MASS DEPENDENCE OF THE HBT RADII OBSERVED IN e^+e^- ANNIHILATION*

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It is shown that the recently established strong mass-dependence of the radii of the hadron sources, as observed in HBT analyses of the e^+e^- annihilation, can be explained by assuming a generalized inside–outside cascade, *i.e.* that *(i)* the *four*-momenta and the space-time position *four*-vectors of the produced particles are approximately proportional to each other and *(ii)* the "freeze-out" times are distributed along the hyperbola $t^2 - z^2 = \tau_0^2$.

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It has been found recently that the parameters describing the B–E interference in e^+e^- annihilation depend strongly on the masses of the particles used in the analysis [1, 2]. One finds r_π between 0.7 and 1 fm; r_K between 0.5 and 0.7 fm; r_Λ between 0.1 and 0.2 fm.

In the present note we suggest that this dependence can be understood if the produced particles satisfy approximately the (generalized) Bjorken–Gottfried conditions [3, 4]:

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(i) The 4-momentum q_μ and the 4-vector x_μ describing the space-time position of the production ("freeze-out") point of a particle are proportional

$$q_\mu = \lambda x_\mu . \tag{1}$$

The proportionality factor λ is a scalar with respect to boosts in the longitudinal direction.

(ii) Particles are produced at a fixed proper time τ_0 after the collision

$$t^2 - z^2 = \tau_0^2 \,, \tag{2}$$

where t, z are time and longitudinal position of the production point.

From (1) and (2) we derive

$$\lambda = \frac{M_\perp}{\tau_0} \,, \tag{3}$$

where $M_\perp^2 = E^2 - q_{||}^2$. Thus finally we have

$$q_\mu = \frac{M_\perp}{\tau_0} x_\mu \,. \tag{4}$$

This picture is, of course, purely classical and can only be treated as a heuristic guide-line when applied to actual production processes. A more adequate formulation of these conditions can be achieved using the Wigner representation $W(P,x)$ of the (single-particle) density matrix which, as is well known (see *e.g.* [5]), corresponds — as close as possible without contradicting quantum mechanics — to the space-time and momentum distribution of the produced particles. To implement the conditions *(i)*, *(ii)* above, we postulate $W(P,x)$ in the form

$$\begin{aligned} W(P,x) &\sim \delta(t^2 - z^2 - \tau_0^2) \exp\left[-\frac{x_\perp^2}{2R_\perp^2} - \frac{P_\perp^2}{2\Delta_\perp^2}\right] \\ &\times \exp\left[-\frac{\left(P_+ - \frac{M_\perp}{\tau_0}x_+\right)^2 + \left(P_- - \frac{M_\perp}{\tau_0}x_-\right)^2}{2\delta_{||}^2} - \frac{\left(P_\perp - \frac{M_\perp}{\tau_0}x_\perp\right)^2}{2\delta_\perp^2}\right] , \end{aligned} \tag{5}$$

where

$$x_\pm = t \pm z; \quad P_\pm = P_0 \pm P_z \tag{6}$$

so that

$$M_\perp^2 = P_+ P_-; \quad \tau_0^2 = x_+ x_- \,. \tag{7}$$

The first exponential represents a standard cylindrically symmetric "longitudinal" distribution in momentum and in configuration space[1]. The new point is the second exponential which introduces correlation between the momentum and the point of emission of the particle, as required by the generalized Bjorken–Gottfried condition (4). Such correlations are known to influence strongly the HBT effect on particle spectra [6]. It is thus this factor which, we think, is responsible for the mass dependence of the observed HBT radii[2].

To derive HBT correlations we need to calculate from (5) the density matrix in momentum space (see *e.g.* [7–10]). This can be done using the relation between $W(P,x)$ and $\rho(q,q')$ which reads

$$\rho\left(q = P + \frac{Q}{2}, q' = P - \frac{Q}{2}\right) = \int d^4x e^{iQx} W(P,x). \tag{8}$$

From (8) we see that now we have to take

$$P = \frac{q+q'}{2}; \quad M_\perp^2 = P_+ P_-; \quad Q = q - q' \tag{9}$$

with the 4-momenta q and q' on the mass-shell.

