$\langle T^{\mu}_{\nu}\rangle_{\text{ren}}$ OF THE QUANTIZED CONFORMAL FIELDS IN THE SCHWARZSCHILD SPACETIME: ISRAEL–HARTLE–HAWKING STATE

Jerzy Matyjasek

Institute of Physics, Maria Curie Skłodowska University
pl. Marii Curie Skłodowskiej 1, 20-031 Lublin, Poland
e-mail: matyjase@tytan.umcs.lublin.pl
or jurek@iris.umcs.lublin.pl

(Received December 23, 1998)

The renormalized expectation value of the stress energy tensor of the conformally invariant massless fields in the Israel–Hartle–Hawking state in the Schwarzschild spacetime is constructed. It is achieved through solving the conservation equation in conformal space and utilizing the regularity conditions in a physical metric. Specifically, the relation of the results of the present approach to the stress tensor constructed within the framework of the Hadamard renormalization is analysed. Finally, the semi-analytic models reconstructing the numerical estimates of the tangential component of the stress-energy tensor with the maximal deviation not exceeding 0.7% are constructed.

PACS numbers: 04.62.+v, 04.70.Dy

In a recent publication [1] we have constructed the approximate mean value of the regularized stress-energy tensor of the massless and conformally invariant quantized scalar field in the Israel–Hartle–Hawking state in the Schwarzschild spacetime. We employed the Hadamard regularization which has proven to be a powerful tool in such calculations [2–8]. In the Hadamard regularization one must solve the constraint equations for three unknown functions of radial coordinate and insert them into the general expression for $\langle T^{\mu}_{\nu}\rangle$ [4]. Unfortunately, since the constraint equation involves three unknown functions, one of which being closely related to the vacuum fluctuation of the quantized field $\langle\phi^2\rangle$, the problem, beside the boundary conditions, must be supplemented by some additional informations regarding the nature of the sought functions [9, 10].

In Ref. [1] we have assumed that unknown functions (and $\langle\phi^2\rangle$) have a simple form

$$\sum_{m=-k}^{k'} (1-x)^m W_m(x), \qquad k, k' \geq 0, \tag{1}$$

where $x = 2M/r$, and $W_m(x)$ for each value of m, is a polynomial in x, and showed that it is relatively easy to construct solutions, which lead to the stress-energy tensor and the vacuum fluctuation which reflects principal features of the exact $\langle\phi^2\rangle_{\rm ren}$, and $\langle T_\nu^\mu\rangle_{\rm ren}$ with a reasonable accuracy. Resulting two-parameter stress-energy tensor may be further determined from the known horizon value of the one of the components of $\langle T_\nu^\mu\rangle_{\rm ren}$, say $\langle T_r^r\rangle_{\rm ren}$, and making use of the equation

$$\langle T_r^r(2M)\rangle_{\rm ren} = \frac{\kappa}{6}\frac{d}{dr}\langle\phi^2\rangle_{\rm ren} + \frac{16}{15}\pi^2 T_H^4, \tag{2}$$

where κ is the surface gravity and T_H stands for the black hole temperature. The remaining free parameter has been fixed by some sort of a best fit argument.

In this note we shall show how the results of Ref. [1] may be obtained and generalized in a more systematic and simpler way, without recourse to $\langle\phi^2\rangle_{\rm ren}$. Moreover, a great advantage of the adopted method is that it could be, contrary to the Hadamard regularization, easily extended to conformally invariant massless spinor and vector fields. Our present approach is based on the method adopted earlier in the different context and uses scaling properties of the one-loop renormalized effective action under the conformal transformations, or more precisely their consequences for appropriate transformations of the renormalized stress-energy tensors [11–15]. The notation is essentially that of Refs. [16, 17], to which the reader is referred for details. In this method, employing the Christensen–Fulling asymptotic analyses [9], one assumes that the tangential component of the stress-energy tensor in the optical companion to the Schwarzschild space has a simple polynomial form

$$\langle\tilde{T}_\theta^\theta\rangle_{\rm ren} = Tp(s)\sum_{n=0}^{N} a_n x^n, \tag{3}$$

