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CLASSICAL DISSIPATIVE FUNCTION AT FINITE MEAN FREE PATH* **

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The dissipative function of slow collective motion in hot nuclei of arbitrary shape is presented in terms of nucleonic trajectories. The expression accounts for finiteness of nucleon mean free path λ. The derivation starts from quantum formula for the dissipation rate of collective energy via the dressed particle-hole propagator. The extreme cases of $\lambda \to \infty$ and $\lambda \to 0$ are studied. As an example, explicit formulas are given for friction coefficients of multipole surface vibrations in spherical leptodermous nuclei.

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1. Introduction

As a starting point for the rate of dissipation $\dot{Q}$ we use the Linear Response Theory expression in terms of the dressed particle–hole propagator [1]

$$\dot{Q} = -\lim_{\omega \to 0} \frac{1}{\omega} \sum_{\mu\nu} \left| \dot{V}_{\mu\nu} \right|^2 (n_\mu - n_\nu) \frac{\Gamma}{(\omega - E_\mu + E_\nu)^2 + \Gamma^2} , \tag{1}$$

where

$$\dot{V}(\boldsymbol{r}) = \frac{\partial V(\boldsymbol{r})}{\partial \sigma} \frac{d\sigma(t)}{dt} , \tag{2}$$

$V(\boldsymbol{r})$ is the mean field depending on the nuclear shape in terms of the collective parameters σ, $\dot{V}_{\mu\nu}$ are the matrix elements of $\dot{V}(\boldsymbol{r})$ on the single-particle states $\psi_\mu(\boldsymbol{r})$, E_μ are the single-particle energies, n_μ are the Fermi gas temperature-dependent occupation numbers, and $\hbar = 1$.

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In the above expression the quantity responsible for the residual interactions is Γ, the spreading width of single-particle states. It can be calculated from

$$\Gamma_\mu = -2 \int d\boldsymbol{r} \psi_\mu^2(\boldsymbol{r}) W(\boldsymbol{r}) \,, \tag{3}$$

where $W(\boldsymbol{r})$ is the imaginary part of the single-particle optical potential. It is available in infinite systems [2] and in finite systems [3] (with simplest versions of Skyrme forces) as a function of particle energy and nuclear temperature T.

2. Quantum expressions for $\dot{Q}$

On introducing the 'soft' δ-function

$$\delta_\Gamma(x) = \frac{1}{2\pi} \frac{\Gamma}{x^2 + \frac{\Gamma^2}{4}} \,, \tag{4}$$

using the identity

$$\delta_{2\Gamma}(\omega - E_\mu + E_\nu) = \int \delta_\Gamma(E - E_\mu + \omega) \delta_\Gamma(E - E_\nu) dE \,,$$

and making the substitution

$$n_\mu - n_\nu \to \frac{\partial n(E_\mu)}{\partial E_\mu}(E_\mu - E_\nu) \tag{5}$$

justified with the quasiclassical accuracy [4], we find

$$\dot{Q} = \dot{Q}_\mathrm{a} + \Gamma \frac{d}{d\Gamma} \dot{Q}_\mathrm{a} \,, \tag{6}$$

where

$$\dot{Q}_\mathrm{a} = -\pi \sum_{\mu\nu} \int_0^\infty dE \frac{\partial n(E)}{\partial E} \left| \dot{V}_{\mu\nu} \right|^2 \delta_\Gamma(E_\mu - E) \delta_\Gamma(E_\nu - E) \,. \tag{7}$$

The dissipative function defined in [5–7] can be written as

$$\dot{Q}^{[0]} = \dot{Q}_\mathrm{a} - \dot{Q}_\mathrm{diag} \,, \tag{8}$$

where

$$\dot{Q}_\mathrm{diag} = -\frac{1}{\Gamma} \sum_{\mu\nu(E_\mu = E_\nu)} \left| \dot{V}_{\mu\nu} \right|^2 \frac{\partial n(E_\mu)}{\partial E_\mu} \,. \tag{9}$$

This $\dot{Q}^{[0]}$ becomes identical to $\dot{Q}$ in the limit $\Gamma \to 0$. At finite Γ the relation between $\dot{Q}$ and $\dot{Q}^{[0]}$ reads

$$\dot{Q} = \dot{Q}^{[0]} + \Gamma \frac{d}{d\Gamma} \dot{Q}^{[0]} \,. \tag{10}$$

Using the finiteness of $\dot{Q}^{[0]}$ at $\Gamma \to 0$ one can find from (8) and (9) the expression

$$\dot{Q}_{\rm diag} = \frac{1}{\Gamma} \lim_{\Gamma' \to 0} \left[\Gamma' \dot{Q}_{\rm a}(\Gamma') \right] \tag{11}$$

which presents $\dot{Q}_{\rm diag}$ in terms of $\dot{Q}_{\rm a}$.