To continue, it is convenient, as usual, to introduce the rapidities

$$Y = \frac{1}{2}\log\frac{P_+}{P_-}; \quad \eta = \frac{1}{2}\log\frac{x_+}{x_-}. \tag{10}$$

The longitudinal integral

$$\begin{aligned} I_{||} \equiv \int d\eta \exp\left(-\frac{M_\perp^2}{2\delta_{||}^2}\left[\left(e^Y - e^\eta\right)^2 + \left(e^{-Y} - e^{-\eta}\right)^2\right]\right) \\ \times \exp\left(i\tau_0\left[m_\perp \cosh(y-\eta) - m'_\perp \cosh(y'-\eta)\right]\right), \end{aligned} \tag{11}$$

where $(m_\perp, y)$ and $(m'_\perp, y')$ are transverse masses and rapidities corresponding to momenta q and q', can be approximated by

$$\begin{aligned} I_{||} \approx \int d\eta \exp\left(-\frac{M_\perp^2}{\delta_{||}^2}(Y-\eta)^2\right) \\ \times \exp\left(i\tau_0\left[m_\perp\left(1 + \frac{(\eta-y)^2}{2}\right) - m'_\perp\left(1 + \frac{(\eta-y')^2}{2}\right)\right]\right). \end{aligned} \tag{12}$$

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[1] To simplify the argument, we ignore the longitudinal momentum and z dependence of the single particle spectrum. This seems a reasonable approximation at high energy.

[2] Admittedly, the form (5) is rather schematic. In particular, gaussians are taken for simplicity and can be replaced if necessary. We also did not include fluctuations of τ_0. These simplifications are not essential for our argument, however.

Ignoring normalization and phase factors, inessential for our argument, we thus obtain

$$I_{||} \sim \exp\left(-i\tau_0 \frac{M_\perp^2}{2B\delta_{||}^2}\left(m_\perp(Y-y)^2 - m'_\perp(Y-y')^2\right)\right)$$
$$\times \exp\left(-\frac{\tau_0^2}{4B} m_\perp m'_\perp (y-y')^2\right), \tag{13}$$

where

$$B = \frac{M_\perp^2}{\delta_{||}^2} - i\frac{\tau_0}{2}(m_\perp - m'_\perp)\,. \tag{14}$$

The transverse integral can be evaluated exactly. Ignoring again the normalization and phase factors we have

$$I_\perp \equiv \exp\left(-\frac{\vec{P}^2}{2\Delta_\perp^2}\right)\int d^2x \exp\left(-\frac{\vec{x}^2}{2R_\perp^2} - \frac{\left(\vec{P} - \frac{M_\perp}{\tau_0}\vec{x}\right)^2}{2\delta_\perp^2} - i\vec{Q}\vec{x}\right)$$
$$\sim \exp\left[-\frac{(\vec{q}+\vec{q}')^2}{8\Delta_{\rm eff}^2} - \frac{(\vec{q}-\vec{q}')^2 R_{\rm eff}^2}{2}\right], \tag{15}$$

where all vectors are two-dimensional (transverse) and

$$\frac{1}{\Delta_{\rm eff}^2} = \frac{\tau_0^2}{M_\perp^2 R_\perp^2 + \tau_0^2\delta_\perp^2} + \frac{1}{\Delta_\perp^2}; \quad R_{\rm eff}^2 = \frac{R_\perp^2\tau_0^2\delta_\perp^2}{M_\perp^2 R_\perp^2 + \tau_0^2\delta_\perp^2}\,. \tag{16}$$

From (15) we find the single particle transverse momentum distribution:

$$\frac{d\sigma}{d^2q_\perp} \sim I_\perp(\vec{q} = \vec{q}' \equiv \vec{q}_\perp) = \exp\left(-\frac{q_\perp^2\tau_0^2}{2(m^2+q_\perp^2)R_\perp^2 + 2\tau_0^2\delta_\perp^2} - \frac{q_\perp^2}{2\Delta_\perp^2}\right). \tag{17}$$

One sees that the average transverse momentum is largely determined by the value of $\Delta_\perp$ which thus cannot be too large if one wants to insure average transverse momentum smaller than, say, 500 MeV.

Let us also note at this point that consistency with uncertainty principle implies the inequality [10]

$$R_{\rm eff}\Delta_{\rm eff} \geq \frac{1}{2}\,. \tag{18}$$

As seen from (16), at large transverse mass $M_\perp$, this inequality can only be satisfied if $\delta_\perp$ is significantly larger than $\Delta_\perp$ (and thus than the average transverse momentum).