with $a_0 = 1$, where $T = \pi^2 T_H^4/90$ and $p(s)$ is a numerical coefficient depending on the spin of the field. Here $p(0) = 1$, $p(1/2) = 7/4$, and $p(1) = 2$. We distinguish quantities evaluated in the conformal space by a tilde. Subsequently, solving the conservation equation in the conformal space for the radial component of the stress tensor and utilizing regularity conditions on the event horizon in the physical space one reduces the number of unknown coefficients a_i. Their number may be substantially reduced accepting one of the two thermal hypotheses, which state that the stress tensor in the Israel–Hartle–Hawking vacuum should have the form

$$\langle T_\nu^\mu \rangle_{\rm ren} = p(s)T\left[1+\sum_{n=1}^{m}(n+1)x^n\right](\delta_\nu^\mu - 4\delta_0^\mu\delta_\nu^0) + O(x^{m+1}), \tag{4}$$

with $m = 2$ for the weak thermal hypothesis and $m = 5$ for the strong one [18]. The weak thermal hypothesis is usually motivated by the observation that since the curvature is proportional to x^3 the curvature corrections to the stress-energy tensor are expected to be of that order. On the other hand, in the strong version of this ansatz one assumes that the curvature corrections are proportional to x^6, *i.e.* (*curvature*^{2}). In practice, it is helpful to invert the order of operations, and to analyse the consequences of the regularity conditions and thermal hypotheses imposed on the tangential component in the Schwarzschild geometry before solving the conservation equation.

The stress-energy tensor under the conformal transformation, $\tilde{g}^{\mu\nu} = \exp(-2\omega)g_{\mu\nu}$, transforms as

$$\langle T_\nu^\mu \rangle_{\rm ren} = \exp(-4\omega)\tilde{T}_\nu^\mu + a(s)A_\nu^\mu + b(s)B_\nu^\mu + c(s)C_\nu^\mu, \tag{5}$$

where

$$\begin{aligned} A^{\mu\nu} &= 8R^{\alpha\mu\nu\beta}\omega_{;\alpha\beta} - \frac{4}{3}\kappa^{;\mu\nu} + 2g^{\mu\nu}\left(2\omega^{;\alpha}\kappa_{;\alpha} + \kappa^2 + \frac{2}{3}\Box\kappa\right) - 8\kappa^{;(\mu}\omega^{;\nu)} \\ &\quad -8\omega^{;\mu}\omega^{;\nu}\kappa, \end{aligned} \tag{6}$$

$$\begin{aligned} B^{\mu\nu} &= 8R^{\alpha\mu\nu\beta}\omega_{;\alpha\beta} + 8R^{\alpha\mu\nu\beta}\omega_{;\alpha}\omega_{;\beta} - 8\omega^{;\mu\alpha}\omega_{;\alpha}{}^{;\nu} - 8\kappa^{;(\mu}\omega^{;\nu)} - 8\kappa\omega^{;\mu}\omega^{;\nu} \\ &\quad +4g^{\mu\nu}\left(\omega_{;\alpha\beta}\omega^{;\alpha\beta} + \kappa_{;\alpha}\omega^{;\alpha} + \frac{1}{2}\kappa^2\right), \end{aligned} \tag{7}$$

$$C^{\mu\nu} = g^{\mu\nu}(2\Box\kappa + 3\kappa^2 + 6\omega_{;\alpha}\kappa^{;\alpha}) - 12\kappa\omega^{;\mu}\omega^{;\nu} - 12\kappa^{;(\mu}\omega^{;\nu)} - 2\kappa^{;\mu\nu} \tag{8}$$

and $\kappa = \omega_{;\alpha}\omega^{;\alpha}$. The numerical coefficients as predicted by $\zeta-$function renormalization are given by

$$a = (2^9 45\pi^2)^{-1}\left[12h(0) + 18h\left(\frac{1}{2}\right) + 72h(1)\right], \tag{9}$$

