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COLLECTIVE EXCHANGE EFFECT IN NUCLEAR COLLECTIVE MODELS* **

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Possible new quantum effect related to quantum exchange force phenomenon is postulated to explain small staggerings observed in superdeformed and ground state rotational bands of nuclei.

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1. Introduction

Since very precise measurements by Eurogamm [1–3] the $\Delta I = 2$ staggering seems to be well founded. This phenomenon observed in superdeformed bands, also called $\Delta I = 4$ bifurcation, has very small amplitude (about 0.3 keV), and it was argued, in principle, as it is a possible evidence for four-fold symmetry C_4 [4]. On the other hand, Dönau, Frauendorf, and Meng [5] gave arguments against the idea that $\Delta I = 2$ staggering can be due to fourth order terms in the rotational nuclear Hamiltonian used in [4] related to hexadecapole deformation of the nucleus [6]. In addition, some authors suggest the existence of staggering effect also in other than superdeformed bands [7] *e.g.*, in ground bands of some even–even nuclei. They even claim that the staggering phenomenon might be widely spread in rotational bands of many nuclei. The remarks above show that the problem of staggering in rotational spectra remains open.

In this paper we shortly describe some foundations of another quantum mechanism related to discrete symmetries that can appear in atomic nuclei. It can possibly explain existence of different kinds of staggerings in nuclear spectra.

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2. Exchange forces

The quantum mechanics formalism predicts an interesting mechanism of exchange of particles. It is able to produce an additional either atractive or repulsive force only due to quantum tunneling effect forbidden in classical mechanics. This mechanism is qualitatively well explained in text books like [8]. In principle, to exploit this quantum mechanical effect it is enough to consider physical states of the system having non-zero overlaps. One can also find, in this case, an analog of exchange forces though one cannot observe a particle to be exchanged. In addition, the exchange type force is sometimes used explicitely in some kinds of interactions, too. An example, can be very well known the phenomenological nucleon–nucleon interaction potential which consists of radial term called Wigner force and three terms connected with operators of positions exchange, spins exchange and isospins exchange:

$$V_C = V_W(\vec{r}) + V_M(\vec{r})P^{\vec{r}} + V_B(\vec{r})P^{\sigma} + V_H(\vec{r})P^{\tau} . \tag{1}$$

They give a contribution to the interaction energy by exchanging positions or quantum numbers of nucleons. It is interesting to realize that the operator $P^{\vec{r}}$ of positions exchange, called Majorana force, is non–local. One needs, however, to note that the origin of this force is slightly of another nature than the quantum effect described above. In this case one can also imagine an exchange term in the Hamiltonian even if the appropriate states do not overlap. This type of force can be used in phenomenological interactions to simulate more fundamental effect of exchange of some properties of the physical system which leads to changes of its energy.

The collective models of nuclei deal with only few variables describing collective motions of nuclei. It is obvious that a single collective configuration can correspond to several nucleon configurations. The configurations, in turn, can have non-zero overlaps and the nucleus can jump among them with some probability amplitudes. This effect is rather ”not seen” by a smooth collective Hamiltonian. To include this phenomenon into the collective model an additional terms should be added to the collective Hamiltonian.

In the figure 1 the process of exchange is figured out in a schematic way, where the letter G stands for a set of new operators which should be introduced to describe the effect that we propose to consider. In analogy to particle exchange, one can call it the collective exchange effect.

3. Collective exchange

In this work we postulate, that if collective states correspond to certain equivalent nucleon configurations, then some terms responsible for additional energy connected with tunneling among these configurations should

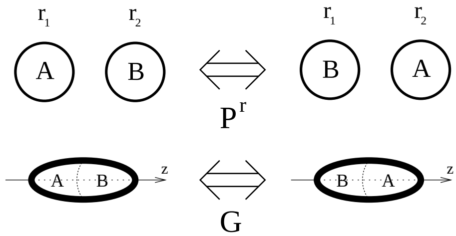

Fig. 1. Upper part – Interaction between two nucleons: A, B - interacting nucleons, r_1,r_2 - their positions, P^r - operator of positions exchange. Lower part — collective energy of nucleus; AB,BA — equivalent configurations; z — an intrinsic axis; G — an exchange operator.

be included to the model Hamiltonian:

$$\hat{H} = \hat{H}_{\text{coll}} + \hat{H}_G \,. \tag{2}$$

In our considerations we assume, that the coupling term between collective degrees of freedom and the collective exchange operators can be neglected.