To proceed, we shall assume that all correlations between particles which are not caused by Bose–Einstein interference can be neglected. Using the formulation of [10] we thus write the two-particle density matrix as a product

$$\rho(q_1, q_2; q_1', q_2') = \rho(q_1, q_1')\rho(q_2, q_2') \,. \tag{19}$$

It then follows from the general theory of HBT effect (see, *e.g.* [9]) that the observed two-particle distribution is given by

$$\Omega(q_1, q_2) = \rho(q_1, q_1)\rho(q_2, q_2) + \rho(q_1, q_2)\rho(q_2, q_1) \equiv \Omega(q_1)\Omega(q_2)(1 \pm C(q_1, q_2)) \,, \tag{20}$$

where

$$\begin{aligned} C(q_1, q_2) &= C_{||}(q_1, q_2) C_{\perp}(q_1, q_2) \\ &= \frac{\mid I_{||}(q_1, q_2) \mid^2}{I_{||}(q_1, q_1) I_{||}(q_2, q_2)} \frac{\mid I_{\perp}(q_1, q_2) \mid^2}{I_{\perp}(q_1, q_1) I_{\perp}(q_2, q_2)} \end{aligned} \tag{21}$$

describes the HBT correlations.

Using (15) we find

$$C_{\perp} = \mathrm{e}^{-(\vec{q}_1 - \vec{q}_2)^2 R^2_{\mathrm{HBT}}} = \mathrm{e}^{-Q_{\perp}^2 R^2_{\perp \mathrm{HBT}}} \,, \tag{22}$$

where

$$R^2_{\perp \mathrm{HBT}} = R^2_{\mathrm{eff}} - \frac{1}{4\Delta^2_{\mathrm{eff}}} = \frac{\tau_0^2}{M_{\perp}^2 R_{\perp}^2 + \tau_0^2 \delta_{\perp}^2} \left(R_{\perp}^2 \delta_{\perp}^2 - \frac{1}{4} \right) - \frac{1}{4\Delta_{\perp}^2} \,. \tag{23}$$

Since

$$M_{\perp}^2 = \left(\frac{m_{1\perp} + m_{2\perp}}{2} \right)^2 + m_{1\perp} m_{2\perp} \sinh^2 \left(\frac{y_1 - y_2}{2} \right) \,, \tag{24}$$

we conclude that indeed $R^2_{\perp \mathrm{HBT}}$ falls with increasing (transverse) mass of the particle.

For $C_{||}$ we have

$$C_{||}(q_1, q_2) = \exp\left(-R^2_{||\mathrm{HBT}} (m_{1\perp} y_1 - m_{2\perp} y_2 + (m_{1\perp} - m_{2\perp}) Y)^2 \right) \,, \tag{25}$$

where

$$R^2_{||\mathrm{HBT}} = \frac{\tau_0^2 M_{\perp}^2}{2|B^2| \delta_{||}^2} \,. \tag{26}$$

From (14), one sees that also $R^2_{||\mathrm{HBT}}$ falls with increasing $M_{\perp}^2$.

If, as is customary (see *e.g.* [1]), one works in the frame where $Y = 0$, (25) can be written as

$$C_{||}(q_1, q_2) \approx \mathrm{e}^{-R^2_{||\mathrm{HBT}}(q_{1z}-q_{2z})^2} = \mathrm{e}^{-R^2_{||\mathrm{HBT}}Q^2_{||}} \,. \tag{27}$$

This completes the qualitative discussion of the mass effect in our approach. It remains to be seen if the values of the HBT radii given by (23) and (26) can be adjusted to be close to the ones obtained from the LEP data [1, 2].

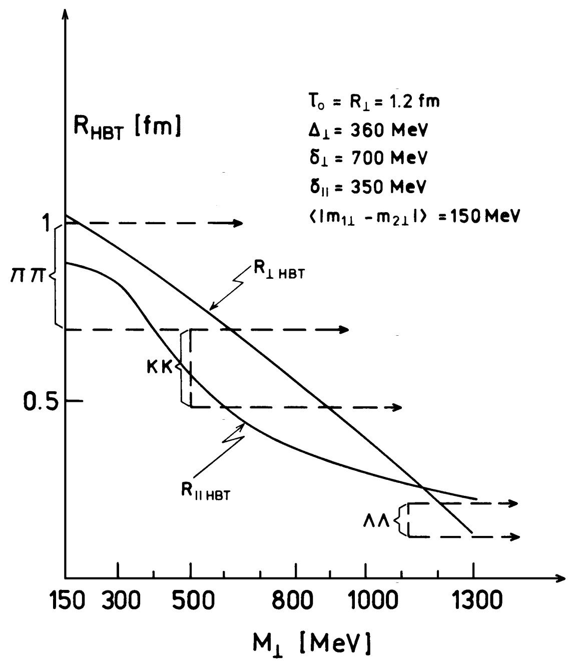

Fig. 1. $R_{\perp\mathrm{HBT}}$ and $R_{||\mathrm{HBT}}$ plotted *versus* $M_\perp$. The parameters are shown in the figure. The data from $\pi\pi$, KK and $\Lambda\Lambda$ correlations are also indicated.

