SHELL-MODEL CALCULATIONS WITH BONN POTENTIAL*

A. Covello, L. Coraggio, A. Gargano, and N. Itaco

Dipartimento di Scienze Fisiche, Università di Napoli Federico II
and Istituto Nazionale di Fisica Nucleare
Complesso Universitario di Monte S. Angelo
Via Cintia, 80126 Napoli, Italy

(Received January 13, 1999)

We present the results of a shell-model study of nuclei having or lacking few identical nucleons with respect to doubly magic ^{208}Pb. Our calculations have been performed by making use of realistic effective interactions derived from the Bonn A nucleon–nucleon potential. The results are in very good agreement with experimental data showing the ability of these effective interactions to describe with quantitative accuracy the properties of heavy-mass nuclei around closed shells.

PACS numbers: 21.60.Cs, 21.30.Fe, 27.80.+w

1. Introduction

The Pb region has long come in focus for shell-model studies of nuclear structure. This is of course related to the fact that ^{208}Pb is a good doubly closed shell nucleus whose neighbors are accessible to detailed spectroscopic studies. From the theoretical point of view the study of nuclei with few valence particles or holes offers the best opportunity to test directly the matrix elements of the two-body residual interaction. In most of the calculations performed so far in the lead region empirical interactions containing adjustable parameters have been used [1–4]. Notable exceptions are the studies of Refs. [5, 6], where a realistic effective interaction derived from the Hamada–Johnston nucleon–nucleon (NN) potential [7] was employed. These calculations, however, date back to some twenty-five years ago and during this time there has been substantial progress towards a microscopic approach to shell-model calculations starting from a free NN potential. On

* Presented at the XXXIII Zakopane School of Physics, Zakopane, Poland, September 1–9, 1998.

the one hand, new high-quality NN potentials have been constructed, on the other hand the many-body methods for deriving a model-space effective interaction $V_{\rm eff}$ from a given NN potential have been largely improved. The major developments in the field of NN potentials are reviewed in Refs. [8] and [9] while the main aspects of the derivation of $V_{\rm eff}$ are discussed in Refs. [10] and [11].

These improvements have set the stage for a new generation of realistic shell-model calculations. Until now, however, attention has been focused on medium-mass nuclei, such as the Sn isotopes and the $N = 82$ isotones [12–17]. In all of our own studies [12–14] we performed shell-model calculations using realistic effective interactions derived from the meson-theoretic Bonn A potential [18].

Motivated by the very good agreement between theory and experiment achieved in these works, we have recently started a similar study on heavy-mass nuclei in the Pb region [19]. In this paper, we present some results of our calculations for the ^{206,205,204}Pb isotopes and the $N = 126$ isotones ^{210}Po, ^{211}At, and ^{212}Rn. These nuclei with two to four identical holes (particles) around the ^{208}Pb core offer the opportunity to put to a test our realistic effective interaction in the $A = 208$ region.

The paper is organized as follows. In Sec. 2 we give a brief description of our calculations. In Sec. 3 we present our results and compare them with the experimental data. Some concluding remarks are given in Sec. 4.

2. Calculations

Our effective interactions were derived from the Bonn A potential using a G-matrix folded-diagram formalism, including renormalizations from both core polarization and folded diagrams. For both the lead isotopes and the $N = 126$ isotones we have considered ^{208}Pb as an inert core. Therefore we have treated neutrons in the former nuclei as valence holes, which implies the derivation of a hole-hole effective interaction. We have chosen the Pauli exclusion operator Q_2 in the G-matrix equation,

$$G(\omega) = V + VQ_2\frac{1}{\omega - Q_2TQ_2}Q_2G(\omega)\,, \tag{1}$$

as specified [11] by $(n_1, n_2, n_3) = (22{,}45{,}78)$ for the neutron orbits and $(n_1, n_2, n_3) = (16{,}36{,}78)$ for the proton orbits. It should be mentioned that in [19] we made the choice $(n_1, n_2, n_3) = (22{,}36{,}66)$ for the neutron orbits and $(n_1, n_2, n_3) = (16{,}28{,}66)$ for the proton orbits, which is consistent with that made in earlier works [10,12,13] on light- and medium-mass nuclei. For the lead region, however, the smaller value of $\hbar\omega$ makes the present choice

more appropriate. Here V represents the NN potential, T denotes the two-nucleon kinetic energy, and ω is the so-called starting energy. We employ a matrix inversion method to calculate the above G matrix in an essentially exact way [20]. The effective interaction, which is energy independent, can be schematically written in operator form as

$$V_{\rm eff} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots , \qquad (2)$$

where $\hat{Q}$ and $\hat{Q}'$ represent the $\hat{Q}$-box, composed of irreducible valence-linked diagrams, and the integral sign represents a generalized folding operation. We take the $\hat{Q}$-box to be composed of G-matrix diagrams through second-order in G; they are just the seven first- and second-order diagrams considered by Shurpin *et al.* [21]. In the calculation of $V_{\rm eff}$ we use an isospin uncoupled representation, where protons and neutrons are treated separately. For the shell-model oscillator parameter $\hbar\omega$ we use the value 6.88 MeV, as obtained from the expression $\hbar\omega = 45A^{-\frac{1}{3}} - 25A^{-\frac{2}{3}}$ for $A = 208$.

