or $f^{(2)}$, which coincides with the predictions of current algebra. For the sake of comparison with Eqs. (24)–(26) in Table I we give a few selected amplitudes corresponding to the symmetry pattern of these references. Table I explicitly demonstrates the antisymmetry of the factor $\{\ldots\}$ under $\Sigma^+ \leftrightarrow p$ ($p \leftrightarrow p$) interchange and the cancellation between the contributions from diagrams 1(a) and 1(b) for $f = i$: for $pp\pi^0$ case antisymmetry ensures vanishing of the total contribution to the parity-violating $pp\pi^0$ coupling. This is also what current algebra gives [13] since $\langle p\pi^0|H_W^-|p\rangle \propto \langle p|[I_3, H_W^+]|p\rangle = 0$. Such vanishing occurs also for $\Sigma^+ \to pU^0$ coupling where $U^0 = (\sqrt{3}\pi^0 + \eta_8)/2$, a U-spin singlet.

TABLE I

Contribution of diagrams 1(a) and 1(b) to selected parity-violating $BB'P^0$ amplitudes.

	diagram 1(a)	diagram1(b)
$\langle p\pi^0\|H_W^-\|\Sigma^+\rangle$	$-\frac{1}{6\sqrt{2}}c$	$\frac{1}{2\sqrt{2}}b$
$\langle \Sigma^+\pi^0\|H_W^-\|p\rangle$	$-\frac{1}{2\sqrt{2}}b$	$\frac{1}{6\sqrt{2}}c$
$\langle p\pi^0\|H_W^-\|p\rangle$	$\left(-\frac{1}{2\sqrt{2}}b - \frac{1}{6\sqrt{2}}c\right)\cot\theta_C$	$\left(\frac{1}{2\sqrt{2}}b + \frac{1}{6\sqrt{2}}c\right)\cot\theta_C$
$\langle pU^0\|H_W^-\|\Sigma^+\rangle$	$-\frac{1}{2\sqrt{6}}b - \frac{1}{6\sqrt{6}}c$	$\frac{1}{2\sqrt{6}}b + \frac{1}{6\sqrt{6}}c$

In Table I, the b-term originates from W-exchange diagrams, while the c-term represents hadronic loop/quark-sea contribution [15]. Although W-exchange seems to contribute to diagram 1(b) only, this does not mean that individual contributions from W-exchange with nonstrange intermediate excited baryons are all zero. They do not vanish but they all cancel

among themselves (*cf.* [14]). Experimental data on NLHD's cannot determine which of the two patterns (corresponding to $f^{(0)}/f^{(2)}$ or $f^{(1)}$) is correct. This is so because in all π^0 emission amplitudes the b-terms come solely from diagrams 1(b) and the c-terms - solely from diagrams 1(a). Since the size and sign of c is a phenomenological parameter it is impossible to differentiate between the two patterns. If η_8 (U^0) emission were kinematically allowed, this would be possible: cancellation of two contributions to the $\langle pU^0|H_W^-|\Sigma^+\rangle$ amplitude would be replaced by constructive interference from diagrams 1(a) and 1(b).

Let us now go back to WRHD's. The connection between NLHD's and WRHD's is achieved in Ref. [7] by considering the combined flavour-spin symmetry $\mathrm{SU}(6)_W$. This symmetry is suited for the description of two-body decays because spin generators of $\mathrm{SU}(2)_W$ commute with Lorentz boosts along decay axis [16]. Consequently, if one wants to apply $\mathrm{SU}(2)_W$ it is appropriate to choose one of Lorentz frames obtained from the initial particle rest frame by boosts along decay axis. Thus, we choose any frame in which $\boldsymbol{p}_i + \boldsymbol{p}_f = \lambda \boldsymbol{q}$ with arbitrary λ.

