# TRANSPORT PROPERTIES AND NEUTRINO EMISSIVITY OF DENSE NEUTRON-STAR MATTER WITH LOCALIZED PROTONS

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As pointed out by Kutschera and Wójcik, very low concentration of protons combined with a specific density dependence of effective neutron–proton interaction could lead to a localization of "proton impurities" in neutron medium at densities exceeding four times normal nuclear matter density. We study consequences of the localization of protons for transport processes in dense neutron star cores, assuming random distribution of proton impurities. Kinetic equations, relevant for the transport of charge, heat and momentum, are solved using variational method. Localization of protons removes a $T^{-2}$ factor from the transport coefficients, which leads, at lower temperatures, to a strong decrease of thermal conductivity, electrical conductivity and shear viscosity of neutron star matter, as compared to the standard case, where protons form a Fermi liquid. Due to the localization of protons a number of conventional neutrino emission processes (including modified URCA process) become inoperative in neutron star cores. On the other hand, the energy loss rate from neutrino–antineutrino pair bremsstrahlung due to electron and neutron scattering off (localized) protons, will have a specific $T^6$ dependence, which could modify the cooling of the neutron star core, as compared to the standard case. Possible astrophysical implications of the localization of protons for neutron star evolution and dynamics are discussed.

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## 1. Introduction

The composition of neutron star matter above three times normal nuclear density ($\rho > 3\rho_0$, where $\rho_0 = 2.8 \times 10^{14}$ g cm$^{-3}$) is largely unknown. Some many-body calculations suggest, that above $3\rho_0$ the baryon component of matter consists nearly exclusively of neutrons, with a small admixture (a few percent) of protons (see, *e.g.* models UV14+TNI, AV14+UVII of [1]); such a composition would be similar to that at $\rho \simeq \rho_0$. A small admixture of protons in high–density neutron matter could behave quite differently than at densities $\rho \simeq \rho_0$. As shown by Kutschera and Wójcik [2], coupling of "proton impurities" to the density waves in neutron matter could lead, above some critical density, to localization of protons in the potential wells associated with the neutron density inhomogeneities; at smaller densities the coupling results in a gradual increase of the proton effective mass [2]. We will show in the present paper, that the localization of protons would change dramatically transport properties of neutron star matter. It would also lead to the "switching on" of two new neutrino emission processes, which would influence cooling of a neutron star.

In order to visualize possible effect of the localization of protons, let us consider the standard case, when nucleons form degenerate normal Fermi liquids. The transport coefficients of neutron star matter exhibit then characteristic temperature dependences, which result from the combined effect of the Pauli principle, and the energy and momentum conservation in the scattering processes [3]. The leading terms in the low–temperature expansions of electrical conductivity, $\sigma$, and shear viscosity, $\eta$, are proportional to $T^{-2}$, while the low–temperature behaviour of thermal conductivity, $\kappa$, is given by $\kappa \propto T^{-1}$.

The conventional neutrino emission processes in the standard *npe* matter include the modified URCA process and the neutrino–antineutrino pair bremsstrahlung from *nn*, *np*, and *pp* collisions. In the absence of nucleon superfluidity the energy emission rates for all these processes vary with temperature as $T^8$ (*e.g.* [4]).

Localized protons will act as scattering centers for neutrons and electrons. In what follows, we assume that the localized protons do not exhibit a long–range crystalline order (this point is discussed in Section 2). In such a case, an elementary consideration indicates, that the localization would give $\sigma^{\rm loc.p.}$, $\eta^{\rm loc.p.} \propto T^0$, and $\kappa^{\rm loc.p.} \propto T$; one may thus expect a strong effect of the proton localization on the transport coefficients at lower temperatures, where scattering off protons dominates. As will be shown in this paper, the localization of protons produces a drastic decrease of the transport coefficients of neutron star cores, as compared to the standard case, when protons form a Fermi liquid: $\sigma^{\rm loc.p.}/\sigma$, $\eta^{\rm loc.p.}/\eta \sim 10^{-5} - 10^{-6}\, T_8^2$,

and $\kappa^{\rm loc.p.}/\kappa \sim 10^{-4}\, T_8^2$, for densities around $4\rho_0$ and for a proton fraction about 1 % ($T_8 \equiv T/10^8$ K).

Furthermore, the emission of neutrino–antineutrino pairs in the $nn$ collisions is the only process of the above mentioned, that survives the localization of protons. Instead, there appear the neutrino–antineutrino pair emission from the scattering of neutrons and electrons off localized protons. In both cases the rate of this process reproduces the temperature dependence of the direct URCA process ($T^6$), which, if operative, accelerates drastically the cooling of a neutron star. As will be seen in Section 6, when neutrons are not superfluid the neutrino emissivity due to $np$ collisions is approximately 3.5 orders of magnitude larger than that due to $ep$ collisions. Despite the same temperature dependence, the neutrino emission due to neutron scattering off localized protons is much less efficient than the direct URCA process. However, the ratio of the emissivity due to the $np$ bremsstrahlung to the emissivity due to the modified URCA process (the most important one among the standard processes in neutron star cores) could be quite large in the temperature range of interest: $Q_{\rm Brem}^{n-{\rm loc.p.}}/Q_{\rm mURCA} \sim 2 \cdot 10^3\, T_8^{-2}$, for the same density and proton fraction. This implies that the proton localization could lead to an intermediate regime of a neutron star cooling: more rapid than the standard cooling, provided by modified URCA process, and less fast than the accelerated cooling due to direct URCA process.

The paper is organized as follows. The physical conditions in neutron star matter with localized protons are discussed in detail in Section 2. In particular, we emphasize the similarity between the behaviour of a proton in neutron matter and a polaron behaviour of slow electrons in solids; we also present some arguments against a crystalline ordering of localized protons, and discuss the importance of relativistic effects in the neutron component of matter. Kinetic equations relevant for the transport of charge, heat and momentum, as well as variational solutions to them, and analytical expressions for the transport coefficients, are derived in Section 3. Angular averages of scattering probabilities, appearing in the expressions for the transport coefficients, are calculated in Section 4. In Section 5, we improve the variational solutions to make them asymptotically exact in the high–temperature regime and present our results for the transport coefficients in the form suitable for practical applications. The neutrino energy emission rates from the $ep$ and $np$ bremsstrahlung are estimated in Section 6. Finally, in Section 7 we discuss some astrophysical implications of these results.

Throughout the paper we will mostly use the units $\hbar = c = k_{\rm B} = 1$ and turn to the normal units whenever presenting the final results.

## 2. Neutron star matter with localized protons

Consider *npe* matter at super nuclear densities. The typical kinetic energy of a proton can be estimated from the uncertainty principle and reads

$$T_p \sim \frac{\hbar^2 n_p^{2/3}}{2m_p^*} = 0.7 \left(\frac{n}{4n_0}\cdot\frac{x_p}{0.01}\right)^{2/3} \left(\frac{m_p}{m_p^*}\right) \ \text{MeV}\,, \tag{1}$$

where $n$ is a mean nucleon density, $x_p$ is a proton fraction (equal to an electron fraction $x_e$), $n_0$ is the normal nuclear number density, $n_0 = 0.16$ fm$^{-3}$, and $m_p^*$ is the proton effective mass resulting from the two–body nucleon–nucleon (NN) interactions.

A proton in neutron matter also has an effective potential energy $V_{\rm eff}$, the value of which depends on the neutron density and ranges, according to different parametrization, from 55 to 75 MeV for $n_n \sim 4n_0$ (Fig. 1 of [2]). Taking into account the possible inhomogeneity of the neutron sea we can write the Hamiltonian of a proton in the form

$$H_p = \frac{-\hbar^2\nabla^2}{2m_p^*} + V_{\rm eff}(n_n) + n_n \frac{\partial V_{\rm eff}}{\partial n_n}\left(\frac{\delta n_n}{n_n}\right)\,, \tag{2}$$

where the last term describes the coupling of the proton to the density waves in neutron matter. The quantity $\sigma(n_n) = n_n \partial V_{\rm eff}/\partial n_n$ plays role of the coupling strength and is of the order 550 – 600 MeV at $n_n \sim 4n_0$. On comparison with the estimate of the kinetic energy, Eq. (1), it becomes clear, that the coupling to neutron density waves might not be neglected, if one aims at a realistic description of a proton behaviour in high–density neutron star matter with low proton fraction [2].

The calculations presented in [2] indicate, that at weak and intermediate couplings the proton, interacting with neutron density waves (which will be referred to as phonons, while, basically, these are acoustical modes of neutron matter), acquires an additional effective mass, which increases gradually with increasing neutron density. The situation reminds the so–called "large" polaron in solids, where a slow electron, moving through a crystal, is dressed into a cloud of virtual phonons, and, consequently, has an effective mass exceeding its "bare" band effective mass. If the coupling strength increases further, the polarization of the ion lattice by the electron, could get very strong. It can eventually be trapped by a local deformation of the lattice, induced by the electron itself [5]. This latter situation corresponds to a "small" polaron. If the temperature is sufficiently high the electron can be then kicked out of the potential well trapping it. However, in the low temperature regime, the electron could only tunnel slowly through the lattice and spends most of the time near one ion.

Let us now come back to the proton moving through the neutron background. The background polarization, induced by the proton, could be characterized by its spatial scale $R_p$. With increasing neutron density the proton–phonon coupling strength $\sigma(n_n)$ increases, whereas the $R_p$ decreases. The conditions for proton self–trapping are roughly given by two inequalities: $m_p^*\sigma(n_n)|\delta n_n|R_p^2/n_n\hbar^2 > 1$, which ensures, that the potential well formed contains a bound state for a proton, and $R_p < n_p^{-1/3}$, which allows each proton to produce its own potential well, and ensures no overlap between wave functions of different protons.