As regards the single-hole and -particle energies, we have taken them from the experimental spectra [22] of ^{207}Pb and ^{209}Bi, respectively. Thus, for the Pb isotopes the adopted single-hole spectrum (in MeV) is: $\varepsilon_{p_{1/2}} = 0.0$, $\varepsilon_{f_{5/2}} = 0.570$, $\varepsilon_{p_{3/2}} = 0.898$, $\varepsilon_{i_{13/2}} = 1.633$, $\varepsilon_{f_{7/2}} = 2.340$, and $\varepsilon_{h_{9/2}} = 3.414$, while for the $N = 126$ isotones we have used the following values: $\varepsilon_{h_{9/2}} = 0.0$, $\varepsilon_{f_{7/2}} = 0.896$, $\varepsilon_{i_{13/2}} = 1.609$, $\varepsilon_{f_{5/2}} = 2.826$, $\varepsilon_{p_{3/2}} = 3.119$, and $\varepsilon_{p_{1/2}} = 3.633$.

3. Results and comparison with experiment

We present here the results of our calculations for the Pb isotopes and the $N = 126$ isotones focusing attention on the lowest-lying states. The calculations have been performed using the OXBASH shell-model code [23]. The calculated excitation energies are displayed and compared with the observed ones in Tables I to VI.

For the Pb isotopes, more results and a detailed comparison between theory and experiment can be found in our recent paper [19]. We should point out, however, that the present results are in a substantially better agreement with experiment than those obtained in [19]. In fact, in that work we found that most of the excited states in the low-energy spectra of ^{206}Pb and ^{204}Pb lay about 200 keV below the experimental ones. Here, from Tables I–III we see that the largest discrepancies are 147 and 114 keV for the 0_2^+ state in ^{206}Pb and the 2_2^+ state in ^{204}Pb, respectively. The reason for this improvements can be traced to the better treatment of the Pauli exclusion operator Q_2 mentioned in Sec. 2.

TABLE I

Experimental and calculated excitation energies for ^{206}Pb. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
0^+	0.000	0.000
2^+	0.803	0.842
0^+	1.166	1.019
3^+	1.340	1.207
2^+	1.467	1.321
4^+	1.684	1.637
1^+	1.703	1.667
2^+	1.784	1.673

TABLE II

Experimental and calculated excitation energies for ^{205}Pb. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
$5/2^-$	0.000	0.018
$1/2^-$	0.002	0.000
$3/2^-$	0.263	0.315
$3/2^-$	0.576	0.727
$7/2^-$	0.703	0.822
$5/2^-$	0.761	0.794

TABLE III

Experimental and calculated excitation energies for ^{204}Pb. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
0^+	0.000	0.000
2^+	0.899	1.004
4^+	1.274	1.245
2^+	1.351	1.237
4^+	1.563	1.554
0^+	1.582	1.493
2^+	1.583	1.496
3^+	1.605	1.562

As regards the $N = 126$ isotones, the states reported in Tables IV–VI all arise from the configuration $h^n_{9/2}$ except the $(^7/_2)_1$ state in ^{211}At, which is dominated by the $h^2_{9/2}f_{7/2}$ configuration. It should be mentioned, however, that our wave functions are not very pure. The contribution coming from

TABLE IV

Experimental and calculated excitation energies for ^{210}Po. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
0^+	0.000	0.000
2^+	1.181	1.130
4^+	1.427	1.395
6^+	1.473	1.493
8^+	1.557	1.555

TABLE V

Experimental and calculated excitation energies for ^{211}At. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
$9/2^-$	0.000	0.000
$7/2^-$	0.674	0.679
$7/2^-$	0.866	0.783
$5/2^-$	0.947	0.955
$13/2^-$	1.067	1.053
$3/2^-$	1.116	1.103
$11/2^-$	1.123	1.098

TABLE VI

Experimental and calculated excitation energies for ^{212}Rn. The experimental data are from [22].

J^π	E (MeV)	
	Exp.	Calc.
0^+	0.000	0.000
2^+	1.274	1.221
4^+	1.502	1.506
6^+	1.640	1.619
8^+	1.694	1.677

other configurations is particularly significant for the ground states (from about 20 to 50 % passing from ^{210}Po to ^{212}Rn) as well as for the $(^7/_2)_1$ state in ^{211}At and all the excited states in ^{212}Rn. The agreement between theory and experiment for the $N = 126$ isotones is even better than that obtained for the Pb isotopes. All the calculated excitation energies differ from the experimental values by less than a few tens of keV, the largest discrepancy being about 80 keV for the $(^7/_2)_2$ state in ^{211}At. For this nucleus, we confirm the tentative spin-parity assignments made in Ref. [22] for the first five excited states, and predict $J^\pi = {}^{11}/_2{}^-$ for the observed $({}^{11}/_2{}^-, {}^{13}/_2{}^-)$ level at 1.123 MeV.