For further discussion let us recall the following identity:

$$\overline{u}_f i\sigma^{\mu\nu}\gamma_5 q_\nu u_i = (m_f - m_i)\overline{u}_f\gamma^\mu\gamma_5 u_i - (p_i + p_f)^\mu \overline{u}_f\gamma_5 u_i\,. \tag{27}$$

After fixing the gauge to be the Coulomb one ($A_0 = 0$, $\boldsymbol{A}\cdot\boldsymbol{q} = 0$), the second term on the right hand side of Eq. (27) decouples from the photon. Thus, in the $\mathrm{SU}(2)_W$-symmetric framework, the terms $\overline{u}_f i\sigma^{k\nu}\gamma_5 q_\nu u_i A_k$ and $\overline{u}_f\gamma^k\gamma_5 u_i A_k$ lead to amplitudes proportional to each other, the coefficient of proportionality being $m_i - m_f$. Consequently, the model of Ref. [4] ($f_1 = 0$, $f_2 \neq 0$) generates the same amplitudes as

$$(m_i - m_f)B_{if}^{(1)}\cdot\overline{u}_f\gamma^k\gamma_5 u_i A_k\,, \tag{28}$$

where $B_{if}^{(1)}$ denotes the term (asymmetric under $i \leftrightarrow f$ interchange) in braces in Eq. (20) with $\tilde{f}_2$ replaced by f_2. For the present paper ($f_1 \neq 0$, $f_2 = 0$), Eq. (16) corresponds to

$$(m_i - m_f)^2 B_{if}^{(2)}\cdot\overline{u}_f\gamma^k\gamma_5 u_i A_k\,, \tag{29}$$

where $B_{if}^{(2)}$ denotes the term (symmetric under $i \leftrightarrow f$) in braces in Eq. (16).

The result of Kamal–Riazuddin [5] corresponds to the expression $(m_i - m_f)^0 B_{if}^{(0)}\cdot\overline{u}_f\gamma^k\gamma_5 u_i A_k$ with some symmetric $B_{if}^{(0)}$.

In general, the factors $B_{if}^{(k)}$ do not vanish for $m_i = m_f$. Symmetry properties of factors $B^{(0)}$ and $B^{(2)}$ are identical and, consequently, they

lead to the same pattern of asymmetries: $(-,+,+,-)$. On the other hand, dominance of the $B^{(1)}$ term would lead to the pattern $(-,-,-,-)$.

If new experiments confirm the pattern $(-,+,+,-)$ which seems to be favoured by the older data (Refs. [1]), it will mean that the dominant parts of all parity violating WRHD amplitudes are proportional to an even power of $m_i - m_f$. Thus, one of two possibilities below must hold. Either

(1) Hara's theorem is violated as in the quark model calculations of Ref. [5] with $B_{if}^{(0)} \neq 0$, or

(2) Hara's theorem is satisfied as a byproduct of vanishing (in the limit $m_i \to m_f$) of all parity-violating WRHD amplitudes. (This vanishing may be approximate for those decays where a nonzero $B^{(1)}$ of Eq. (28) may contribute). This corresponds to $B^{(0)} = 0$, $(m_\Sigma - m_N)B^{(2)} \gg B^{(1)} \approx 0$. In this case, the observed large asymmetry of $\Sigma^+ \to p\gamma$ decay should not surprise us too much. To say that the size of the relevant parity-violating amplitude is "large" means that we have to compare it with some standard size. Thus, we should compare the $\Sigma^+ \to p\gamma$ amplitude with other parity-violating amplitudes of WRHD's. However, since they all vanish in the SU(3) limit in the same way as the $\Sigma^+ \to p\gamma$ amplitude does, the relative size of the latter amplitude is large indeed.

Within the $SU(6)_W\times$ VMD approach one expects that in NLHD's and WRHD's the terms of the same order in $m_i - m_f$ are symmetry-related. Thus, if $SU(6)_W\times$ VMD predictions for the WRHD asymmetries are borne out by the data and one insists that Hara's theorem is to be satisfied, this would mean that only the contributions from $f^{(2)}$ terms should be present in NLHD's and that, consequently, the parity-violating NLHD amplitudes should vanish in the SU(3) limit.