In order to check if this "small" polaron regime occurs for a proton in neutron matter one have to perform a detailed calculation. In particular, one should compare the energy of the state with trapped proton and that of the state, in which the proton is not localized. The results of calculations of this type has been reported in Ref. [2]. These authors evaluated the energy of a Wigner–Seitz cell with homogeneous distributions of neutrons and a proton versus that of the cell with proton wave function localized near the center of the cell and a neutron distribution having minimum at the center. The parameters, characterizing the deviation of those distributions from the homogeneous ones, has been treated as variational. It was found, that above some critical density (ranging from 4 to 9 $n_0$ depending on the model; from now on we will adopt the most optimistic model and assume the critical density to be $\simeq 4n_0$) there existed a domain of the parameters, where the localized state was energetically preferable to the uniform one. Therefore, above this density, and provided $R_p$, the rms radius of the proton probability distribution corresponding to the minimum energy, is sufficiently small, the protons most likely are in effective potential wells trapping their wave functions. The typical value of $R_p$ at $4n_0$ is about 1 fm and typical depth of the well is $\sim 100$ MeV. The mean distance between protons

$$a_p \simeq n_p^{-1/3} = 5.4 \left(\frac{n}{4n_0}\cdot\frac{x_p}{0.01}\right)^{-1/3} \text{ fm} , \tag{3}$$

while the energy of the zero–point vibration of a proton in the well is $T_0 \sim 42(R_p/1\text{ fm})^2\ (m_p/m_p^*)$ MeV. Thus, we see that at densities above $4n_0$ and for sufficiently low proton fraction, say 1–5 %, the wave functions of the protons are well localized around the neutron density minima, and there is no overlap between them. This, in turn, means that there is no need to invoke Fermi statistics to describe the proton system. The temperature, when falls below, say 10 MeV, is negligible compared to the relevant energy scale in the proton–trapping well, so that the protons occupy the lowest available energy levels, and other (degenerate) particles scatter off them elastically. The proton could then only tunnel through the neutron background with

very low probability, and in what follows, we will neglect the possibility of such events.

Let us discuss in some detail the astrophysical scenario of the formation of a dense neutron star core, where one might expect the localization of protons. According to the standard scenario of the formation of neutron stars in a gravitational collapse of massive stellar cores, initial temperature in the central core of a newly born neutron star is $T \simeq 30$ MeV. Moreover, due to neutrino trapping in dense hot plasma, initial proton fraction is very high, $x_p \simeq 0.3$. Both a high value of $x_p$ and a high value of $T$ exclude possibility of the localization of protons. It may take place after the neutron star core becomes transparent to neutrinos, and the core temperature falls to the range of $\lesssim 10$ MeV. One could ask a question, whether the proton impurities could be ordered during the localization process due to some long–range interaction? We immediately find, that Coulomb energy, estimated as,

$$E_{Cpp} \sim \frac{e^2}{a_p} \simeq 0.3 \left( \frac{n}{4n_0} \cdot \frac{x_p}{0.01} \right)^{1/3} \text{ MeV} , \tag{4}$$

is negligible compared to the potential energy, felt by a localized proton in its effective well, or the energy, resulting from the Fermi motion of protons before the localization point. However, it is not excluded, that some correlation of proton sites could be induced by a long–range correlation resulting from (strong) nuclear interactions in the neutron sea. A detailed discussion of this possibility is difficult and goes beyond the scope of the present paper. One could recall only, that the localization occurs at rather high temperature, implying large thermal fluctuations, and it is uneasy to imagine a long–range correlation, originating from strong (that is short–range) forces, with energy scale of several MeVs. Therefore, we think that neutron star matter with localized protons is formed as a disordered system and remains locked in this disordered state during subsequent cooling. In what follows, we will assume that the positions of localized protons show no long–range crystalline order: localized protons will be treated as randomly distributed, spatially fixed scattering centers for neutrons and electrons.

Transport processes in neutron star matter with localized protons are carried out by electrons and neutrons. For simplicity, we will not allow for the presence of muons. Their inclusion would lead to some modifications of the kinetic equation formalism (see, for details, the paper by Gnedin and Yakovlev [6], where the thermal conductivity of $pe\mu$ component of neutron star core matter was studied under standard assumptions) but would not alter qualitatively the main results of this paper.

The Fermi energy of electrons is given by

$$\varepsilon_{Fe} = 113.4 \left( \frac{n}{4n_0} \cdot \frac{x_p}{0.01} \right)^{1/3} \text{ MeV} . \tag{5}$$

Typical energy of Coulomb $ee$ and $ep$ interactions are on the order of $E_{Cpp}$ (4), that is much smaller than $\varepsilon_{Fe}$. Therefore, electrons should be treated as a free, uniform, ultrarelativistic, and strongly degenerate ($\varepsilon_{Fe} \gg T$) Fermi gas. Transport of energy, momentum and charge is carried out by the elementary excitations in electron gas — thermal electron quasiparticles. Electron quasiparticles are approximated by the excited single–particle states in a free electron gas, close to the Fermi surface [*i.e.* with momenta close to $k_{Fe} = \left(3\pi^2 n_e\right)^{1/3}$], with effective mass given by $m_e^* = \varepsilon_{Fe}$. The pair interaction between electron quasiparticles is described in the formalism of the dielectric function, the details of which are outlined in Section 3. Here we mention only, that since usually we have $T \ll 0.1\varepsilon_{Fe}$ in the neutron stars cores, the effect of dynamical screening can be neglected.

Neutrons, by contrast, form a strongly interacting Fermi system. We will consider the case, when neutrons are normal (non–superfluid). Transport processes involving neutrons can then be described in the spirit of the Landau theory of normal Fermi liquids. Namely, transport of energy and momentum is carried out by the neutron quasiparticles — elementary single–particle excitations in the vicinity of the Fermi surface. As we restrict ourselves to strongly degenerate neutron matter, the gas of neutron quasiparticles is dilute. The Fermi momentum of neutron quasiparticles coincides with that of the real neutron matter, $k_{Fn} = \left(3\pi^2 n_n\right)^{1/3}$. The neutron quasiparticle velocity at the Fermi surface is given by

$$v_{Fn} = \frac{\hbar k_{Fn}}{m_n^*} = 0.56 \left[ \frac{n}{4n_0}(1 - x_p) \right]^{1/3} \frac{m_n}{m_n^*} , \tag{6}$$

where $m_n^*$ is the neutron quasiparticle effective mass. We see, that at densities of interest (we will confine ourselves to densities below $5n_0$), and with $x_p \ll 1$, neutrons gradually become relativistic, although not very much; nevertheless, we will take neutron relativism into account. In the non–relativistic system the effective mass of neutron quasiparticles $m_n^* = p_{Fn}/v_{Fn}$ differs from bare nucleon mass $m_N$ due to many–body effects. Here it should also include relativistic effect. Using Lorentz invariance arguments one obtains, in the reference frame of neutron matter, $m_n^*/\varepsilon_{Fn} = 1 + F_1^s/3$, where $\varepsilon_{Fn}$ is the neutron chemical potential, which includes neutron rest mass, and $F_1^s$ is the Landau parameter.

Actually, the presence of proton impurities leads to the appearance of neutron density inhomogeneities. These inhomogeneities will slightly modify the mean–field, felt by an incident neutron quasiparticle (the de Broglie

wavelength of which is generally smaller than the sizes of the inhomogeneities), inducing a continuous drift of the quasiparticle in the momentum space. We note, that these inhomogeneities are restricted to a very small fraction of the volume $(R_p/a_p)^3 \sim 0.01$, and are concentrated around the proton impurities. The latter produce sudden changes in neutron momenta, thus making a dominant contribution to the scattering. The contribution from the inhomogeneities would account for a correction to the off–proton scattering, which, in principle, could be included into $np$ scattering transition probability. However, in view of the anticipated smallness of this effect, we will neglect it.