4. Concluding remarks

We have presented here some results of a shell-model study of nuclei having or lacking few nucleons with respect to doubly magic ^{208}Pb. They have been obtained by employing two-particle and two-hole effective interactions derived from the Bonn A NN potential. We have shown that the low-energy spectra of all the six nuclei considered are very well reproduced by the theory. It should be emphasized that these are the first shell-model calculations in the lead region where the effective interaction is derived from a modern NN potential by means of a G-matrix folded-diagram method.

A more complete and detailed presentation of the results of our study of nuclei around ^{208}Pb will be made in a forthcoming paper. Here, we conclude that the success achieved by the present calculations are quite in line with the findings of our earlier works [12–14, 19] indicating that the Bonn potential is quite suitable for use in shell-model studies of nuclear structure properties.

The results presented in this paper are part of a research project carried out in collaboration with T.T.S. Kuo. This work was supported in part by the Italian Ministero dell'Università e della Ricerca Scientifica e Tecnologica (MURST).

REFERENCES

[1] See Ref. [6] for a comprehensive list of references through 1973.

[2] D. Zwarts, P.W.M. Glaudemans, *Z. Phys.* **A320**, 487 (1985).

[3] D. Wang, M.T. McEllistrem, *Phys. Rev.* **C42**, 252 (1990), and references therein.

[4] C.A.P. Ceneviva, L. Losano, N. Teruya, H. Dias, *Nucl. Phys.* **A619**, 129 (1997), and references therein.

[5] G.H. Herling, T.T.S. Kuo, *Nucl. Phys.* **A181**, 113 (1972).

[6] J.B. McGrory, T.T.S. Kuo, *Nucl. Phys.* **A247**, 283 (1975).

[7] T. Hamada, I.D. Johnston, *Nucl. Phys.* **34**, 382 (1962).

[8] R. Machleidt, G.Q. Li, *Phys. Rep.* **242**, 5 (1994).

[9] R. Machleidt, in *Highlights of Modern Nuclear Structure*, Proceedings of the Sixth International Spring Seminar on Nuclear Physics, S. Agata sui Due Golfi, 1998, ed. A. Covello, World Scientific, Singapore, 1999, in press.

[10] M.F. Jiang, R. Machleidt, D.B. Stout, T.T.S. Kuo, *Phys. Rev.* **C46**, 910 (1992).

[11] T.T.S. Kuo, in *New Perspectives in Nuclear Structure*, Proceedings of the Fifth International Spring Seminar on Nuclear Physics, Ravello, 1995, ed. A. Covello, World Scientific, Singapore, 1996, p. 159.

[12] F. Andreozzi, L. Coraggio, A. Covello, A. Gargano, T.T.S. Kuo, Z.B. Li, A. Porrino, *Phys. Rev.* **C54**, 1636 (1996).

[13] F. Andreozzi, L. Coraggio, A. Covello, A. Gargano, T.T.S. Kuo, A. Porrino, *Phys. Rev.* **C56**, R16 (1997).

[14] A. Covello, F. Andreozzi, L. Coraggio, A. Gargano, T.T.S. Kuo, A. Porrino, *Prog. Part. Nucl. Phys.* **38**, 165 (1997).

[15] A. Holt, T. Engeland, E. Osnes, M. Hjorth-Jensen, J. Suhonen, *Nucl. Phys.* **A618**, 107 (1997).

[16] J. Suhonen, J. Toivanen, A. Holt, T. Engeland, E. Osnes, M. Hjorth-Jensen, *Nucl. Phys.* **A628**, 41 (1998).

[17] A. Holt, T. Engeland, M. Hjorth-Jensen, E. Osnes, *Nucl. Phys.* **A634**, 41 (1998).

[18] R. Machleidt, K. Holinde, Ch. Elster, *Phys. Rep.* **149**, 1 (1987).

[19] L. Coraggio, A. Covello, A. Gargano, N. Itaco, T.T.S. Kuo, *Phys. Rev.* **C58**, 3346 (1998).

[20] E.M. Krenciglowa, C.L. Kung, T.T.S. Kuo, E. Osnes, *Ann. Phys.* (N.Y.) **101**, 154 (1976).

[21] J. Shurpin, D. Strottman, T.T.S. Kuo, *Nucl. Phys.* **A408**, 310 (1983).

[22] Data extracted using the NNDC On-Line Data Service from the ENSDF database, file revised as of November 10, 1998. M.R. Bhat, *Evaluated Nuclear Structure Data File (ENSDF)*, *Nuclear Data for Science and Technology*, p. 817, edited by S.M. Qaim, Springer-Verlag, Berlin, Germany, 1992.

[23] B.A. Brown, A. Etchegoyen, W.D.M. Rae, The computer code OXBASH, MSU-NSCL report number 524.