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ROTATION OF SUPERDEFORMED EVEN–EVEN NUCLEI *

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Starting from the microscopic Hamiltonian, we generalized the Bohr–Mottelson equation to describe both the rotation and β-vibrations of superdeformed even-even axially symmetric nuclei.

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A lot of papers (see *e.g.* [1]) are devoted to investigation of the rotation with high spins of superdeformed nuclei, which are characterized by the quadrupole deformation parameter $\beta_0 \sim 1$. Their energy levels are usually calculated in the frameworks of the cranked shell model. But the familiar cranking model deals only with static deformation of the nuclei and do not take into consideration any relation of the rotation and vibrations of the nuclear shape. At the same time, this relation is described by the Bohr-Mottelson equation [2]. Most explicitly the dependence of the rotation of the normally deformed nuclei ($\beta_0 \sim 0.2 - 0.3$) on β-vibrations is revealed in the Davydov-Chaban model [3]. But its application to superdeformed nuclei faces with difficulty that the Bohr-Mottelson equation is derived assuming small deviations of the nuclear shape from the sphere, when $\beta \ll 1$. Therefore their generalization to the case of arbitrary deformations seems to be actual.

For this aim we shall use the kinetic energy operator expressed [4] in terms of independent collective variables. As usually two frames were introduced. One of them x, y, z is the laboratory coordinate system and another ξ, η, ζ is the moving one with the axes directed along the principal axes of the inertia tensor of the nucleus. Then the projections of the Jakobi vectors

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of the nucleons $\vec{q}_i$ on these axes obey the following constraints:

$$\sum_{i=1}^{A-1} q_{i\xi} q_{i\eta} = \sum_{i=1}^{A-1} q_{i\xi} q_{i\zeta} = \sum_{i=1}^{A-1} q_{i\eta} q_{i\zeta} = 0, \tag{1}$$

where A is the number of nucleons in the nucleus.

The rotation of the nucleus is identified with the rotation of the coordinate frame ξ, η, ζ , whose orientation with respect to x, y, z is determined by the Euler angles φ, θ, ψ. Equation (1) is formally considered as the orthogonality condition for three vectors $\mathbf{A}_\xi = (q_{1\xi}, q_{2\xi}, \ldots, q_{A-1,\xi})$, $\mathbf{A}_\eta = (q_{1\eta}, q_{2\eta}, \ldots, q_{A-1,\eta})$, and $\mathbf{A}_\zeta = (q_{1\zeta}, q_{2\zeta}, \ldots, q_{A-1,\zeta})$ in an abstract $(A-1)$-dimensional space. Three of intrinsic coordinates are defined as lengths of these vectors:

$$a = \sqrt{\sum_i q_{i\xi}^2} \ , \quad b = \sqrt{\sum_i q_{i\eta}^2} \ , \quad c = \sqrt{\sum_i q_{i\zeta}^2}. \tag{2}$$

Others are the angles to specify rotation in the abstract space. We shall consider only axially symmetric prolate nuclei ($\gamma = 0$) and introduce the nuclear radius ρ and deformation parameter β:

$$a = b = \frac{\rho}{\sqrt{3}} \left[1 - \frac{\beta}{1+\beta}\right]^{\frac{1}{2}}, \quad c = \frac{\rho}{\sqrt{3}} \left[1 + \frac{2\beta}{1+\beta}\right]^{\frac{1}{2}}, \tag{3}$$

where β varies from 0 to ∞. Neglecting in the kinetic energy operator, derived in [4]], all the terms which depend on the intrinsic angular variables we find

$$\begin{aligned} \hat{T} \ &= \ -\frac{\hbar^2}{m\rho_0^2} \frac{(1+\beta)^{3A/2-3}}{(1+3\beta)^{A/2-2}} \frac{1}{\beta^3} \frac{\partial}{\partial\beta} \frac{(1+3\beta)^{A/2-1}}{(1+\beta)^{3A/2-5}} \beta^3 \frac{\partial}{\partial\beta} \\ &\quad + \frac{\hbar^2}{3m\rho_0^2\beta^2} (1+3\beta/2)(1+\beta) \left(\hat{I}_\xi^2 + \hat{I}_\eta^2\right). \end{aligned} \tag{4}$$

The Hamiltonian of the nucleus will be

$$\hat{H} = \hat{T} + V(\beta), \tag{5}$$

where $V(\beta)$ is a potential for β-vibrations.