## 3. Kinetic equation

Transport of charge, heat or momentum is limited by scattering of the elementary excitations (quasiparticles) off localized protons and by their mutual collisions. Among the latter we neglect those caused by weak $en$ electromagnetic interaction, which enables us to study transport properties of electrons and neutrons separately. The distribution function of electrons satisfies the standard time–independent Boltzmann equation, valid for ideal gases, (*e.g.* Ref. [7]), whereas the neutron distribution is governed by the time–independent Landau equation, which takes into account a self–consistent mean field depending on the quasiparticle distribution itself. However, a proper linearization (*e.g.* Ref. [8]) reveals that the quantity of real significance for the transport processes (the deviation of the distribution function from the local equilibrium one) satisfies the linearized Boltzmann equation, which allows a unified treatment for electrons and neutrons. We have

$$\boldsymbol{v}_1 \partial_{\boldsymbol{r}} f_{\boldsymbol{k}_1} = I_{ii} + I_{ip}\,, \tag{7}$$

$$e\boldsymbol{E}\partial_{\boldsymbol{k}_1} f_{\boldsymbol{k}_1} = I_{ee} + I_{ep}\,, \tag{8}$$

where $f_{\boldsymbol{k}_1}$ is the distribution function in question, $i = \{n, e\}$, $\boldsymbol{E}$ is an external electric field, $\boldsymbol{v}_1$ is the velocity of quasiparticles, which is assumed to be independent of coordinates, and $\boldsymbol{k}_1$ is the quasiparticle wavevector. On the right hand sides of these equations we have integrals of particle–particle and particle–proton collisions which can be written as

$$I_{ii} = \frac{(2\pi)^4}{(2\pi)^9} \int \mathrm{d}\boldsymbol{k}_2 \mathrm{d}\boldsymbol{k}_1' \mathrm{d}\boldsymbol{k}_2'\; \delta(\mathcal{E}' - \mathcal{E})\, \delta(\boldsymbol{K}' - \boldsymbol{K})\; \mathcal{L}_{ii}\; \frac{1}{2} \sum_{\sigma_2 \sigma_1' \sigma_2'} |A_{ii}|^2, \tag{9}$$

$$I_{ip} = \frac{2\pi n_p}{(2\pi)^3} \int \mathrm{d}\boldsymbol{k}_1'\; \delta(\varepsilon_1' - \varepsilon_1)\; \frac{1}{2} \sum_{\sigma_p \sigma_1' \sigma_p'} |A_{ip}|^2 \times \left[ f_{\boldsymbol{k}_1'}(1 - f_{\boldsymbol{k}_1}) - f_{\boldsymbol{k}_1}(1 - f_{\boldsymbol{k}_1'}) \right]. \tag{10}$$

In this case, non–primed and primed variables correspond to particles before and after a collision, respectively, $A_{ii}$ and $A_{ip}$ are the transition amplitudes of the $ii$ and $ip$ scattering processes, containing all exchange contributions, $\mathcal{L}_{ii}$ is the standard "two fermion" Pauli factor, and $n_p$ is the density of localized protons. In the first integral the factor $1/2$ serves to avoid double counting of the final states in a collision event. In the second one the same factor accounts for the fact that the density $n_p$ already includes protons with both possible spin orientations.

It is useful to cast the spin sums in the preceding equations in a more symmetric form:

$$\frac{1}{2}\sum_{\sigma_2\sigma_1'\sigma_2'}|A_{ii}|^2=\frac{1}{4}\sum_{\sigma_1\sigma_2\sigma_1'\sigma_2'}|A_{ii}|^2\equiv Q_{ii}, \tag{11}$$

with analogous expression for $Q_{ip}$.

If the distribution functions are known, the kinetic coefficients can be found by calculating the currents induced in the system by the spatial gradients or the external field. We have

$$\boldsymbol{j}_{\mathrm{T}} = \frac{2}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}\; f_{\boldsymbol{k}}\,(\varepsilon-\mu)\,\boldsymbol{v}=-\kappa\boldsymbol{\nabla}T, \tag{12}$$

$$\boldsymbol{j}_{\mathrm{e}} = \frac{2}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}\; f_{\boldsymbol{k}}\,e\,\boldsymbol{v}=\sigma\boldsymbol{E}, \tag{13}$$

$$-\sigma_{\mathrm{xy}} = \frac{2}{(2\pi)^3}\int \mathrm{d}\boldsymbol{k}\; f_{\boldsymbol{k}}\,k_{\mathrm{x}}\,v_{\mathrm{y}}=-\eta\frac{\partial u_{\mathrm{x}}}{\partial y}, \tag{14}$$

where $\kappa$ is the thermal conductivity, $\sigma$ is the electrical conductivity, $\eta$ is the shear viscosity, and $\boldsymbol{u}$ is a bulk velocity, which is assumed to be directed along the $x$–axis and to be dependent only on the $y$ coordinate (thus satisfying $\mathrm{div}\;\boldsymbol{u}=0$). Further, $\boldsymbol{j}_{\mathrm{T}}$, $\boldsymbol{j}_{\mathrm{e}}$, and $\sigma_{\mathrm{xy}}$ are, respectively, the heat flux, the charge flux and the $xy$–component of the dissipative part of the stress tensor.

In the presence of the spatial gradients we approximate the true distribution functions on the left hand side of the Boltzmann equation (8) by the local equilibrium (le) distributions as

$$f_{\kappa}^{\mathrm{le}}(\varepsilon) = \left[1+\exp\frac{\varepsilon-\mu}{T(\boldsymbol{r})}\right]^{-1}, \tag{15}$$

$$f_{\eta}^{\mathrm{le}}(\varepsilon) = \left[1+\exp\frac{\varepsilon-\mu-\boldsymbol{k}\boldsymbol{u}(\boldsymbol{r})}{T}\right]^{-1}. \tag{16}$$

Simple estimates [8] show that it is a plausible approximation provided the typical scales of the gradients of temperature or bulk velocity are large

compared to the quasiparticle mean free path $l$. On the left hand side of Eq. (8) we use the global equilibrium distribution $f^0(\varepsilon)$. Corresponding condition of applicability reads $|eEl| \ll T$.

On the right hand side we adopt the following decomposition of the true distribution functions in terms of the $f^0$:

$$f_{\kappa,\sigma,\eta} = f^0 + \frac{f^0\,(1-f^0)}{T}\Phi_{\kappa,\sigma,\eta}. \tag{17}$$

In this case $\Phi_\alpha$ is an unknown function of energy and angular variables, which represents a nonequilibrium correction to the distribution function. It is clear that the nonequilibrium term should be tied to the Fermi surface.

The dependences on angular and energy variables can be separated and the functions $\Phi$ can be looked for in the form

$$\begin{aligned}
\Phi_\kappa &= -\tau_{ii}\Psi_\kappa(x)\,\boldsymbol{v}\boldsymbol{\nabla}T,\\
\Phi_\sigma &= \tau_{ii}\Psi_\sigma(x)\,\boldsymbol{v}\boldsymbol{E}e,\\
\Phi_\eta &= -\tau_{ii}\Psi_\eta(x)\,\boldsymbol{v}\boldsymbol{\nabla}(\boldsymbol{k}\boldsymbol{u}).
\end{aligned} \tag{18}$$

In this case a dimensionless function $\Psi_\alpha$ represents a dependence on energy $[x = (\varepsilon-\mu)/T]$, while angular dependences are uniquely determined by the left hand sides of Eqs. (7), (8); $\tau_{ii}$ is a constant, which has the meaning of a typical time between the collisions of identical particles.

Omitting standard details which include the linearization of the collision integrals Eqs. (9), (10) with respect to the small nonequilibrium corrections (17) and integration of the momentum and energy conserving delta functions, we arrive at the one–dimensional equation for the functions $\Psi_\alpha$:

$$\begin{aligned}
g_\alpha(x_1)f_1^0(1-f_1^0) &= \frac{x_1^2+\pi^2}{2}f_1^0(1-f_1^0)\Psi_\alpha(x_1)\\
&\quad+\lambda_\alpha\int\limits_{-\infty}^{\infty}\mathrm{d}x_2\;f_1^0f_2^0\frac{x_1+x_2}{1-\mathrm{e}^{-x_1-x_2}}\Psi_\alpha(x_2)\\
&\quad+\frac{\tau_{ii}}{\tau_{ip}}\,f_1^0(1-f_1^0)\,(1-\omega_\alpha)\,\Psi_\alpha(x_1),
\end{aligned} \tag{19}$$

where $\alpha$ runs over $\kappa,\sigma$, and $\eta$, $g_\kappa(x_1) = x_1$, and $g_{\sigma,\eta} = 1$. The expression for the characteristic collisional time is given by

$$\tau_{ii} = \frac{8\pi^3}{m_i^{*3}T^2\langle Q_{ii}\rangle}, \tag{20}$$

where the angle brackets denote an angular averaging of the form

$$\langle Q_{ii}\rangle = \int\limits_{(4\pi)} \frac{\mathrm{d}\theta\mathrm{d}\phi}{4\pi}\,\frac{\sin\theta}{\cos\theta/2}\,Q_{ii}(\theta,\phi). \tag{21}$$

In this case, the angles $\theta$ and $\phi$ specify a collision of identical particles in the Abrikosov–Khalatnikov frame of reference. Namely, $\theta$ is an angle between momenta of first and second incident particles, and $\phi$ is an angle between planes, containing two incident and two final momenta. The other quantities characterizing $ii$–type collisions are

$$\begin{aligned}
\lambda_\kappa &= \frac{1}{\langle Q_{ii}\rangle}\,\langle Q_{ii}(\theta,\phi)\,(1+2\cos\theta)\rangle,\\
\lambda_\eta &= \frac{1}{\langle Q_{ii}\rangle}\,\langle Q_{ii}(\theta,\phi)\,\left(3\sin^4\theta/2\;\sin^2\phi-1\right)\rangle,
\end{aligned} \tag{22}$$

and $\lambda_\sigma = -1$. The latter relation corresponds to the fact that $ee$ collisions do not limit the transport of electric charge.