$$b = (2^9 45\pi^2)^{-1}\left[-4h(0) - 11h\left(\frac{1}{2}\right) - 124h(1)\right], \tag{10}$$

and

$$c = -(2^9 45\pi^2)^{-1} 120 h(1), \tag{11}$$

where $h(s)$ denotes the number of helicity states for fields of spin s, while the dimensional renormalization gives

$$c(1) = 0. \tag{12}$$

Since the transformational rule for a general geometries is much more complicated we restricted ourselves to the Ricci-flat metrics. The stress tensor in the optical space naturally splits into two parts:

$$\langle \tilde{T}^{\mu}_{\nu} \rangle_{\rm ren} = \mathcal{T}^{\mu}_{\nu} + \frac{9c}{8M^4} x^6 \delta^{\mu}_{0} \delta^{0}_{\nu}, \tag{13}$$

where $\mathcal{T}^{\mu}_{\nu}$ is a conserved traceless tensor and the second term in the right hand side of (13) is constructed from the trace anomaly.

Now, we assume $N = 10$ and restrict ourselves to the scalar field. Hence, taking $\omega = 1/2 \ln(|g_{tt}|)$ and making use of the regularity condition $|\langle T^{\theta}_{\theta} \rangle_{\rm ren}| < \infty$, one obtains

$$a_9 = -\sum_{n=1}^{8} (10 - n) a_n, \tag{14}$$

and

$$a_{10} = \sum_{n=1}^{8} (9 - n) a_n. \tag{15}$$

Moreover, accepting the strong thermal hypothesis one concludes that the coefficients a_i for $i \leq 5$ should vanish.

Further, solving the conservation equation for $\langle \tilde{T}^{r}_{r} \rangle_{\rm ren}$ in the optical space, transforming the resulting tensor back to the physical space and employing the Christiensen–Fulling conditions, *i.e.* the conditions which guarantee the regularity of the stress-energy tensor in the local frames on the event horizon one has

$$\langle T^{\mu}_{\nu} \rangle_{\rm ren} = \langle T^{\mu}_{\nu} \rangle^{P} + \Delta^{\mu}_{\nu}, \tag{16}$$

where

$$\langle T^{\mu}_{\nu} \rangle^{P} = T \left\{ \frac{1 - x^6 (4 - 3x)^2}{(1 - x)^2} {\rm diag}[-3, 1, 1, 1]^{\mu}_{\nu} + 24 x^6 {\rm diag}[3, 1, 0, 0]^{\mu}_{\nu} \right\}, \tag{17}$$

is the stress tensor evaluated within the framework of the Page approximation [14], and the conserved and traceless tensor Δ_ν^μ is given by

$$\Delta_t^t = -3T\left[\frac{a_6}{2}x^6 + \frac{1}{366}(9a_6 - 64a_8)x^7 - \frac{11}{61}(3a_6 - a_8)x^8\right], \tag{18}$$

$$\Delta_r^r = -T\left[\frac{a_6}{2}x^6 - \frac{1}{122}(13a_6 - 16a_8)x^7 - \frac{9}{61}(3a_6 - a_8)x^8\right], \tag{19}$$

and

$$\Delta_\theta^\theta = \Delta_\phi^\phi = T\left[a_6x^6 - \frac{1}{61}(a_6 + 20a_8)x^7 - \frac{21}{61}(3a_6 - a_8)x^8\right]. \tag{20}$$

Further determination of the model requires two pieces of numerical data. Taking, for example, a horizon value of the tangential component of the stress tensor (the radial component and hence $\langle T_t^t(1)\rangle_{\text{ren}}$ may be easily obtained from the trace anomaly) one gets

