

COLLECTIVE QUADRUPOLE EXCITATIONS IN  
EVEN–EVEN Ru ISOTOPES\* \*\*

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Quadrupole excitations in Ru isotopes are described within microscopic approach including the effect of pairing dynamics. The observed collective energies and transition probabilities of  $^{104,112}\text{Ru}$  are reproduced in the calculation containing no free parameters.

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Collective properties of even-even neutron-rich Ru isotopes have been recently investigated and discussed [1, 2]. However, the models applied up to now to study the role of the  $\gamma$  deformation in this nuclear region made use of parameters fitted to the data [3]. The aim of this work is to describe collective excitations in Ru isotopes in the frame of a microscopic approach containing no free parameters [4] and to show that including the effect of pairing vibrations we are able to reproduce the experimental data.

Our calculations are based on the microscopically derived Hamiltonian of the generalized Bohr collective model [5–7] expressed in terms of the intrinsic variables  $\beta$  and  $\gamma$  (parametrizing the nuclear shape) and the Euler angles denoted in short as  $\Omega$  :

$$\hat{\mathcal{H}}_{\text{Bohr}} = \hat{\mathcal{T}}_{\text{vib}}(\beta, \gamma) + \hat{\mathcal{T}}_{\text{rot}}(\beta, \gamma, \Omega) + V_{\text{coll}}(\beta, \gamma). \quad (1)$$

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The vibrational  $\hat{\mathcal{T}}_{\text{vib}}$  and rotational  $\hat{\mathcal{T}}_{\text{rot}}$  kinetic energy operators depend on the set of inertial functions of intrinsic variables: mass parameters  $B_{\beta\beta}$ ,  $B_{\beta\gamma}$ ,  $B_{\gamma\gamma}$  and moments of inertia  $\mathcal{J}_k$ , ( $k = 1, 2, 3$ ). All these functions as well as the collective potential  $V_{\text{coll}}$  can be determined microscopically assuming that the nucleus is a system of nucleons moving in a deformed mean field and interacting through state-independent pairing forces. We applied here known [4, 6] cranking formulas and the standard Nilsson single-particle potential [8]. The collective potential was evaluated within macroscopic-microscopic Strutinsky method [4].

Within this approach one can describe quadrupole excitations in all types of nuclei but, as is known for a long time, energies yielded by the model are in general of about 50% too large with respect to the experimental data (see Fig. 2). The discrepancy is connected with a strong influence of pairing correlations on collective quadrupole degrees of freedom [4, 7]. In consequence, the gap parameter  $\Delta$  should not be artificially fixed at its equilibrium point (found from the BCS equations) but it rather takes the role of a collective variable too.

For a given number of particles  $N$  (protons or neutrons) the Hamiltonian of collective pairing vibrations takes the form [9, 10]:

$$\hat{\mathcal{H}}_{\text{pair}} = -\frac{\hbar^2}{2\sqrt{g(\Delta)}} \frac{\partial}{\partial \Delta} \frac{\sqrt{g(\Delta)}}{B_{\Delta\Delta}(\Delta)} \frac{\partial}{\partial \Delta} + V_{\text{pair}}(\Delta). \quad (2)$$

The pairing mass function  $B_{\Delta\Delta}$ , the metric tensor determinant  $g$  and the collective pairing potential  $V_{\text{pair}}$  can be evaluated microscopically at each deformation point according to the formulas given in Ref. [10] (see Fig. 1). It should be mentioned that we apply the approximate projection of the BCS wave function on the correct particle number [11]. All calculations are made with the standard pairing strength  $G = g_0 N^{-2/3}$ , where  $g_0 = 0.26 \hbar\omega_0$  and  $N$  means the number of protons or neutrons [4, 12].

Solving the eigenproblem of Eq. (2) we can find the the ground state-function  $\Psi_0(\Delta)$  and the most probable value of the pairing gap  $\Delta_0$  (which corresponds to the maximum of the function  $g(\Delta)|\Psi_0(\Delta)|^2$ ). Usually the most probable gap is shifted towards smaller  $\Delta$  values from the equilibrium point as it is exemplified in Fig. 1. On average the ratio of  $\Delta_0$  to the equilibrium gap value is of about 0.7.

This general behaviour of the pairing vibrational ground-state function is due to the rapid increase of the inertial function  $B_{\Delta\Delta}$  and it allows us to approximate the complete "quadrupole + pairing" collective Hamiltonian (difficult to solve because of nine degrees of freedom) by the expression

$$\hat{\mathcal{H}}_{\text{coll}} = \hat{\mathcal{H}}'_{\text{Bohr}}(\beta, \gamma, \Omega; \Delta_0^p, \Delta_0^n) + \hat{\mathcal{H}}_{\text{pair}}. \quad (3)$$

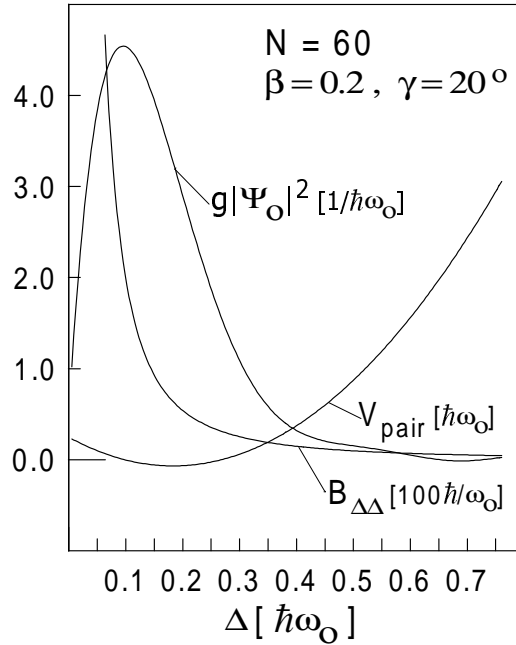


Fig. 1. Pairing vibrations for neutrons in  $^{104}\text{Ru}$  at deformation  $\beta = 0.2$ ,  $\gamma = 20^\circ$ . The equilibrium value of the energy gap is  $\Delta_{\text{eq}} \approx 0.14\hbar\omega_0$ , the most probable one is  $\Delta_0 \approx 0.09\hbar\omega_0$ .