For simple collective model like rigid rotator (in the body-fixed frame) the form of collective exchange Hamiltonian is strongly dependent on intrinsic structure of the rotating nucleus, which in turn can be approximately replaced by a shape of the nucleus. As in figure 1 if shape of the nucleus is elipsoidal the symmetry group of the shape and the group symmetry of the rotator Hamiltonian coincide. This suggests to use as the collective exchange Hamiltonian $\hat{H}_G$ a linear combination of the symmetry group D_{2h} operators which tranform among the equivalent positions of the nucleus:

$$\hat{H}_G = \sum_{k=1}^{\text{card}(G)} \gamma_k(\hat{H}')\boldsymbol{g}_k \,, \tag{3}$$

where $\boldsymbol{g}_k$ are elements from the symmetry group, card(G) is the order of this group and $\gamma_k(\hat{H}')$ are coupling coefficients, in general state dependent.

It seems that the additional to the collective Hamiltonian $\hat{H}_{\text{coll}}$ the collective exchange Hamiltonian $\hat{H}_G$ could be responsible for different types of more or less regular staggerings in the collective spectra of nuclei. The deviations are expected to be very small because the transitions amplitudes among the eqivalent configurations are small.

The staggering effect in some superdeformed band Hamamoto and Mottelson have tried to explain as effect of fourfold symmetry adding to the rotator some additional terms constructed from higher powers of the angular momentum operators [4]. However, in the paper [6] it has been shown that the interpretation of these additional terms as terms related to hexadecapole deformations of nuclei and $\mathcal{C}_{4v}$ seems to be questionable. Our idea leads to another picture of the process. The staggering effects are not due to dependence of the moment of inertia tensor on angular momentum caused by a hexadecapole deformation, but due to a collective exchange effect induced by quantum exchange of equivalent nucleon configurations.

One of the most difficult problems in this case is to calculate the coupling coeficients gamma responsible for final effect of the Hamiltonian (3) on the collective spectrum of a nucleus under consideration. Full microscopical calculations of γ's seems to be as difficult as the solution of the many-body problem, but one can propose some approximate methods that can be useful in practical applications. Some examples are listed below:

1. $\gamma_n =$const can be fit to experimental spectra.
2. Approximate value of γ_n can be found with help of semiclasical approximation and WKB approach to calculate the tunneling amplitudes.
3. γ_n can be calculated from projection of a generating Hamiltonian $\hat{H}'$ related in the appropriate way to required effect.
4. Making use of the Generator Coordinator Method for derivation of the collective Hamiltonian taking into account the equivalent generating functions corresponding to different nucleon configurations.
5. γ_n can be also evaluated for example, by Variable Moment of Inertia (VMI) method.

The third method can be used to interpret an arbitrary Hamiltonian $\hat{H}'$ in terms of collective exchange operators. In order to do that it is projected onto space of irreducible representations of considered symmetry group. For the generating Hamiltonians non-invariant or invariant under G either first or second formula should be used, respectively

$$\begin{aligned}
\gamma_n(\nu) &= \frac{1}{\mathrm{card}(G)} \sum_{\Gamma} \chi_G^{(\Gamma)}(\dot{g}_n)^\star \sum_a \langle \nu; \Gamma a | \hat{H}' | \nu; \Gamma a \rangle \,, \\
\gamma_n(\nu) &= \frac{1}{\mathrm{card}(G)} \sum_{\Gamma} \dim(\Gamma) \chi_G^{(\Gamma)}(\dot{g}_n)^\star \langle \nu; \Gamma a_0 | \hat{H}' | \nu; \Gamma a_0 \rangle \,.
\end{aligned} \tag{4}$$

In both formulas $|\nu;\Gamma a\rangle$ are generating states, Γ labels the irreducible representations of the group G, a dinstinguishes state vectors within a given irreducible representation Γ and ν describes the additional quantum numbers required for physical states. $\chi_G^{(\Gamma)}$ denotes the irreducible characters of the group G.