A characteristic time between collisions of a particle with localized protons reads

$$\tau_{ip} = \frac{2}{n_p m_i^* k_{Fi}\langle Q_{ip}\rangle}, \tag{23}$$

where in the case of $ip$–collisions the angle brackets stand for an averaging according to

$$\langle Q_{ip}\rangle = \frac{1}{\pi}\int\limits_0^\pi \mathrm{d}\chi\,\sin\chi\,Q_{ip}(\chi). \tag{24}$$

$\chi$ is a scattering angle of a particle colliding with a proton. Finally, the efficiency of $ip$–collisions is described by the quantities $\omega_\alpha$ defined as:

$$\begin{aligned}
\omega_\kappa &= \omega_\sigma = \frac{1}{\langle Q_{ip}\rangle}\,\langle Q_{ip}(\chi)\cos\chi\rangle,\\
\omega_\eta &= \frac{1}{\langle Q_{ip}\rangle}\,\langle Q_{ip}(\chi)\,\tfrac{1}{2}\,(3\cos^2\chi-1)\rangle.
\end{aligned} \tag{25}$$

Eqs. (19) can be solved exactly if either particle–proton collisions dominate ($\tau_{ii} \gg \tau_{ip}$) or in the opposite case. In the former situation we have simply $\Psi_\alpha(x) = g_\alpha(x)\tau_{ip}[\tau_{ii}(1-\omega_\alpha)]^{-1}$. If, in contrast, the $ip$–collisions are negligible the exact solution is nontrivial and can be obtained with the aid of a method developed by Brooker and Sykes [9] and by Højgaard Jensen *et al.* [10]. Finally, it is possible to find variational solutions [7] for the functions $\Psi_\alpha$ at arbitrary value of the ratio $\tau_{ii}/\tau_{ip}$. This is done by assuming specific

dependences of these functions on $x$ ($\Psi_\kappa \propto x, \Psi_{\sigma,\eta} = \mathrm{const}$), which are consistent with symmetries of Eq. (19), and by subsequent determination of the unknown coefficients on integration of the equation over $x_1$ within infinite limits (the thermal conduction equation must be, in addition, multiplied by $x_1$). The resulting expressions are:

$$\Psi_\kappa(x) = \left[\frac{2}{5}\,\pi^2\,(3-\lambda_\kappa) + \frac{\tau_{ii}}{\tau_{ip}}\,(1-\omega_\kappa)\right]^{-1} x, \tag{26}$$

$$\Psi_{\sigma,\eta}(x) = \left[\frac{2}{3}\,\pi^2\,(1+\lambda_{\sigma,\eta}) + \frac{\tau_{ii}}{\tau_{ip}}\,(1-\omega_{\sigma,\eta})\right]^{-1}. \tag{27}$$

When the functions $\Psi_\alpha$ are known the calculation of the transport coefficients becomes straightforward. We insert Eqs. (17), (18), (26), and (27) into Eqs. (12)–(14) and find

$$\kappa_i = \frac{\pi^2\, n_i\, T}{3\, m_i^*\,(\nu_{\kappa ii} + \nu_{\kappa ip})}, \tag{28}$$

$$\sigma_e = \frac{e^2\, n_e}{m_e^* \nu_{\sigma ep}}, \tag{29}$$

$$\eta_i = \frac{m_i^*\, n_i\, v_{Fi}^2}{5\,(\nu_{\eta ii} + \nu_{\eta ip})}. \tag{30}$$

In this case $\nu_{\alpha ii}$ are the effective frequencies of particle–particle collisions:

$$\nu_{\kappa ii} = \frac{m_i^{*3} T^2}{20\pi}\,\langle Q_{ii}\rangle\,(3-\lambda_\kappa) = \frac{3}{5}\,\nu_{\eta ii}\,\frac{3-\lambda_\kappa}{1+\lambda_\eta}, \tag{31}$$

while $\nu_{\alpha ip}$ are those of particle–proton collisions:

$$\begin{aligned} \nu_{\kappa ip} &= \frac{1}{2} n_p m_i^* k_{Fi}\,\langle Q_{ip}\rangle\,(1-\omega_\kappa) = \nu_{\eta ip}\,\frac{1-\omega_\kappa}{1-\omega_\eta}, \\ \nu_{\sigma ep} &= \nu_{\kappa ep}. \end{aligned} \tag{32}$$

The above variational expressions for the transport coefficients are exact in the limit when $ip$–collisions dominate. However, in the opposite limit the variational method appears to be not very accurate (most notably for the thermal conductivity). The exact formulae of refs. [9,10] differs from ours by some factors, depending on the parameters $\lambda$. We will come back to those factors and correct our equations in Section 5. Now we turn to a calculation of the angular integrals which appear in the above equations for the effective frequencies.

## 4. Scattering probabilities

### *4.1. Electrons*

Let us summarize briefly the main assumptions that facilitate the angular averaging for electrons in Eqs. (21)–(22), and (24)–(25). The electron–electron scattering is described adequately by a Coulomb potential, screened by modifications of the charge density induced in the vicinity of each charge. This screening, being purely electron (as the protons are fixed and in no way respond to a small perturbation), is represented by a dielectric function $\epsilon(\omega, \boldsymbol{q})$ [$(\omega, \boldsymbol{q})$ — is a 4-momentum transfer in a collision event] and results mainly in a strong suppression of the collision probability when the momentum transfer $q$ is smaller than $q_{\rm TF} = 2\sqrt{\alpha/\pi v_{Fe}}\, k_{Fe}$ — the Thomas–Fermi wave number (here $\alpha$ is the fine-structure constant). When $\omega \ll qv$ (where $v$ is a typical velocity of electrons) the screening is static and the dielectric function may be approximated by [12]

$$\epsilon(0, \boldsymbol{q}) = 1 + \frac{\alpha}{\pi u^2}\left(\frac{2}{3} - \frac{1-3u^2}{6u}\ln\left|\frac{1-u}{1+u}\right| - \frac{u^2}{3}\ln\left|\frac{1-u^2}{u^2}\right|\right), \tag{33}$$

where $u = q/2k_{Fe}$, and we took into account that electrons are ultrarelativistic. In practice, the required smallness of energy transfers for a degenerate system puts limit on its temperature: $T \ll qv_{Fe} \sim q_{\rm TF}v_{Fe} = \sqrt{3}T_{pe}$, where $T_{pe}$ is an electron plasma temperature. In terms of the degeneracy parameter for the ultrarelativistic system this means that $T \ll 0.1\varepsilon_{Fe}$ — the condition we assume to be met.

Furthermore, one can easily verify that it is reasonable to approximate the right hand side of the Eq. (33) for any $q < 2k_{Fe}$, by the formula

$$\epsilon(0, \boldsymbol{q}) = 1 + \frac{q_{\rm TF}^2}{q^2}, \tag{34}$$

valid strictly only for small momentum transfers $q \ll 2k_{Fe}$. Even with the latter simplification the integrations in Eqs. (21), and (22) remain cumbersome. However, for ultrarelativistic particles the ratio $y = q_{\rm TF}/2k_{Fe}$ is a small number and, for practical applications, it is sufficient to retain only the lowest order terms of expansions of the sought-for quantities in $y$. In this way we obtain

$$\langle Q_{ee}\rangle = \frac{3\pi^3}{2}\frac{e^4}{\varepsilon_{Fe}^4 y^3}, \quad \lambda_\kappa = -\frac{1}{3}, \quad \lambda_\eta = -1 + 10y^2. \tag{35}$$

The resulting expressions for partial electron–electron thermal conductivity and viscosity coincide with those obtained by other authors (*e.g.* Refs. [6,11]) in the ultrarelativistic limit.

The treatment of the electron–proton collisions is somewhat simpler. First of all, here, the electron screening is always static (unless, of course, the temperature is large enough to excite the protons at their sites). The situation with proton–proton correlations is not quite certain. According to the arguments given in Section 2, we assume that the proton system is completely disordered. Under such conditions the problem of $ep$ scattering is very similar to the scattering of electrons off impurities which was studied *e.g.* by Flowers and Itoh [11]. In this case the integrations can be performed exactly at any degree of electron relativism and at any $y$, but again only the lowest order terms in $y$ in Eqs. (24), and (25) are needed. The exact expressions can be found in [11], however, the formula for viscosity obtained in this work is slightly inaccurate. For this reason we give here both exact and approximate expressions for the shear viscosity and an approximate formula for the thermal and electrical conductivities:

$$\langle Q_{ep}\rangle = \frac{2\pi e^4}{k_{Fe}^4 y^2}, \quad \omega_\kappa = 1+4y^2(\ln y+1), \quad \omega_\eta = 1+12y^2(\ln y+1.25). \tag{36}$$

The exact expression for the viscosity reads

$$\langle Q_{ep}\rangle\,(1-\omega_\eta) = \frac{12\pi e^4}{k_{Fe}^4}\left\{[1+2y^2(1+\beta^2)+3y^4\beta^2]\,\ln\frac{1+y^2}{y^2} - 2 - \frac{\beta^2}{2} - 3y^2\beta^2\right\}, \tag{37}$$

where $\beta = v_{Fe}/c$.

Finally, we note that the results obtained for $ep$ scattering, remain unchanged, if the protons are fully spin polarized.

### *4.2. Neutrons*

Contributions of neutrons and protons to the thermal conductivity and viscosity for the standard non-localized model of matter in neutron star cores were studied by Flowers and Itoh [13]. In the case of nucleon contributions a great deal of uncertainty is related to the description of the scattering processes. It is impossible to make use of the Landau theory for $nn$ scattering, since at present we do not know values of the momentum dependent quasiparticle amplitudes in the density range of interest. Yet even worse is our understanding of scattering of a neutron quasiparticle at the Fermi surface off a localized proton.

Another approach, fundamentally cruder, but able to supply us with the desired physical input, consists of a neglect of an influence of many-body effects on the scattering amplitudes. This means that we base our

consideration on the data on nucleon–nucleon (NN) scattering *in vacuum*. An obvious disadvantage of this approach is an inappropriate treatment of the short-range repulsive part of the NN interaction: being to some extent screened in a collision of quasiparticles, it is well sampled in a collision of bare nucleons. Therefore, such a method is expected to overestimate the role of the collisional processes in limiting the neutron transport, thus underestimating the transport coefficients. Although, following this second way, we hope that it must yield correct order-of-magnitude estimate of the neutron transport coefficients, we warn the reader that our results can be a few times less than the actual values of the quantities in question.