3. Classical approximation for $\dot{Q}$

The $\dot{Q}_{\rm a}$ can be rewritten in the form

$$\dot{Q}_{\rm a} = -\pi \int d\boldsymbol{r} d\boldsymbol{r'} \dot{V}(\boldsymbol{r}) \dot{V}(\boldsymbol{r'}) \int dE \frac{\partial n(E)}{\partial E} \rho^2(\boldsymbol{r}, \boldsymbol{r'}; E), \tag{12}$$

where

$$\rho(\boldsymbol{r}, \boldsymbol{r'}; E) = -\frac{1}{\pi} {\rm Im} G(\boldsymbol{r}, \boldsymbol{r'}; E) \tag{13}$$

is the single-particle spectral density, $G(\boldsymbol{r}, \boldsymbol{r'}; E)$ being the 1-particle Green function:

$$G(\boldsymbol{r}, \boldsymbol{r'}; E) = \sum_{\mu} \frac{\psi_\mu(\boldsymbol{r}) \psi_\mu(\boldsymbol{r'})}{E - E_\mu + i\Gamma/2}.$$

Employing the quasiclassical Van Fleck expression for the time dependent 1-particle Green function we obtain the classical approximation for the spectral density

$$\rho^2(\boldsymbol{r}, \boldsymbol{r'}; E) = \frac{1}{\pi} \frac{1}{(2\pi)^3} \int_0^\infty dt {\rm e}^{-\Gamma t} \int d\boldsymbol{p} \delta \left[\boldsymbol{r'} - \boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t) \right] \delta \left(E - H_{\boldsymbol{r},\boldsymbol{p}} \right) . \tag{14}$$

The phase space trajectory $\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)$, $\boldsymbol{P}_{\boldsymbol{r},\boldsymbol{p}}(t)$ obeys the Hamilton's equations with the Hamiltonian

$$H_{\boldsymbol{r},\boldsymbol{p}} = \frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r})$$

subject to the initial conditions

$$\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t=0) = \boldsymbol{r} \,, \qquad \boldsymbol{P}_{\boldsymbol{r},\boldsymbol{p}}(t=0) = \boldsymbol{p} \,. \tag{15}$$

Inserting (14) into (12) and integrating over E leads to

$$\dot{Q}_{\mathrm{a}} = -\int_0^\infty dt \mathrm{e}^{-\Gamma t} \int \frac{d\boldsymbol{r}\, d\boldsymbol{p}}{(2\pi)^3} \dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)] \dot{V}(\boldsymbol{r}) \frac{\partial n(H_{\boldsymbol{r},\boldsymbol{p}})}{\partial H_{\boldsymbol{r},\boldsymbol{p}}}. \tag{16}$$

The phase space integral in (16) is the autocorrelation function for $\dot{V}(\boldsymbol{r})$. Hence the subscript 'a' in $\dot{Q}_{\mathrm{a}}$.

Inserting (16) into (6) we find

$$\dot{Q} = -\int_0^\infty dt(1-\Gamma t) \mathrm{e}^{-\Gamma t} \int \frac{d\boldsymbol{r}\, d\boldsymbol{p}}{(2\pi)^3} \dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)] \dot{V}(\boldsymbol{r}) \frac{\partial n(H_{\boldsymbol{r},\boldsymbol{p}})}{\partial H_{\boldsymbol{r},\boldsymbol{p}}}. \tag{17}$$

This expression can be used for practical calculations of dissipation rates in hot nuclei of arbitrary shape. It is seen from (17) that the ratio $\lambda = v_{\mathrm{F}}/\Gamma$, where v_{F} is the Fermi velocity, plays the role of a mean free path.