The Schrödinger equation with the Hamiltonian given above coincides with the Bohr-Mottelson equation for prolate axially symmetrical nucleus only if $\beta \ll 1$. Its solution may be written as a product of two factors depending on β and θ:

$$\Psi_I(\beta, \theta) = \left[\frac{\beta^3 (1+3\beta)^{A/2-1}}{(1+\beta)^{3A/2-5}}\right]^{-1/2} \varphi_I(\beta) |IM0\rangle, \tag{6}$$

where the function

$$|IM0>=\frac{1}{2}\sqrt{\frac{2I+1}{8\pi^2}}\left(D^I_{0M}(\theta)+(-1)^I D^I_{0M}(\theta)\right) \tag{7}$$

describes the rotation of an axially symmetrical rigid rotor with spin I, its projection M on the axis z, and projection $K=0$ on the symmetry axis ζ; D^I_{0M} are the Wigner functions. The function $\varphi_I(\beta)$ obeys the boundary condition $\varphi_I(0)=0$ and the normalization constraint

$$\int\limits_0^\infty \varphi^2(\beta)\alpha(\beta)d\beta = 1, \tag{8}$$

where

$$\alpha(\beta) = (1+3\beta)^{-1}(1+\beta)^{-2}. \tag{9}$$

Substitution of (6) into (4) yields the equation for $\varphi_I(\beta)$:

$$\left\{-\frac{\hbar^2}{2B(\beta)}\frac{\partial^2}{\partial\beta^2}+W_I(\beta)-E_I\right\}\varphi_I(\beta)=0, \tag{10}$$

where the effective potential energy is

$$W_I(\beta) = W_0(\beta)+\frac{\hbar^2}{6B(0)\beta^2}(1+\frac{3}{2}\beta)(1+\beta)I(I+1), \tag{11}$$

with

$$\begin{aligned} W_0(\beta) \;&=\; V(\beta)+\frac{(1+\beta)^{3A/4-1/2}}{\beta^{3/2}(1+3\beta)^{A/4-3/2}} \\ &\times\frac{\partial}{\partial\beta}\left\{\frac{\beta^3(1+3\beta)^{A/2-1}}{(1+\beta)^{3A/2-5}}\frac{\partial}{\partial\beta}\left[\frac{(1+\beta)^{3A/4-5/2}}{\beta^{3/2}(1+3\beta)^{A/4-1/2}}\right]\right\}. \end{aligned} \tag{12}$$

For the mass parameter $B(\beta)$ we find the expression

$$B(\beta)=B(0)\alpha(\beta) \;\;, \quad B(0)=\tfrac{1}{2}m\rho_0^2. \tag{13}$$

Hereafter we neglect the dependence of α on β and take the value $\alpha = \alpha(\beta_0)$. The corresponding mass parameter is designated by $B = B(\beta_0)$. This procedure gives the relative error of the order of μ, which stands for the softness parameter.

Besides, we approximate $W_0(\beta)$ by the function

$$W_0(\beta) = C_0 + C\beta_0^2\left(\frac{\beta_0^2}{2\beta^2} - \frac{\beta_0}{\beta}\right), \tag{14}$$

where the constant C_0 determines the position of the potential well bottom.

Introducing the notations

$$\beta = \beta_{00}\zeta \quad \beta_{00} = \sqrt[4]{\frac{\hbar^2}{BC}}, \quad \omega = \sqrt{\frac{C}{B}}, \quad \mu = \frac{\beta_{00}}{\beta_0}, \tag{15}$$

$$2Z_I = \frac{2}{\mu^3} - \frac{5}{6}\beta_{00}\alpha I(I+1),$$
$$l = \frac{1}{2}\left[\sqrt{1 + \frac{4}{\mu^4} + \frac{4\alpha}{3}I(I+1)} - 1\right],$$

one can rewrite equation (10) as

$$\left(\frac{\partial^2}{\partial\zeta^2} - \frac{l(l+1)}{\zeta^2} + \frac{2Z_I}{\zeta} + 2\varepsilon_I\right)\varphi_I(\beta_{00}\zeta) = 0. \tag{16}$$

Here $2\varepsilon_I$ is related to the energy by

$$2\varepsilon_I = \frac{2E_I}{\hbar\omega} - \frac{1}{2}\beta_{00}^2\alpha I(I+1). \tag{17}$$

We see that (16) is formally the equation for the radial part of the wave function of a charged particle bound in the Coulomb potential (see *e.g.* [5]). This enables us to write immediately the energies as

$$E_{In_\beta} = -\frac{\hbar\omega}{2}\left\{\frac{Z_I^2}{n^2} - \frac{1}{2}\beta_{00}\alpha I(I+1)\right\} + C_0, \tag{18}$$

where $n_\beta = 0, 1, 2, \ldots$ determines the number of phonons for β-vibrations, and $n = n_\beta + l + 1$. Note that the numbers l and n are not integers.

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