However, since the mass of the decaying particle is not a free parameter, one cannot differentiate between contributions of type $f^{(0)}$ and $f^{(2)}$ using data on hyperon nonleptonic decays alone. Nonetheless, instead of considering $\Delta S = 1$ decays, one may study $\Delta S = 0$ parity-violating NNM couplings, and try to see if mass-dependence characteristic of $f^{(2)}$ (or perhaps $f^{(1)}$) terms is present in these couplings. In theoretical calculations mass-dependence characteristic of the $f^{(1)}$ term was obtained in the past [17], leading to $A(n_-^0)$ of order $(m_n - m_p)/(m_\Sigma - m_p) \approx 10^{-2}$ times the "best values" of Ref. [13]. If the NLHD and WRHD amplitudes are indeed proportional to $(m_i - m_f)^2$ as the signature $(-,+,+,-)$ and insistence on satisfying Hara's theorem would demand, then one would expect totally negligible weak parity-violating $NN\pi$ and NNV couplings. At present, data seem to indicate that these couplings, although somewhat smaller than the "best value"

prediction of Ref. [13], are nonetheless of the same order [18, 19]. Totally negligible value of weak $NN\pi$ coupling is also possible [20]. However, the general order of magnitude of NNM couplings is consistent with the lack of the $(m_i - m_f)^2$ factor [18, 19]. Hence, although in principle it is possible that the signature $(-,+,+,-)$ for the WRHD asymmetries is consistent with Hara's theorem, the underlying approach leads then to negligible weak NNM couplings in disagreement with experiment.

5. Conclusions

We have studied parity-violating WRHD amplitudes in the pole model. In this model the properties of these amplitudes depend on the properties of the parity-conserving $1/2^- - 1/2^+ - \gamma$ couplings. Two different conserved electromagnetic local baryonic currents have been used for the description of the transition of an on-shell ground-state baryon into an off-shell excited baryon (or vice versa). Although the two currents become indistinguishable for a transition between on-shell baryons, they are inequivalent when baryons are off-shell. As a result, the two currents lead to different patterns of asymmetries in weak radiative hyperon decays. We have shown that in the pole model with Hara's theorem explicitly satisfied it is still possible to obtain the asymmetry pattern $(-,+,+,-)$ that is characteristic of the quark model. Thus, the pattern $(-,+,+,-)$ is not an unmistakable sign of the violation of Hara's theorem. Phenomenological success of the $\mathrm{SU}(6)_W\times$ VMD approach to WRHD's may be understood as being consistent with Hara's theorem if the dominant parts of all WRHD and NLHD parity-violating amplitudes vanish in the SU(3) limit. Although the success of the $\mathrm{SU}(6)_W\times$ VMD approach does not necessarily demand violation of Hara's theorem, it requires totally negligible weak NNM couplings if Hara's theorem is to be satisfied. Data on hadronic parity violation indicate that no such suppression of NNM couplings occurs in reality, however. Thus, if the pattern $(-,+,+,-)$ of WRHD asymmetries (*i.e.*, especially, the positive sign of the $\Xi^0 \to \Lambda\gamma$ asymmetry) is confirmed, then, together with the non-negligible size of weak NNM couplings this would indicate violation of Hara's theorem.

A Hara's-theorem-saving alternative to this conclusion is to accept non-negligible NNM couplings but to relinquish the VMD assumption. That is, one has to accept that although VMD works for WRHD's (if it does indeed), it cannot be used for the determination of parity-violating coupling of photon to proton. The success of VMD for WRHD's looks then more like an accident. Although such a standpoint is a logical possibility it leaves us in a *very bad* situation because it means that our most cherished and successful models of electromagnetic properties of hadrons (*i.e.* both the VMD *and* the

quark model which violates Hara's theorem explicitly) are useless when the parity-violating coupling of photon to proton is to be described. Therefore, rather than to disregard the predictions of both the quark model and the VMD, I prefer to accept that Hara's theorem is indeed violated and to study the meaning and origin of this violation.

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