TOWARDS NEW UNDERSTANDING OF NUCLEAR ROTATION*

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Properties of time evolution of wave packets built up from rotator eigenstates are discussed. The mechanism of perfect cloning of the initial wave packet for *circular states* at fractional revival times is explained. The smooth transition from *circular* to *linear* through intermediate *elliptic states* is described. Examples of time evolution of a nuclear wave packet created in Coulomb excitation mechanism are presented.

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1. Introduction

The advent of experimental techniques (mainly short, tunable laser pulses) resulted in rapidly growing interest in time evolution of Wave Packets (WP) in many fields of physics and chemistry. Nine years ago Averbukh and Perelman [1] discovered and explained a universal scenario of this time evolution applicable to a wide class of quantum systems. The general requirements for their scenario are very weak: *(i)* the initial WP have to be a superposition of bound states of system's Hamiltonian, *(ii)* the weights of the superposition should be strongly peaked around the mean value ($\psi(t=0)=\sum_n c_n\phi_n$ where $H\phi_n=E_n\phi_n$). In such conditions, as we already explained during the previous conference [2], the evolution is governed basically by two time scales, $T_{\rm cl}=2\pi\hbar/|E'_n|_{n=\bar{n}}$ — the period of motion of the corresponding classical system with $E=E_{\bar{n}}$, and

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$T_{\rm rev} = 2\pi\hbar/|E''_n|_{n=\bar{n}}$ — revival time, after which the WP reassembles to a shape reminding the initial one. In general the longer time scales also exist and usually $T_{\rm cl} << T_{\rm rev} << \ldots$. However, there exist systems where E_n depends quadratically on the quantum number n (infinite square well, ideal rotator). In such cases the quantum evolution is exactly periodic with $T = T_{\rm rev}$. The general scenario of [1] exhibits in these cases some particularly interesting features.

2. Time evolution of WP in quantum rotator, clones and mutants

The time evolution of the WP is given by the formula:

$$\Psi(t) = \sum_n c_n\, \phi_n\, \exp\left(\frac{-iE_n t}{\hbar}\right) . \tag{1}$$

Let us expand energies in Taylor series $E_n = E_{\bar{n}} + E'_{\bar{n}}\,(n-\bar{n}) + E''_{\bar{n}}\,(n-\bar{n})^2 + \ldots$. If E_n is only quadratic function of quantum number n, the series terminates at $E''_{\bar{n}}$. This is the case for quantum rotator, $H = (\hbar^2/2J)\, I^2$, with $E_I = (\hbar^2/2J)\, I(I+1)$. Introducing time scales mentioned above one can rewrite (1) in the form ($k = (I - \bar{I})$):

$$\Psi(t) = \sum_I c_I\, \phi_I\, \exp\left[-2\pi i\left(\frac{kt}{T_{\rm cl}} + \frac{k^2 t}{T_{\rm rev}}\right)\right] . \tag{2}$$

For all times of the form $t = (m/n)T_{\rm rev}$ (where m and n are mutually prime numbers) it is possible to use Gauss sum and rewrite (2) in the form:

$$\Psi\left(t = \frac{m}{n}T_{\rm rev}\right) = \sum_{s=0}^{l-1} a_s\, \Psi^s_{\rm cl}\, , \tag{3}$$

where $l = n/2$ for n–multiple of 4, $l = n$ otherwise, $\Psi^s_{\rm cl}$ is either identical (*clone*) or similar (*mutant*) to the initial WP, depending on topology of the motion.

The initial WP can be created as a Coherent State (CS) of angular momentum [3, 4], fulfilling during evolution the minimum uncertainty condition: $\Delta L_x^2\, \Delta L_y^2 = \frac{1}{4}\langle L_z\rangle^2$. One of the best possible choice of such CS can be written in an exponential form depending on 2 real parameters N and η:

$$\Psi_{N,\eta}(\theta,\phi) = \sqrt{\frac{N}{2\pi\sinh(2N)}}\, {\rm e}^{N\sin\theta(cos\phi + i\eta\sin\phi)}\, , \tag{4}$$

with $\langle L_z\rangle \stackrel{N\to\infty}{\longrightarrow} \eta\,(N - \frac{1}{2})$ and $\eta = \langle L_z\rangle/(2\Delta L_y^2) = \pm\sqrt{\Delta L_x^2/\Delta L_y^2}$. There are 2 special cases, particularly interesting: *(i)* $\eta = 1$, corresponding to *circular states*, and *(ii)* $\eta = 0$, so called *linear states.* In the former case the expansion of the initial WP in spherical harmonics contains only functions with maximal $M = I$, in the latter only those with $M = 0$. In general case, $\eta \neq 0$, additional sumation over M is necessary and the motion is called *elliptic*:

$$|CS, \eta, t = 0\rangle = \sum_{IM} b_{IM}(N, \eta)\, Y_M^I(\theta, \phi)\,. \tag{5}$$