$$a_6 = \frac{1}{3}(61\Theta - 732 - a_8), \tag{21}$$

where $\Theta = 1/T\langle T_\theta^\theta(1)\rangle_{\text{ren}}$. Finally, the remaining constant a_4 may be fixed by some sort of the best fit argument. Here however we proceed differently: we perform the least-quare fit to the available numerical data. There are two published sources of information: the numerical estimates carried out by Candelas and Howard [19, 20], and more recently by Anderson, Hiscock, and Samuel [21, 22]. Although the latter authors presented their results only graphically some of their results concerning the tangential component may by found in Ref. [18]. In the region $[2M, 5M]$ we adopted the Anderson, Hiscock, and Samuel data as presented in Ref. [18], whereas for $r > 5M$ we accept the results of numerical calculations carried out by Howard [20]. We discarded 3 points because the numerically determined trace exceeded the exact one by more than 1%. Performing the linear least-square analysis we obtained

$$a_6 = -74.230, \tag{22}$$

and

$$a_7 = -329.135. \tag{23}$$

It should be noted that the logarithmic term appearing in the solution of the conservation equation *i.e.* $\langle \tilde{T}_r^r\rangle_{\text{ren}}$ survives if the regularity condition of $\langle T_\theta^\theta\rangle_{\text{ren}}$ is imposed. However, since the coefficient in front of the logarithmic term involves a_1 and a_2 such a term by the thermal hypotheses vanish.

We complete the discussion of the scalar $N = 10$ case by comparing obtained approximation (16)–(20) to the model developed earlier. Introducing a new set of parameters α_4 and A_8 in place of a_6 and a_8

$$a_6 = \frac{17}{12}A_8 - 2\beta, \tag{24}$$

$$a_8 = \frac{469}{96}A_8 - 6\beta \tag{25}$$

one obtains precisely the results of Ref. [1]. Indeed, inserting (24), (25) into (17)–(20) after simple rearrangements one has

$$\begin{aligned} 8\pi^2\Delta_t^t &= \frac{M^2}{r^6}\left(\frac{\beta}{240} + \frac{17}{6}\alpha_4\right) - \frac{M^3}{r^7}\left(\frac{433}{27}\alpha_4 + \frac{\beta}{120} + \frac{4}{405}A_8\right) \\ &\quad + \frac{M^4}{r^8}\left(\frac{11}{540}A_8 + \frac{385}{18}\alpha_4\right), \end{aligned} \tag{26}$$

$$\begin{aligned} 8\pi^2\Delta_r^r &= \frac{M^2}{r^6}\left(\frac{\beta}{720} + \frac{17}{18}\alpha_4\right) - \frac{M^3}{r^7}\left(\frac{121}{27}\alpha_4 + \frac{\beta}{360} + \frac{A_8}{405}\right) \\ &\quad + \frac{M^4}{r^8}\left(\frac{35}{6}\alpha_4 + \frac{A_8}{180}\right), \end{aligned} \tag{27}$$

and

$$\begin{aligned} 8\pi^2\Delta_\theta^\theta &= -\frac{M^2}{r^6}\left(\frac{17}{9}\alpha_4 + \frac{\beta}{360}\right) + \frac{M^3}{r^7}\left(\frac{277}{27}\alpha_4 + \frac{\beta}{180} + \frac{A_8}{162}\right) \\ &\quad - \frac{M^4}{r^8}\left(\frac{245}{18}\alpha_4 + \frac{7}{540}A_8\right), \end{aligned} \tag{28}$$

where α_4 by (2) is given by

$$\alpha_4 = -\frac{1}{960}A_8. \tag{29}$$

For completeness we write out the general expression for the field fluctuation.

$$\langle\phi^2\rangle_{\text{ren}} = \frac{T_H^2}{12}(1 + x + x^2 + x^3 + \alpha_4 x^4 - \alpha_4 x^5). \tag{30}$$

which is necessary ingredient of the Hadamard regularization.

Now let us consider the consequences of the assumption that the curvature corrections to the stress-energy tensor are of order x^3. From the analyses carried out by Jensen and Ottewill [23] we know that analytical approximation of the stress-energy tensor of the vector field satisfies the energy condition in its weak form.