Within this framework the transition probabilities for $nn$ collisions at any degree of relativism are easily reconstructed from vacuum differential cross sections in the center-of-mass (cm) reference frame, namely:

$$Q_{nn}(\theta,\phi) = \frac{16\pi^2\hbar^4}{m_N^2 + k_{Fn}^2 \sin^2\theta/2} \frac{\mathrm{d}\sigma_{nn}}{\mathrm{d}\Omega_{\mathrm{cm}}}(E_{\mathrm{lab}},\phi_{\mathrm{cm}}), \tag{38}$$

where

$$E_{\mathrm{lab}} = \frac{k_{Fn}^2}{m_N}(1-\cos\theta), \quad \text{and} \quad \phi_{\mathrm{cm}} = \phi, \tag{39}$$

are the collision energy in the laboratory (lab) reference frame and the cm scattering angle.

The situation is more problematical for the case of $np$ collisions. Under the condition that neutrons are not relativistic, the transition probability for the scattering of a neutron off a localized proton (which can be thought of as a neutron scattering off an external field describing by the same $np$ potential) can be again derived from the differential cross sections of $np$ scattering in vacuum as

$$Q_{np}(\theta) = \frac{16\pi^2\hbar^4}{m_N^2} \frac{\mathrm{d}\sigma_{np}}{\mathrm{d}\Omega_{\mathrm{cm}}}(E_{\mathrm{lab}},\phi_{\mathrm{cm}}), \tag{40}$$

where

$$E_{\mathrm{lab}} = \frac{2k_{Fn}^2}{m_N}, \quad \text{and} \quad \phi_{\mathrm{cm}} = \theta. \tag{41}$$

However, with growing relativism, such a procedure ceases to be adequate, as the scattering off an instantaneous external potential becomes different from the scattering of two particles (the case studied in a laboratory). But for the densities considered ($\leq 5n_0$) the neutrons are only moderately relativistic, and, consequently, we may adopt the above formalism with the natural modification of the Eq. (40):

$$Q_{np}(\theta) = \frac{16\pi^2\hbar^4}{m_N^2 + p_{Fn}^2} \frac{\mathrm{d}\sigma_{np}}{\mathrm{d}\Omega_{\mathrm{cm}}}(E_{\mathrm{lab}},\phi_{\mathrm{cm}}). \tag{42}$$

We have calculated the required angular averages of the transition probabilities [those appearing in the Eqs. (31), and (32)] using the tables of vacuum cross sections in the cm reference frame as functions of lab energy (in the range from 10 to 700 MeV) and cm scattering angle. The tables themselves for *pp* and *np* scattering were obtained by using the partial wave solution WI96 available in the SAID database [14] developed at the Virginia Polytechnic Institute by R.A. Arndt with collaborators. To construct the *nn* cross sections from the *pp* ones the following procedure was used. First of all we subtracted from the table values the well known values of the Coulomb cross sections. This gave us reasonable estimates of the *nn* cross sections at larger angles but made the tables inapplicable at smaller angles (*e.g.* at energies less than 400 MeV the table values at $5^{\rm o}$ became negative, indicating the importance of the interference terms). It was tempting then to extrapolate smoothly the values of the *nn* cross sections at larger angles ( $\gtrsim 20^{\rm o} - 40^{\rm o}$ depending on energy) to the domain of small angles, which yielded a reasonable estimate of the *nn* cross sections over the entire angle range. Finally, at zero energy we have used the value of 3030 mb that followed from the theory of *nn* scattering length (*e.g.* Ref. [15]).

The calculations were done for the values of neutron Fermi wave vector $k_{Fn}$ from 1.1 to 2.9 fm$^{-1}$ ($n_n$ from 0.5 to 5 $n_0$). To interpolate between the neighbour nodes of the tables of the cross sections the bilinear interpolation was used. The results of our calculations are fitted with the mean and maximum errors of the fits less than 1.5 % by the following analytical expressions:

$$\begin{aligned}
S_{\kappa n} &= \frac{m_N^2\,\langle Q_{nn}\rangle\,(3-\lambda_\kappa)}{256\,\pi\hbar^4\,1\mathrm{mb}} = \frac{0.3833}{z_n^3\sqrt{z_n}} + 3.652 z_n^{0.4},\\
S_{\eta n} &= \frac{m_N^2\,\langle Q_{nn}\rangle\,(1+\lambda_\eta)}{192\,\pi\hbar^4\,1\mathrm{mb}} = \frac{0.1152}{z_n^3} + \frac{3.965 z_n}{2.499 + z_n^6\sqrt{z_n}},\\
S_{\kappa p} &= \frac{m_N^2\,\langle Q_{np}\rangle\,(1-\omega_\kappa)}{16\,\pi\hbar^4\,1\mathrm{mb}} = \frac{1.833}{z_n^2} + \frac{1.430 z_n^2}{0.3958 + z_n^8},\\
S_{\eta p} &= \frac{m_N^2\,\langle Q_{np}\rangle\,(1-\omega_\eta)}{16\,\pi\hbar^4\,1\mathrm{mb}} = \frac{0.5663}{z_n^3} + \frac{4.545 z_n}{1.276 + z_n^6}.
\end{aligned} \tag{43}$$

In all these expressions the quantity $z_n$ is the neutron Fermi wave vector in units of 2.666 fm$^{-1}$ (corresponding to density $4n_0$).

## 5. Practical formulae

Let us summarize the results, derived in the previous sections, and present them in the form useful for practical applications. First of all, we note that the variational solutions obtained could be corrected to yield the

exact asymptotes in the limit when particle–particle collisions dominate. This is done customarily by multiplying the partial particle–particle variational transport coefficients by well-known factors $C_\alpha$ depending on the parameters $\lambda_\alpha$. In the case of electrons $C_\kappa(-1/3) = 1.3$, while $C_\eta(-1+y^2) \approx 1$, *i.e.* the variational solution for the shear viscosity is exact for ultrarelativistic electrons. For neutrons the coefficients $C_\alpha$ are weakly varying functions of density. For the considered density range it is a very good approximation to adopt fixed values of the correction factors $C_\kappa = 1.2$ and $C_\eta = 1.05$.

Bringing together Eqs. (28)–(32), (35), (36), and (43), and incorporating the above factors, we may write for the thermal conductivity:

$$\begin{aligned}
\kappa_i^{-1} &= \kappa_{ii}^{-1} + \kappa_{ip}^{-1}, \\
\kappa_{ee} &= 2.2 \cdot 10^{23} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right) \frac{1}{T_8} \quad \text{ergs s}^{-1}\text{cm}^{-1}\text{K}^{-1}, \\
\kappa_{ep} &= 2.5 \cdot 10^{19} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right)^{1/3} T_8 \quad \text{ergs s}^{-1}\text{cm}^{-1}\text{K}^{-1}, \\
\kappa_{nn} &= 8.3 \cdot 10^{23} \left(\frac{m_N}{m_n^*}\right)^4 \left(\frac{n_n}{4n_0}\right) \frac{1}{T_8} \frac{1}{S_{\kappa n}} \quad \text{ergs s}^{-1}\text{cm}^{-1}\text{K}^{-1}, \\
\kappa_{np} &= 8.9 \cdot 10^{17} \left(\frac{m_N}{m_n^*}\right)^2 \left(\frac{n_n}{4n_0}\right)^{2/3} \\
&\quad \times \left(\frac{n}{4n_0} \cdot \frac{x_p}{0.01}\right)^{-1} \frac{T_8}{S_{\kappa p}} \quad \text{ergs s}^{-1}\text{cm}^{-1}\text{K}^{-1},
\end{aligned} \tag{44}$$

for the electrical conductivity:

$$\sigma_e = \sigma_{ep} = 9.2 \cdot 10^{23} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right)^{1/3} \text{ s}^{-1}, \tag{45}$$

and for the shear viscosity:

$$\begin{aligned}
\eta_i^{-1} &= \eta_{ii}^{-1} + \eta_{ip}^{-1}, \\
\eta_{ee} &= 1.7 \cdot 10^{19} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right)^{5/3} \frac{1}{T_8^2} \quad \text{g cm}^{-1}\text{s}^{-1}, \\
\eta_{ep} &= 1.1 \cdot 10^{13} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right) \quad \text{g cm}^{-1}\text{s}^{-1}, \\
\eta_{nn} &= 1.5 \cdot 10^{19} \left(\frac{m_N}{m_n^*}\right)^4 \left(\frac{n_n}{4n_0}\right)^{5/3} \frac{1}{T_8^2} \frac{1}{S_{\eta n}} \quad \text{g cm}^{-1}\text{s}^{-1}, \\
\eta_{np} &= 2.2 \cdot 10^{13} \left(\frac{m_N}{m_n^*}\right)^2 \left(\frac{n_n}{4n_0}\right)^{4/3} \\
&\quad \times \left(\frac{n}{4n_0} \cdot \frac{x_p}{0.01}\right)^{-1} \frac{1}{S_{\eta p}} \quad \text{g cm}^{-1}\text{s}^{-1}.
\end{aligned} \tag{46}$$

Let us remind the conditions of applicability of the above expressions. Both electrons and neutrons must be strongly degenerate. The neglect of the dynamical screening effect is based on a more stringent condition for electrons: $T \ll 0.1\varepsilon_{Fe}$. The electron number density must ensure their ultrarelativism. Finally, the density of neutrons is restricted to the range from 0.5 to 5 $n_0$.