In all cases the coefficients b_{IM} are given analytically. For $\eta = 1$ all fractional revivals (3) are copies of the initial WP (clones). Their number is equal $q = n/2$ for even n or $q = n$ for odd n. For $\eta \neq 1$ fractional revivals have in general different shapes (from crescents on a sphere to rings at $\eta = 0$) than the initial WP (we call them mutants). However if the position of fractional revival coincides with that of the initial WP the clone is always built. The example of WP shapes for the *elliptic state* with $N = 20, \eta = 0.3$ is shown in Fig. 1.

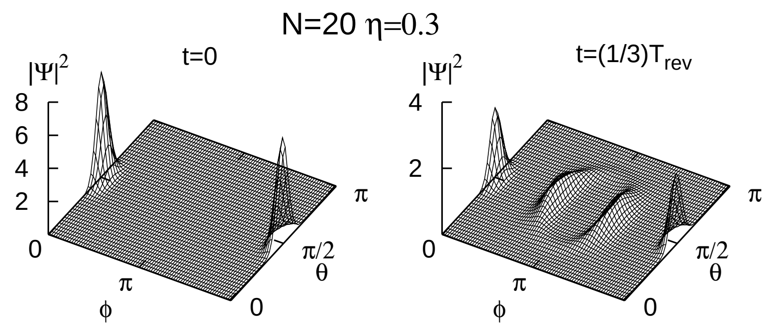

Fig. 1. Initial WP (left) and evolved one at $t = (1/6)T_{\rm rev}$ or $(1/3)T_{\rm rev}$ (right). One clone and two mutants (crescent shaped) are clearly visible in the right figure.

3. Nuclear rotation

It is well known [5, 6] that during Coulomb excitation (CE) a deformed nucleus is excited to a coherent mixture of rotational states. This superposition is also peaked around a mean value of angular momentum, so one can expect similar features as predicted by scenario of Averbukh and Perelman. The most clear case is CE with backscattering as in this case excited WP has cylindrical symmetry (only Y_0^I components, as *linear* CS). The partial waves and fractional revivals have then topology of rings on a sphere. Therefore for presentation of shapes only one angular variable (θ) is sufficient. In Fig. 2 we present the time evolution of WP obtained by CE of ^{238}U bombarded by ^{40}Ar at $E = 170$ MeV. The amplitudes of excitation of given I angular momentum eigenstates have been calculated within semiclassical theory of CE [5]. The left part shows the 'ideal case', *i.e.* when E_I's follow perfect rotor dependence $I(I+1)$, the right corresponds to time evolution (1) with energies taken from experiment. Although the 'carpet' for experimen-

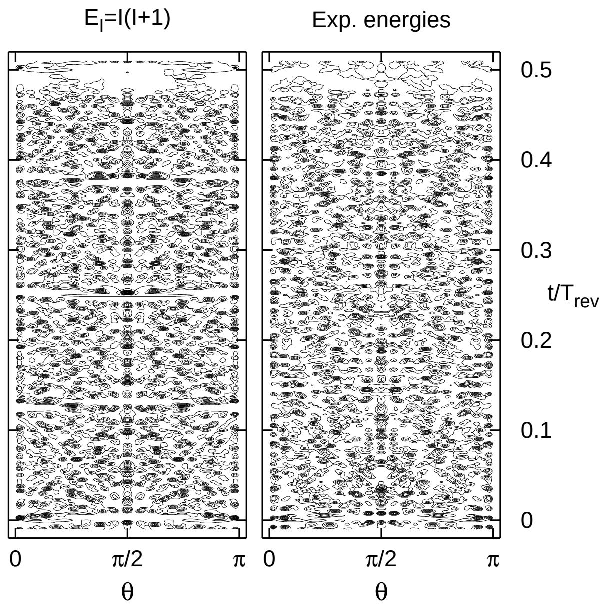

Fig. 2. Time evolution of nulear rotational wave packet obtained in CE of ^{238}U presented in 'quantum carpet' representation. Contours of $2\pi \sin\theta |\Psi|^2$ are plotted.

tal energies is not as regular as that for 'ideal' ones, still strong revivals of WP occur. The absolute time scales for nuclear rotation are very short ($T_{\rm rev} \sim 10^{-19}$s, $T_{\rm cl} \sim 10^{-20}$s) and are still beyond time resolution of present experimental techniques. For more details see [3, 4].

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