Let $\wp$ be the order of the polynomials that describe resulting stress-energy tensor. Taking $N = 10$ and repeating the calculations for fields of arbitrary spin one obtains $\wp = 8$ involving 5 unknown parameters. On the other hand $N = 8$ yields $\wp = 6$ polynomials with 3 undetermined constants. To simplify our discussion, in the further analyses we take $N = 8$. Since in the optical space the trace anomaly of the conformally coupled massless vector field does not vanish one has to take into account an analog of the Zannias term. After some algebra one finds for scalar, spinor, and vector fields

$$\begin{aligned}\langle T^{t}_{t}\rangle_{\rm ren} = &-3pT\left\{1+2x+3x^2+4x^3\right.\\&-x^4\left(\frac{97}{3}-\frac{176\alpha}{9p}-\frac{64\beta}{3p}+\frac{32\gamma}{3p}+\frac{11a_4}{9}+\frac{17a_5}{18}+\frac{a_6}{3}\right)\\&+x^5\left(-\frac{730}{9}+\frac{1232\alpha}{27p}+\frac{448\beta}{9p}-\frac{224\gamma}{9p}-\frac{74a_4}{27}-\frac{95a_5}{54}-\frac{7a_6}{9}\right)\\&\left.+x^6\left(-\frac{1225}{9}+\frac{1448\alpha}{27p}+\frac{520\beta}{9p}-\frac{224\gamma}{9p}-\frac{119a_4}{27}-\frac{70a_5}{27}-\frac{7a_6}{9}\right)\right\},\end{aligned} \tag{31}$$

$$\begin{aligned}\langle T^{r}_{r}\rangle_{\rm ren} = &\,pT\left\{1+2x+3x^2\right.\\&+x^3\left(\frac{460}{3}-\frac{704\alpha}{9p}-\frac{256\beta}{3p}+\frac{128\gamma}{3p}+\frac{56a_4}{9}+\frac{34a_5}{9}+\frac{4a_6}{3}\right)\\&+x^4\left(\frac{575}{3}-\frac{880\alpha}{9p}-\frac{320\beta}{3p}+\frac{160\gamma}{3p}+\frac{61a_4}{9}+\frac{85a_5}{18}+\frac{5a_6}{3}\right)\\&+x^5\left(\frac{578}{3}-\frac{880\alpha}{9p}-\frac{320\beta}{3p}+\frac{160\gamma}{3p}+\frac{58a_4}{9}+\frac{73a_5}{18}+\frac{5a_6}{3}\right)\\&\left.+x^6\left(175-\frac{248\alpha}{3p}-\frac{88\beta}{p}+\frac{32\gamma}{p}+\frac{17a_4}{3}+\frac{10a_5}{3}+a_6\right)\right\},\end{aligned} \tag{32}$$

and

$$\begin{aligned}\langle T^{\theta}_{\theta}\rangle_{\rm ren} = &\,pT\left\{1+2x+3x^2\right.\\&+x^3\left(-\frac{212}{3}+\frac{352\alpha}{9p}+\frac{128\beta}{3p}-\frac{64\gamma}{3p}-\frac{28a_4}{9}-\frac{17a_5}{9}-\frac{2a_6}{3}\right)\\&+x^4\left(-\frac{433}{3}+\frac{704\alpha}{9p}+\frac{256\beta}{3p}-\frac{128\gamma}{3p}-\frac{47a_4}{9}-\frac{34a_5}{9}-\frac{4a_6}{3}\right)\\&+x^5\left(-218+\frac{352\alpha}{3p}+\frac{128\beta}{p}-\frac{64\gamma}{p}-\frac{22a_4}{3}-\frac{14a_5}{3}-2a_6\right)\end{aligned}$$

$$+x^6\left(-\frac{875}{3}+\frac{1312\alpha}{9p}+\frac{464\beta}{3p}-\frac{160\gamma}{3p}-\frac{85a_4}{9}-\frac{50a_5}{9}-\frac{5a_6}{3}\right)\Bigg\}, \quad (33)$$

where $\alpha = a(64M^4T)^{-1}$, $\beta = b(64M^4T)^{-1}$, and $\gamma = c(64M^4T)^{-1}$. Surprisingly, the time component, $\langle T_\nu^\mu\rangle_{\rm ren}$ is of the form (4) with $m = 3$. It could be shown that similar approximate stress-energy tensor may be obtained from the formulae derived by Visser [18].