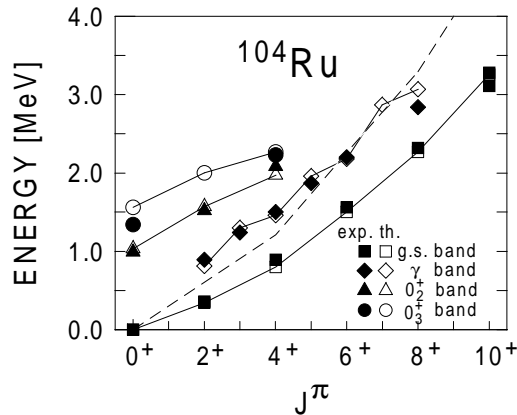


Fig. 2. Experimental [13,14] and calculated excitation energies in  $^{104}\text{Ru}$  versus the angular momentum  $J^\pi$ . Theoretical bands are marked with full lines, the dashed line connects the energies of the g.s. band calculated without pairing vibrations effect taken into account.

Here the quadrupole part  $\hat{\mathcal{H}}'_{\text{Bohr}}$  has the same structure as usual Bohr Hamiltonian (1) but the potential and all inertial functions appearing in it are calculated using the most probable gap values  $\Delta_0^p$  and  $\Delta_0^n$  (for protons and neutrons respectively) instead the equilibrium ones.

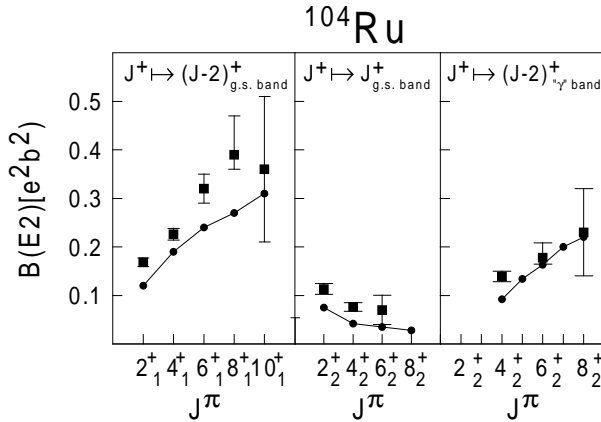


Fig. 3. Experimental [13] and calculated reduced  $E2$  transition probabilities for ground-state and  $\gamma$ - bands in  $^{104}\text{Ru}$ . Theoretical values are connected by straight lines.

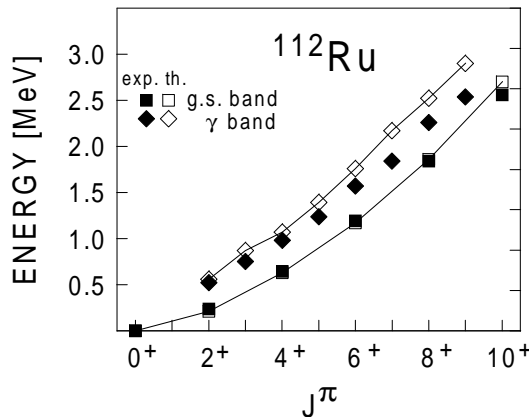


Fig. 4. Experimental [1] and calculated energies of ground-state and  $\gamma$ - bands in  $^{112}\text{Ru}$ . Theoretical values are marked with straight lines.

Diagonalizing the Hamiltonian (3) in the complete basis discussed in Ref. [4] we have obtained excitation energies and other collective characteristics of  $^{100-112}\text{Ru}$ . Some examples of our results are presented here. The theoretical and experimental energies of excitations with positive parity of

$^{104}\text{Ru}$  [13, 14] are compared in Fig. 2. The agreement is surprisingly good in spite of the calculation has been done with no free parameters. Some small discrepancies (*e.g.* for  $0_3^+$  and  $8_2^+$  levels) could be connected with the approximate treatment of pairing vibrations or an admixture of two-particle modes.

Investigating quadrupole moments and E2 transitions (Fig. 3) we can confirm the quadrupole nature of reproduced states. The calculated probabilities of E2 transitions in the ground-state band are too small indeed (this tendency can be connected with absence of the pairing-quadrupole mixed terms in Hamiltonian (3) [4]) but nevertheless, they are close to the measured ones. The results obtained for other Ru isotopes are similar. As an example we present in Fig. 4 the energies of  $^{112}\text{Ru}$  ground-state and  $\gamma$ - bands in comparison to the experimental values. The agreement for the ground-state band is still very good whereas the theoretical moment of inertia of  $\gamma$ -band is a little bit too big in this case.

We may conclude that the influence of pairing vibrations appears to be very important in describing the quadrupole excitations of triaxially deformed neutron-rich nuclei like Ru isotopes. Taking it into account we can reproduce quadrupole energy levels and transition probabilities in the frame of the microscopic model with no need of introducing free parameters.

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