## 6. Neutrino losses

In this section we study two processes, contributing to neutrino cooling, which are specific for neutron star matter with localized protons. The first of these processes is the neutrino–antineutrino pair bremsstrahlung due to electron scattering off localized protons. The second process is the strong-interaction analog of the first one: it is the neutrino–antineutrino pair bremsstrahlung accompanying scattering of neutrons off protons localized in neutron medium. The temperature dependence of both these processes is $T^6$, that is the same as for the direct URCA process. Therefore, the localization of protons may imply non-standard (accelerated) cooling of a neutron star. Below, we will estimate the rate of the energy losses in these processes under some model assumptions.

*(a) Electrons.* In the case of electrons we can use the results of Ref. [16]. This paper was concerned with the neutrino-pair bremsstrahlung due to electron–nucleus scattering in the liquid phase of the neutron star crusts. The authors derived the following general expression for the energy loss rate (Eq. (8) of [16])

$$Q^e_{\rm Brem} = \frac{8\pi G_{\rm F}^2 Z^2 e^4 C_+^2}{567\hbar^9 c^8}(k_B T)^6 n_i L \quad \text{ergs s}^{-1}\ \text{cm}^{-3}, \tag{47}$$

where $G_F = 1.436 \cdot 10^{-49}$ ergs cm$^3$ is the Fermi weak coupling constant, factor $C_+^2 = 1.675$ takes into account the emission of $\nu_e, \nu_\mu$, and $\nu_\tau$, $Z$ is the nucleus charge, $n_i$ is the number density of nuclei, and the quantity $L \approx 1$, interpreted as a Coulomb logarithm, is a weakly varying function of $Z$, $T$, and $n_i$. The authors derived also the general formula for $L$ [their Eq. (19)], which takes into account the nucleus electromagnetic formfactor, the static structure factor of nuclei, static electron screening, non-Born corrections and incorporates accurately the thermal effects. They also proposed an analytical fitting formula for $L$ [Eq. (25)]. In our case we have obviously $Z = 1$ and $n_i = n_p$. The fit of [16] does not apply here (the authors were interested in the crust and considered $Z \geq 10$). Besides, in our situation the temperature is much lower than the screening momentum, *the low-temperature case* of [16]. In this regime the thermal effects are not important and $L$ is

given by much simpler formula (21) of [16]. In this expression we can neglect non-Born corrections and nucleus formfactor as well as nuclei structure factor (because our proton system is assumed to be fully disordered), and approximate static dielectric electron screening function by Eq. (34). Performing one-dimensional integration for ultrarelativistic electrons, we obtain $L = 1.755$. Inserting this value into Eq. (47) we obtain

$$Q_{\rm Brem}^{e-{\rm loc.p.}} = 6.0 \cdot 10^{18}\, T_9^6 \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right) \quad \text{ergs s}^{-1}\ \text{cm}^{-3}, \tag{48}$$

where $T_9 \equiv T/10^9$ K.

*(b) Neutrons.* In this subsection we will obtain an expression, which will enable us to estimate the energy loss rate due to neutrino pair emission from neutron scattering off a localized proton. To simplify the derivation we will, first, regard the neutrons as fully non-relativistic, and, second, will treat $np$-interaction in a very crude manner, assuming it to be given by a contact spin-independent potential, which will be treated in the Born approximation. The interaction strength $U$ will further be made density-dependent by fitting it at a given density (or, equivalently, at a given collision energy) to the total vacuum $np$ cross-section, known experimentally or theoretically (see below). No correction of the $np$ scattering rate to account for the presence of the medium (except for the exclusion principle in the initial and final neutron states) will be made.

The process in question, in the formalism of Feynman diagrams, is described (to lowest order) by two diagrams, (A) and (B). In both cases, we consider an initial neutron with 4-momentum $k = (\varepsilon, \boldsymbol{k})$. In the case of diagram (A), this neutron first emits a neutrino–antineutrino pair with total 4-momentum $p = (\omega, \boldsymbol{p}) = k_1 + k_2$, where $k_1 = (\omega_1, \boldsymbol{k}_1)$ and $k_2 = (\omega_2, \boldsymbol{k}_2)$ are, respectively, 4-momenta of neutrino and antineutrino, then propagates with 4-momentum $k - p$ and finally interacts strongly with a localized proton, absorbing 4-momentum $q = (0, \boldsymbol{q})$ and ending in the final state with 4-momentum $k' = (\varepsilon', \boldsymbol{k}')$. Diagram (B) corresponds to the situation, in which neutron first interacts strongly with a localized proton, then propagates with 4-momentum $k' + p$, and, finally, emits a neutrino–antineutrino pair of total 4-momentum $p$, ending in the final state $k'$.

We will use nonrelativistic formalism (for neutrons) to describe the weak-interaction vertex, take non-relativistic neutron Green functions, when dealing with the intermediate states, and nonrelativistic spinors for neutron initial and final states. Then the first order matrix element assumes the form

$$M = M_A + M_B = -\frac{iG_F U}{2\sqrt{2}}\, \chi'^{\dagger}(\delta_{\mu 0} - g_A \delta_{\mu i}\sigma_i)\chi\, [G(k-p) + G(k'+p)]\, l^{\mu}, \tag{49}$$

where $g_A \approx 1.26$ is the axial renormalization constant, greek indices take the values 0, 1, 2, 3, and latin indices take the values 1, 2, 3, $\sigma_i$ are the standard

Pauli matrices, $\chi$ and $\chi'$ are the Pauli spinors, describing the initial and final neutron states. Further, $l^\mu$ is the neutrino neutral current given by

$$l^\mu = \bar{u}_1 \gamma^\mu (1+\gamma^5) u_{-2}, \tag{50}$$

where $u_1 = u(k_1)$ and $u_{-2} = u(-k_2)$ are respectively neutrino and antineutrino bispinors, and bar means Dirac conjugate. In our problem the denominators of the neutron propagators $G$ cannot be zero. Hence, we can replace them by the vacuum propagators, in spite of the fact, that our process goes in the presence of neutron Fermi sea. Nonrelativistic neutron propagator takes therefore the form

$$G(\omega, \boldsymbol{k}) = \frac{1}{\omega - \varepsilon_{\boldsymbol{k}} + i0}\,. \tag{51}$$

The neutrino pair energy loss rate (in erg cm$^{-3}$ s$^{-1}$) is given by

$$\begin{aligned} Q_{\rm Brem}^{n-\rm loc.p.} &= 3n_p \int \frac{\mathrm{d}\boldsymbol{k}\,\mathrm{d}\boldsymbol{k}'\,\mathrm{d}\boldsymbol{k}_1\,\mathrm{d}\boldsymbol{k}_2\,\mathrm{d}\boldsymbol{q}}{4\omega_1\omega_2\,(2\pi)^{15}}\,(2\pi)^4\,\delta^{(4)}(k+q-k'-k_1-k_2) \\ &\quad \times f(1-f')\,\omega \sum_{ss'} |M|^2, \end{aligned} \tag{52}$$

where $n_p$ is the number density of localized protons, $f = f(\varepsilon)$ and $f' = f(\varepsilon')$ are the neutron Fermi–Dirac distribution functions, and the summation runs over final and initial neutron spin states. A factor of 3 accounts for the emission of $\nu_e, \nu_\mu$, and $\nu_\tau$ pairs.

Calculating the spin-summed squared matrix elements in the straightforward manner and using the identity [17]

$$\int \frac{\mathrm{d}\boldsymbol{k}_1\,\mathrm{d}\boldsymbol{k}_2}{\omega_1\omega_2}\,k_1^\mu k_2^\nu \delta^{(4)}(p-k_1-k_2) = \frac{\pi}{6}\,(p^2 g^{\mu\nu} + 2p^\mu p^\nu), \tag{53}$$

we obtain the following expression for the emissivity

$$\begin{aligned} Q_{\rm Brem}^{n-\rm loc.p.} &= \frac{\pi G_F^2 U^2 n_p}{(2\pi)^{11}} \int \mathrm{d}\boldsymbol{k}\,\mathrm{d}\boldsymbol{k}'\,\mathrm{d}\boldsymbol{p}\,f(1-f')\,\omega\,[3g_A^2\omega^2 + (1-2g_A^2)\boldsymbol{p}^2] \\ &\quad \times 4m_n^2 \left(\frac{1}{2m_n\omega - 2\boldsymbol{p}k' - \boldsymbol{p}^2} - \frac{1}{2m_n\omega - 2\boldsymbol{p}k + \boldsymbol{p}^2}\right)^2, \end{aligned} \tag{54}$$

where we have replaced the integration over $\mathrm{d}\boldsymbol{q}$ by that over $\mathrm{d}\boldsymbol{p}$. In this expression we must integrate over the domain, where $\varepsilon' < \varepsilon$ and for $|\boldsymbol{p}| < \omega = \varepsilon - \varepsilon'$. In the denominators we can safely omit the terms $\boldsymbol{p}^2/2m_n$ which are negligible compared to $\omega$, and using the fact that neutrons are

nonrelativistic, we can expand the denominators to first order in $\boldsymbol{p}k'/m_n$ and $\boldsymbol{p}k/m_n$. Then we direct the $z$-axis of the spherical reference frame along the vector $\boldsymbol{k}$ and place the vector $\boldsymbol{k}'$ into the $xz$-plane. Integrations over d$\boldsymbol{p}$ and over angle between vectors $\boldsymbol{k}$ and $\boldsymbol{k}'$ yield the emissivity in the form of two-dimensional "Fermi" integral