In Fig. 1 $R_{||\mathrm{HBT}}$ and $R_{\perp\mathrm{HBT}}$ are plotted *versus* $M_\perp$, the transverse mass of the two-particle system. The values of other parameters were taken as follows : $\Delta_\perp = 360$ MeV, $\tau_0 = R_\perp = 1.2$ fm, $\delta_\perp = 700$ MeV, $\delta_{||} = 350$ MeV, $|m_{1\perp} - m_{2\perp}| = 150$ MeV. One sees a rather strong mass dependence of both longitudinal and transverse radii. We did not try to fit the obtained values to the data as this would require working directly with data themselves and thus goes beyond the scope of the present investigation. It is nevertheless recomforting to observe that the HBT radii, obtained with "reasonable" values of the model parameters, are not far from the ones found in LEP experiments.

We thus conclude that the existing data on HBT radii are consistent with the hypothesis that — in e^+e^- annihilation at high energy — 4-momentum of a produced particle is approximately proportional to its space time position 4-vector at the freeze-out time[3].

This proportionality is of course well-known for the *longitudinal* components [3,4], and is exhibited explicitly in numerous models [11]. At this point our approach is similar to the one proposed for a longitudinally expanding fireball [9, 13], although the mass dependence following from our Eq. (5) seems somewhat stronger. On the other hand, a rather novel feature following from our analysis is that the original Gottfried–Bjorken proportionality relation should be extended to include also the *transverse* components of the 4-vectors, as explicitly expressed in (4).

Several comments are in order.

(i) It should be emphasized that our argument is only semi-quantitative and can be improved in many details when applied to real data. In particular, the gaussians in the Wigner function (5) can be replaced by more realistic functions for numerical analysis. Also, the Fourier transform (11) can be calculated numerically without approximations shown in (12), which were introduced simply to obtain an analytic result. Finally, including a distribution of τ_0 is probably needed to obtain a good description of data. We feel, however, that all this necessary fine tuning does not invalidate our main conclusion, summarized in Eq. (4).

(ii) As we already mentioned, the results shown in Fig. 1 do not represent a fit to experimental data which we think would be premature at the present stage. Therefore, the values of the parameters used to produce this figure are by no means final. Some of them seem rather stable, however. In particular, $\Delta_\perp$ is closely related to the average transverse momentum and thus cannot be arbitrarily changed. Also a rather large value of $\delta_\perp$ seems necessary to satisfy the consistency condition (18). This means that the correlation between the transverse momentum and transverse position of a particle at freeze-out is fairly weak. It is remarkable that such a weak correlation is sufficient to create a strong variation of $R_{\perp\mathrm{HBT}}$ with the transverse mass of the investigated two-particle system.

(iii) From the point of view of data analysis, our argument emphasizes the importance of the investigation of the HBT correlations as function of the transverse mass of the pion pair.

[3] Recently an alternative interpretation has been proposed in [12].

(iv) Relation (4), when applied to transverse directions, implies the existence of an important "collective transverse flow" in the system of particles produced in e^+e^- annihilation[4]. It would be interesting to search for other evidence of such a "flow" in the data.

(v) A natural modification of the relation (2) is to consider freeze-out times given by the fully Lorentz-invariant formula

$$t^2 - z^2 - y^2 - x^2 = \tau_0^2\,, \qquad (28)$$

which leads to qualitatively similar results as those discussed in the present paper. It is not clear if the present data can distinguish between (2) and (28) but investigation of this question is certainly a challenging issue for future work.

(vi) The recent data of L3 coll. [15] show a strong dependence of the transverse $\pi\pi$ HBT radius (and a somewhat weaker dependence of the longitudinal radius) on the average transverse mass of the two pions $m_\perp = \frac{1}{2}(m_{1\perp} + m_{2\perp})$. This seems not inconsistent with our results, although more work is needed to establish a closer connection between $M_\perp$ and the average transverse mass $m_\perp$ which is used to parametrize the data. Thus before more detailed calculations (including a realistic single particle distribution) are performed, it is not clear to what extent the results shown in Fig. 1 are related to the observations of [15].

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Note added in proof: After this paper has been sent to printers, we have learned that the consequences of Eq. (1) for HBT correlations were discussed earlier [16]. We would like to thank T. Csorgo for calling our attention to this reference.

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