Consider the $\Gamma \to 0$ ($\lambda \to \infty$) limit of Eq. (17). On using the identity

$$\int_0^\infty dt \mathrm{e}^{-\Gamma t} f(t) = \int_0^\infty dt \mathrm{e}^{-\Gamma t} \frac{d}{dt} \int_0^t dt' f(t') = \Gamma \int_0^\infty dt \mathrm{e}^{-\Gamma t} \int_0^t dt' f(t')$$

in the first term of (17) and taking into account the relation

$$\lim_{t\to\infty} f(t) = \lim_{\Gamma\to 0} \Gamma \int_0^\infty dt \mathrm{e}^{-\Gamma t} f(t)\,, \tag{18}$$

one obtains

$$\lim_{\Gamma\to 0} \dot{Q} = -\lim_{t\to\infty} \int \frac{d\boldsymbol{r}\, d\boldsymbol{p}}{(2\pi)^3} \left[\int_0^t dt' \dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t')] - t \dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)] \right] \dot{V}(\boldsymbol{r}) \frac{\partial n(H_{\boldsymbol{r},\boldsymbol{p}})}{\partial H_{\boldsymbol{r},\boldsymbol{p}}}\,. \tag{19}$$

This expression is equivalent to the Koonin–Randrup formula for classical dissipation rate in the long mean-free-path regime [8].

Using (11), (16) and (18), we find that in the classical approximation

$$\dot{Q}_{\mathrm{diag}} = -\frac{1}{\Gamma} \lim_{t\to\infty} \int \frac{d\boldsymbol{r}\, d\boldsymbol{p}}{(2\pi)^3} \dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)] \dot{V}(\boldsymbol{r}) \frac{\partial n(H_{\boldsymbol{r},\boldsymbol{p}})}{\partial H_{\boldsymbol{r},\boldsymbol{p}}}\,. \tag{20}$$

Since $\dot{V}[\boldsymbol{R}_{\boldsymbol{r},\boldsymbol{p}}(t)]$ is finite at large t whereas $\lim_{t\to\infty} t = \frac{1}{\Gamma}$ (see (18)), we conclude that the so-called convergence term of Koonin and Randrup (the second term in (19)) is nothing else but a classical counterpart of $\dot{Q}_{\mathrm{diag}}$.

In the opposite extreme $\Gamma \to \infty$ ($\lambda \to 0$), Eq. (17) is conveniently to study using the leptodermous approximation. Then, following [8], one can decompose $\dot{Q}$ into a sum of the wall formula dissipation rate $\dot{Q}_{\rm wall}$ [9] and a multireflection series. In the latter, the contribution of a path of length s is weighted with $\exp[-s/\lambda]$.

When $\Gamma \to \infty$, the $\dot{Q} - \dot{Q}_{\rm wall}$ decreases exponentially while $\dot{Q}^{[0]} - \dot{Q}_{\rm wall}$ tends to zero as $1/\Gamma$, except for non compressing systems with nondegenerate 1-particle spectrum, when $\dot{Q}_{\rm diag} = 0$ [8]. One should remember that the condition $\Gamma < T$ for Eq. (1) to be valid, does not allow for too small λ.

4. Illustrative example

For multipole surface vibrations in spherical leptodermous nuclei, $\dot{Q}^{[0]}$ becomes

$$\dot{Q}^{[0]} = \rho \bar{v} R^4 \sum_{LM} \gamma_L^{[0]} \left|\dot{\alpha}_{LM}\right|^2 , \tag{21}$$

where $\dot{\alpha}_{LM}$ are the collective velocities, R the radius of the nucleus, ρ is the matter density, $\bar{v} = (3/4)v_{\rm F}$,

$$\gamma_L^{[0]} = \frac{16\pi}{2L+1} \sum_{N=-L}^{L} \left| Y_{LN}\left(\frac{\pi}{2}, \frac{\pi}{2}\right)\right|^2 \int_0^{\pi/2} d\phi \sin^3\phi \cos\phi \, \gamma_N^{[0]}(x,\phi) \tag{22}$$

with

$$\gamma_N^{[0]}(x,\phi) = \begin{cases} \coth\beta - \frac{1}{\beta}, & N = 0 \\ \left(1 - 2{\rm e}^{-2\beta}\cos 2N\phi + {\rm e}^{-4\beta}\right)^{-1}\left(1 - {\rm e}^{-4\beta}\right), & N \neq 0 \end{cases} \tag{23}$$

and $\beta \equiv x \sin\phi,\ \ x \equiv R/\lambda$.

Friction coefficients $\gamma^L_{\rm wall}$ associated with $\dot{Q}_{\rm wall}$ are equal to 1 while friction coefficients γ_L associated with $\dot{Q}$ can be found from the relation

$$\gamma_L = \gamma_L^{[0]} + x \frac{d}{dx} \gamma_L^{[0]}$$

which follows from (10).