$$\begin{aligned} Q_{\rm Brem}^{n-\rm loc.p.} &= \frac{8\pi G_F^2 U^2 n_p}{21(2\pi)^8}\,(1+2.2g_A^2)\,\left(\frac{m_n^*}{m_n}\right)^2 \\ &\quad\times\int {\rm d}\varepsilon\,{\rm d}\varepsilon' |\boldsymbol{k}||\boldsymbol{k}'|(\boldsymbol{k}^2+\boldsymbol{k}'^2) f(1-f')\omega^4 \\ &= \frac{16\pi G_F^2 U^2 n_p}{21(2\pi)^8}\,(1+2.2g_A^2)\,\left(\frac{m_n^*}{m_n}\right)^2 k_{Fn}^4 T^6 \\ &\quad\times\int\limits_{-\infty}^{\infty}{\rm d}x\int\limits_0^{\infty}{\rm d}\omega\,\frac{\omega^4}{(e^x+1)(e^{\omega-x}+1)}, \end{aligned} \tag{55}$$

where we have taken into account that neutrons are strongly degenerate and have pulled all the smooth functions of the neutron momenta out of the integral at the Fermi surface. The latter integral is standard and is equal to $(2\pi)^6/504$. Substituting this value into Eq. (55) we obtain

$$Q_{\rm Brem}^{n-\rm loc.p.} = \frac{G_F^2 U^2}{2646\pi}\,(1+2.2g_A^2)\,\left(\frac{m_n^*}{m_n}\right)^2 n_p k_{Fn}^4 T^6. \tag{56}$$

The last step is to specify the value of $U^2$. For the contact spin-independent $np$ interaction $U^2$ gives differential probability of elastic neutron scattering off a localized proton (the same for all angles $\theta$). Our approximation consists in expressing $U^2$ in terms of the total elastic cross section for $np$ scattering in vacuum, using Eqs. (40, 41). Integrating over the solid angle we have

$$4\pi U^2 = \frac{16\pi^2\hbar^4}{m_N^2}\,\sigma_{np}^{\rm tot.el.}\left(\frac{2k_{Fn}^2}{m_N}\right). \tag{57}$$

In this case, $\sigma_{np}^{\rm tot.el}$ is the total elastic $np$ cross section which can be obtained from the same theoretical model as in Section 4.2., WI96, in the SAID database. We propose the following fitting formula for $U^2$ in the energy range from 100 to 700 MeV:

$$U^2 = \frac{4\pi\hbar^4\ \ 1\ {\rm mb}}{m_N^2}\,S_{np}^{\rm t},\quad \text{where}\quad S_{np}^{\rm t} = 26.09 + \frac{3.444}{z_n^3}, \tag{58}$$

and $z_n$ is defined below Eq. (43). The mean error of this expression is $\approx 1.2\%$ and the maximum error occurring at 700 MeV is $\approx 2.9\%$. Combining

Eqs. (56) and (58), we obtain the following numerical result:

$$Q_{\rm Brem}^{n-\rm loc.p.} = 1.3\cdot 10^{21}\, S_{np}^{\rm t} \left(\frac{n}{4n_0}\cdot\frac{x_p}{0.01}\right)\left(\frac{n_n}{4n_0}\right)^{4/3}\left(\frac{m_n^*}{m_n}\right)^2 \times T_9^6 \quad {\rm ergs\,s^{-1}\,cm^{-3}}. \tag{59}$$

Medium effects can be expected to modify both the magnitude and the density dependence of $U^2$, as compared to that given by our simple prescription, Eq. (58). Therefore, expression (59) should be treated as a rough estimate of $Q_{\rm Brem}^{n-\rm loc.p.}$.

## 7. Astrophysical implications

As one can see from the results obtained in the preceding sections, presence of randomly distributed localized protons, removing the $T^{-2}$ factor from the transport coefficients, leads at lower temperatures to a dramatic decrease of $\kappa$, $\sigma$, and $\eta$, as compared to the standard case, where protons form a Fermi liquid. Let us discuss in some detail the astrophysical implications of this effect for neutron stars.

*(a) Thermal conductivity.* In our case transport of heat is dominated by electrons, which scatter predominantly off localized protons, so that $\kappa^{\rm loc.p.} \simeq \kappa_e^{\rm loc.p.} \simeq \kappa_{ep}^{\rm loc.p.}$. Let us compare this result with the values of $\kappa$ corresponding to the standard case. The latter are expressed by the formulae derived in [6]:

$$\kappa_e = \frac{7.6\cdot 10^{24}}{T_8}\left(\frac{m_p}{m_p^*}\right)^{1/2}\left(\frac{n_e}{n_0}\right)^{7/6} \ {\rm ergs\ cm^{-1}s^{-1}K^{-1}} \tag{60}$$

for non-superfluid $p$, and

$$\kappa_e = \frac{5.6\cdot 10^{24}}{T_8}\left(\frac{n_e}{n_0}\right) \ {\rm ergs\ cm^{-1}s^{-1}K^{-1}} \tag{61}$$

for highly superfluid $p$, the decisive difference between the two regimes being the presence or absence of proton screening. At $n = 4n_0$ and $x_e = 0.01$ we thus get

$$\frac{\kappa_e}{\kappa^{\rm loc.p.}} \simeq \frac{(5\div 10)\cdot 10^3}{T_8^2}\ . \tag{62}$$

The diffusive thermal conductivity due to neutrons is generally on the order of or somewhat smaller than that of electrons. (The actual ratio between them depends sensitively on *(i)* the fraction of electrons, *(ii)* the nucleon effective masses, and *(iii)* the strength of quasiparticle interactions; in the

non-superfluid regime $\kappa_{nn}^{\rm loc.p.}$ may serve as a good approximation of the neutron conductivity.) Thus we can accept the value of $10^4/T_8^2$ as a reasonable estimate of the ratio $\kappa/\kappa^{\rm loc.p.}$.

Neutron stars are born as very hot objects which cool subsequently due to neutrino losses from their interior. The cooling is accompanied by thermal equilibration of the stellar interior with a typical time scale, determined by the size of the core $R_{\rm core}$, the specific heat per unit volume $C$, and the thermal conductivity of matter $\kappa$ as

$$\tau_{\rm t.e.} \sim \frac{R_{\rm core}^2 C}{\kappa}. \tag{63}$$

For estimates, in the standard non-superfluid case we can set $C = C_n$, the specific heat of degenerate neutrons,

$$C_n = 2.6 \cdot 10^{20} \left(\frac{m_n^*}{m_N}\right) \left(\frac{n_n}{4n_0}\right)^{1/3} T_9 \quad \text{ergs} \quad \text{cm}^{-3}\text{K}^{-1}, \tag{64}$$

and approximate $\kappa$ by $\kappa_e$ (60). Approximating further all the effective masses by bare masses and assuming a core of constant density ($n \simeq n_n = 4n_0$) and composition ($x_e = 0.01$) we obtain $\tau_{\rm t.e.} \sim 460\ T_9^2\ (R_{\rm core}/10\ \text{km})^2$ years. If the core is superfluid we may use for $C$ the specific heat of electrons $C_e$:

$$C_e = 6.6 \cdot 10^{18} \left(\frac{n}{4n_0} \cdot \frac{x_e}{0.01}\right)^{2/3} T_9 \quad \text{ergs} \quad \text{cm}^{-3}\text{K}^{-1}, \tag{65}$$

and take $\kappa$ from Eq. (61). This gives us $\tau_{\rm t.e.} \sim 10\ T_9^2\ (R_{\rm core}/10\ \text{km})^2$ years (the latter relation should be regarded as a lower bound, as the neutron specific heat could decrease rather slowly depending on the type of superfluidity [18]). The temperature dependence of both those expressions is removed by the localization of protons. Multiplying $\tau_{\rm t.e.}$ by the factor $\kappa/\kappa^{\rm loc.p.}$ (62) we obtain $\tau_{\rm t.e.}^{\rm loc.p.}$ equal to $4.6 \cdot 10^4\ (R_{\rm core}^{\rm loc.p.}/10\ \text{km})^2$ years and $10^3\ (R_{\rm core}^{\rm loc.p.}/10\ \text{km})^2$ years for non-superfluid and strongly superfluid cases, respectively.

The thermal conduction becomes important when the above time scales are comparable or smaller than a characteristic time of thermal evolution of matter due to neutrino losses. Otherwise, the temperature of a given element of matter is determined locally. The neutrino cooling time scale could be estimated as $CT/\varepsilon_\nu$, where $\varepsilon_\nu$ is the total neutrino emissivity. The temperature dependences of $\varepsilon_\nu$ for various neutrino-emission processes are usually very strong. In view of this, the equilibration temperature for the case of proton localization will not be much smaller, than that for the

standard case; in both cases the temperatures fall to the range of $10^8$–$10^9$ K. However, the time required to reach the thermal equilibrium in a core, containing localized protons, will be two orders of magnitude longer than that for a standard liquid core. The latter conclusion will not be changed by the appearance of neutron superfluidity.

Another difference between the two cases comes from the simple idea that the thermal equilibration time in the localized protons case is temperature independent. While in the standard case the core below $\sim 10^8$ K could be treated as perfectly isothermal, it will still take $\sim 10^3 - 10^4$ years to wash out any accidental temperature inhomogeneity in the core with localized protons.