It is easy to show, that if the generating Hamiltonian $\hat{H}_G$ is invariant under transformations from the group G and the generating states $|\nu;\Gamma a\rangle$ are its eigenstates:

$$\hat{H}'|\nu;\Gamma a\rangle = E(\nu,\Gamma)|\nu;\Gamma a\rangle, \tag{5}$$

then the hamiltonian $\hat{H}_G$ based on the group of invariance G reproduces energy spectrum of the generating Hamiltonian:

$$H_G(\nu) = \sum_{\Gamma} E(\nu,\Gamma)P^{(\Gamma)}, \tag{6}$$

where $P^{(\Gamma)}$ is a projection operator projecting onto the carrier space of the irreducible representation Γ of the group G. So, we can interpretate a Hamiltonian invariant under considered symmetry in terms of the collective exchange effect.

To show the simplest example of behavior of the collective exchange terms let us consider slightly non–axial rotational Hamiltonian with exchange group described by the point symmetry group D_{2h} in the body–fixed frame of reference:

$$D_{2h} = \{e_G, C_2^x, C_2^y, C_2^z, \Pi C_2^z, \Pi C_2^y, \Pi C_2^x, \Pi\}, \tag{7}$$

where e_G is the neutral element of the group, C_2^w denotes a rotation around axis w about the angle π and Π is an operator of inversion. In this case the Hamiltonian $\hat{H}$ has the following form:

$$\hat{H} = \hat{H}_{Rot} + \hat{H}_G = \frac{\hat{I}_x^2}{2\Theta_x} + \frac{\hat{I}_y^2}{2\Theta_y} + \frac{\hat{I}_z^2}{2\Theta_z} + \sum_{n=0}^{7} \gamma_n \boldsymbol{g}_n, \tag{8}$$

where $\boldsymbol{g}_n \in D_{2h}$.

A little of algebra leads to the following matrix element of $\hat{H}_G$:

$$\begin{aligned} \langle \pi J K'|\hat{H}_G|\pi J K\rangle \; = \; & (\gamma_0 + (\gamma_3+\gamma_4\pi)(-1)^K + \gamma_7\pi)\boldsymbol{\delta}_{K'K} + \\ & (\gamma_1 + (\gamma_2+\gamma_5\pi)(-1)^K + \gamma_6\pi)(-1)^J\boldsymbol{\delta}_{-K'K}. \end{aligned} \tag{9}$$

An example of the contribiution of the collective exchange energy understood as the difference between the energy obtained from (8) and the rigid rotator itself is shown in the figure 2. The behavior of this additional energy is strongly dependent on relations among the moments of inertia Θ_k. One can see, for example, that for axially symmetric rotator only shift of even angular momenta levels is observed and the odd levels disappear due to symmetry requirements.

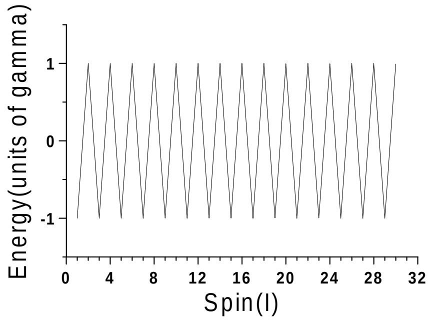

Fig. 2. Staggering energy given by D_{2h} collective exchange terms for $\Theta_x = 2\Theta_z$ and $\Theta_y = 2.1\Theta_z$.

4. Conclusions

This is a preliminary report on the hypothesis of the collective exchange effect. Further investigations are needed to find a good approximation for microscopic description of the coupling coefficients gamma in the Hamiltonian $\hat{H}_G$. They are needed to go beyond a phenomenology and to confirm or not the hypothesis on the collective exchange effect and its relation to the staggering effects in rotational (more generally collective) excitations of nuclei. The investigations are in progress.

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