As seen from Fig. 1, γ_L and $\gamma_L^{[0]}$ tend to $\gamma^L_{\rm wall}$ at $\lambda \to 0$ with γ_L achieving this limit much faster. At $\lambda \geq R$, $\gamma_L^{[0]}$ strongly differ from γ_L. It is only at very large λ that $\gamma_L^{[0]} \approx \gamma_L$ and both are close to the values 0, 0.85 and 0.45 predicted in [8] for $L = 2, 3, 4$, respectively. The corresponding Γ however are so small that quantum calculations would lead to vanishing friction [5].

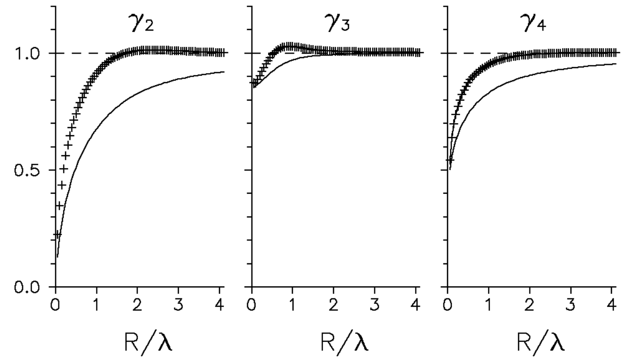

Fig. 1. Friction coefficients $\gamma^L_{\rm wall}$ (- - -), γ_L (+ + +), and $\gamma^{[0]}_L$ (—) for L=2 (left), 3 (middle), and 4 (right) as functions of the ratio R/λ.

Figure 2 shows the imaginary parts of the optical potentials and the corresponding spreading widths in ^{208}Pb at E equal to the chemical potential. To take into account in (23) the dependence of Γ on the nucleon angular momentum l, we used the substitution $l = l_{\rm F}\cos\phi$, where $l_{\rm F} = mv_{\rm F}R$.

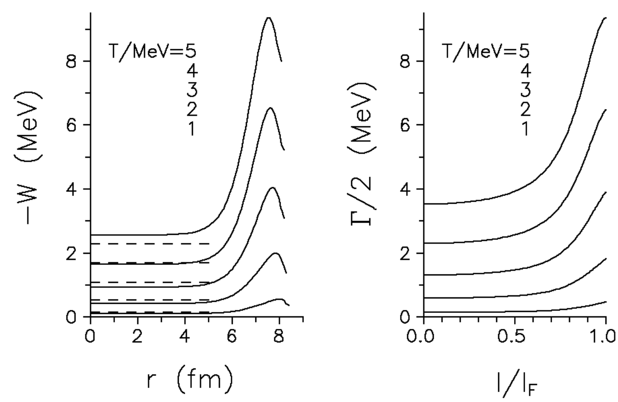

Fig. 2. The imaginary part of the nucleon-nucleus potential (left) and the single-particle spreading width (right) in ^{208}Pb at $T = 1, 2, 3, 4, 5$ MeV. The dashed lines represent the infinite matter results.

Figure 3 demonstrates the temperature dependence of the friction coefficients found with Γ shown in Fig. 2. One concludes that friction coefficients γ_L corresponding to the dressed particle-hole propagator achieve the wall formula limit at the temperatures about 3–4 MeV.

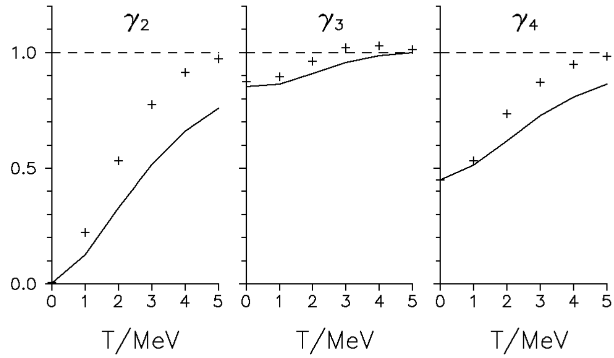

Fig. 3. Temperature dependence of friction coefficients $\gamma^L_{\rm wall}$ (- - -), γ_L (+ + +), and $\gamma^{[0]}_L$ (—) for $L=2$ (left), $L=3$ (middle), $L=4$ (right) in ^{208}Pb.

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