*(b) Electrical conductivity.* The electrical conductivity is relevant for the ohmic dissipation of internal magnetic fields in neutron stars. In the standard case of *npe* matter with non-superfluid protons the charge transport is dominated by ultrarelativistic electrons, scattering off protons, which results in an electrical conductivity $\sigma_{ep} = 2.1 \times 10^{31}\ (n_e/n_0)^{3/2}/T_8^2\ \mathrm{s}^{-1}$ [19]. Localization of protons prevents the appearance of a proton superconductor (as the localization temperature is rather high, of the order of $10^{10} - 10^{11}$ K, which is commonly believed to be higher than $T_{cp}$) and at $n = 4n_0$ and $x_e = 0.01$ reduces the electrical conductivity of neutron star matter by a large factor

$$\frac{\sigma_{ep}}{\sigma_e^{\mathrm{loc.p.}}} \simeq \frac{2 \cdot 10^5}{T_8^2}. \tag{66}$$

Such a low value of electrical conductivity would lead to a significant decay of the electric currents, circulating within the core with localized protons, over a time scale of

$$\tau_{\mathrm{d}}^{\mathrm{loc.p.}} \sim \frac{\sigma_e^{\mathrm{loc.p.}} (R_{\mathrm{core}}^{\mathrm{loc.p.}})^2}{c^2} \simeq 3 \cdot 10^7 \left( \frac{R_{\mathrm{core}}^{\mathrm{loc.p.}}}{10\ \mathrm{km}} \right)^2 \ \text{years}. \tag{67}$$

Current analyses of the population of radio pulsars do not show any evidence of magnetic field decay during active lifetime of a normal radio pulsar [20]. Specific *lower bounds* on the magnetic field decay timescale, obtained using various types of statistical analyses, range from $2 \times 10^7$ years to $10^8$ years [20]. To be consistent with this observational fact, we should assume that either the core with localized protons is small, and the bulk of the field is sustained by the currents, circulating in the outer part of the core; either the external field is separated from the internal field, and the observable bounds for the decay of the surface field put no evident constraints on the evolution of the core field; or that the core magnetic field is due not to electric currents but results from a permanent ferromagnetic magnetization of the matter. Actually, as demonstrated by Kutschera and Wójcik [21,22],

ferromagnetism due to a complete spin polarization of protons and a partial spin polarization of neutrons (of the order of $x_p$) could be a generic property of the neutron star matter with localized protons (for a confrontation of this theoretical prediction with observations of radio pulsars, see [23]).

*(c) Shear viscosity.* The localization of protons leads also to a strong decrease of the shear viscosity of neutron star matter. Assuming normal neutrons, we can estimate the value of $\eta$ of standard *npe* matter by the quantity $\eta_{nn}^{\text{loc.p.}}$. The localization of protons will result in a decrease of $\eta$ by a factor

$$\frac{\eta}{\eta^{\text{loc.p.}}} \simeq \frac{7 \cdot 10^5}{T_8^2} \ . \tag{68}$$

In contrast to the standard case $\eta^{\text{loc.p.}}$ is temperature independent. This might be relevant for stability of rapidly rotating neutron stars. In the standard case of the *npe* matter, $\eta$ increases with decreasing temperature as $T^{-2}$. Dissipative effects due to $\eta$ contribute to damping of the secular instability driven by the gravitational radiation reaction (GRR) [24,25] in rapidly rotating neutron stars. Detailed calculations show, that viscous effects of $\eta$ damp completely the GRR secular instability, if internal temperature falls below $10^7$ K [26]. However, within a neutron star core with localized protons, the shear viscosity remains constant and close to the value of the shear viscosity of standard *npe* matter at $T \simeq 10^{11}$ K [see Eq. (68)]. Such a low value of the shear viscosity could not prevent the growth of the GRR secular instability at any internal temperature of a neutron star [26].

*(d) Neutrino cooling.* At the earlier stages of the evolution matter with localized protons cools emitting neutrinos via two reactions involving only nucleons, and several reactions, which involve also electrons. The two nucleonic reactions are the neutrino–antineutrino pair bremsstrahlung in the *nn* and *np* collisions. The first process (*e.g.* [4]) is common for both standard and localized protons models of matter, and the rate of energy emission in this process varies with temperature as $T^8$. The second nucleonic process is modified drastically by the localization of protons: its emissivity becomes proportional to $T^6$, reproducing the temperature dependence of the direct URCA process. Nevertheless, the *np* bremsstrahlung remains several orders of magnitude less efficient than the direct URCA ($Q_{\text{Brem}}^{n-\text{loc.p.}}/Q_{\text{dURCA}} \sim 3 \cdot 10^{-5}$). On the other hand, at temperatures below $\sim 10^9$ K its emissivity exceeds that of the modified URCA ($Q_{\text{Brem}}^{n-\text{loc.p.}}/Q_{\text{mURCA}} \sim 2 \cdot 10^3 \ T_8^{-2}$) process, which, in turn, is thought to govern the cooling in the standard (non-localized) model, when the direct URCA is forbidden by the momentum conservation law. Thus, one might expect that cooling of a neutron star with significant fraction of mass being in the phase with localized protons would follow some intermediate path between the curves describing the standard (modified URCA) and accelerated (direct URCA) cooling.

As is any nucleonic process, the $np$ bremsstrahlung is subject to strong suppression, if the neutron superfluidity appears. In this case, the neutrino pair emission from the scattering of electrons off localized protons serves as a dominant mechanism of cooling of a neutron star core.

*(e) Final remarks.* In this subsection we will come back to the problem of a crystalline ordering of localized protons and comment on the values of the transport coefficients in this case. As discussed in Section 2, we consider this possibility unlikely, however, see Ref. [27]. For temperatures below 1 MeV the phase space available for phonons (in the proton crystal) is small (as $T^3$). Then the transport of energy, charge and momentum would be limited by $ee$, $nn$, and, in the case of the electrical conductivity, by $en$ collisions. This means that all the transport coefficients would be even larger than those in the standard case and would reproduce the standard (Fermi liquid) temperature dependences. The situation would be complicated by the band structure of quasiparticle states, however, this would further increase the transport coefficients by reducing the phase space available for scattering quasiparticles.

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## REFERENCES

[1] R.B. Wiringa, V. Fiks, A. Fabrocini, *Phys. Rev.* **C38**, 1010 (1988).

[2] M. Kutschera, W. Wójcik, *Phys. Rev.* **C47**, 1077 (1993).

[3] E.M. Lifshitz, L.P. Pitaevskii, *Physical Kinetics*, Pergamon Press, Oxford 1981.

[4] B.L. Friman, O.V. Maxwell, *Astrophys. J.* **232**, 541 (1979).

[5] Y. Toyozawa, *Prog. Theor. Phys.* **26**, 29 (1961).

[6] O.Y. Gnedin, D.G. Yakovlev, *Nucl. Phys.* **A582**, 697 (1995).

[7] J.M. Ziman, *Electrons and Phonons*, Oxford University Press, Oxford 1960.

[8] G. Baym, C. Pethick, *Landau Fermi-Liquid Theory*, John Wiley, New York 1991.

[9] G.A. Brooker, J. Sykes, *Phys. Rev. Lett.* **21**, 279 (1968); J. Sykes, G.A. Brooker, *Ann. Phys.* (N.Y.) **56**, 1 (1970).

[10] H. Højgaard Jensen, H. Smith, J.W. Wilkins, *Phys. Lett.* **A27**, 532 (1968); *Phys. Rev.* **185**, 323 (1969).

[11] E. Flowers, N. Itoh, *Astrophys. J.* **206**, 218 (1976).

[12] B. Jancovici, *Nuovo Cimento* **25**, 428 (1962).

[13] E. Flowers, N. Itoh, *Astrophys. J.* **230**, 847 (1979).

[14] R.A. Arndt, L.D. Roper, R.A. Bryan, R.B. Clark, B.J. VerWest, P. Signell, *Phys. Rev.* **D28**, 97 (1983).

[15] M.A. Preston, R.K. Bhaduri, *Structure of the Nucleus*, Addison-Wesley, Reading, Massachusetts 1975.

[16] P. Haensel, A.D. Kaminker, D.G. Yakovlev, *Astron. Astrophys.* **314**, 328 (1996).

[17] V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, *Quantum Electrodynamics*, Pergamon Press, Oxford 1982.

[18] K.P. Levenfish, D.G. Yakovlev, *Astron. Rep.* **38**, 247 (1994).

[19] G. Baym, C. Pethick, D. Pines, *Nature* **224**, 674 (1969).

[20] D. Bhattacharya, G. Srinivasan, in *X–ray Binaries*, W.H.G. Lewin, J. Van Paradijs, E.P.J. Van den Heuvel (eds), Cambridge University Press, Cambridge 1995.

[21] M. Kutschera, W. Wójcik, *Phys. Lett.* **B223**, 11 (1989).

[22] M. Kutschera, W. Wójcik, *Acta. Phys. Pol.* **B23**, 947 (1992).

[23] P. Haensel, S. Bonazzola, *Astron. Astrophys.* **314**, 1017 (1996).

[24] J.L. Friedman, *Phys. Rev. Lett.* **51**, 11 (1983).

[25] R.V. Wagoner, *Astrophys. J.* **278**, 345 (1984).

[26] L. Lindblom, *Astrophys. J.* **438**, 265 (1995).

[27] M. Kutschera, W. Wójcik, *Nucl. Phys.* **A581**, 706 (1995).