Inspection of (31)–(33) shows that for $N \le 8$ the result is described by polynomials of order 6, because of the geometrical terms that contribute to $\langle T_\nu^\mu\rangle_{\rm ren}$. Indeed, for example making use of the additional constraints (obtained from equations $a_7 = 0$ and $a_8 = 0$)

$$a_5 = -66+\frac{35\alpha}{p}+\frac{39\beta}{p}-\frac{39\gamma}{2p}-2a_4 \quad (34)$$

$$a_6 = 45-\frac{47\alpha}{2p}-\frac{51\beta}{2p}+\frac{51\gamma}{4p}+a_4 \quad (35)$$

one obtains a simple stress-energy tensor that depends on undetermined parameter a_4 [16–25]. The free parameter may be fixed from the known value of one component of the stress-energy tensor, say, $\langle T_\theta^\theta\rangle_{\rm ren}$ on the event horizon. Taking, for example, in the vector case,

$$\langle T_\theta^\theta\rangle_{\rm ren} = -\frac{1}{240\pi^2M^4} \quad (36)$$

results in the approximation that coincides with the analytic part of the Jensen and Ottewill evaluation of the stress-energy tensor [23, 25]. The interesting property of $N = 6$ model is that the difference $\langle T_t^t\rangle_{\rm ren} - \langle T_r^r\rangle_{\rm ren}$ does not depend on the parameter a_4 and consequently the entropy of quantized fields could be constructed [16].

To this end, we report the results of our $N = 11$ calculations which generalize Eqs. (16)–(20). The stress tensor of the scalar field is described by $\wp = 9$ polynomials with 3 free parameters and has a general form (16) with

$$\begin{aligned}\frac{1}{T}\Delta_\theta^\theta &= \frac{1}{17}\left(5844-487\Theta-17a_9-27a_{10}\right)x^6\\ &+ \frac{1}{17}\left(252\Theta-3024+17a_9+22a_{10}\right)x^7\\ &+ \frac{3}{17}\left[56\Theta+3\left(a_{10}-224\right)\right]x^8+\frac{4}{17}\left(21\Theta-252-a_{10}\right)x^9, \quad (37)\end{aligned}$$

$$\begin{aligned}
\frac{1}{T}\Delta_r^r &= \frac{1}{34}\left(487\Theta - 5844 + 17a_9 + 27a_{10}\right)x^6 \\
&+ \frac{1}{34}\left(3588 - 299\Theta - 17a_9 - 23a_{10}\right)x^7 \\
&+ \frac{1}{34}\left(1908 - 159\Theta - 7a_{10}\right)x^8 + \frac{3}{34}\left(252 - 21\Theta + a_{10}\right)x^9, \quad (38)
\end{aligned}$$

and

$$\begin{aligned}
&- \frac{1}{3T}\Delta_t^t = \frac{1}{34}\left(5844 - 487\Theta - 17a_9 - 27a_{10}\right)x^6 \\
&+ \frac{1}{102}\left(709\Theta - 8508 + 51a_9 + 65a_{10}\right)x^7 \\
&+ \frac{1}{102}\left(513\Theta - 6156 + 29a_{10}\right)x^8 + \frac{13}{102}\left(21\Theta - 252 - a_{10}\right)x^9. \quad (39)
\end{aligned}$$

Although generalization of the method to $N > 11$ is obvious it seems that such models are of little use.

Final determination of the model is achieved by performing the least square fit of the $\langle T_\theta^\theta \rangle_{\rm ren}$ to the numerical data discussed earlier. Performing the linear least-square fit we find

$$a_9 = 123.347\,, \quad (40)$$

$$a_{10} = -1.071\,, \quad (41)$$

and

$$\Theta = 10.245. \quad (42)$$

Note that one of the parameters is the horizon value of the tangential component of the stress-energy tensor. The maximal deviation of obtained $\langle T_\theta^\theta \rangle_{\rm ren}$ does not exceed 0.3%. It is belived that the Anderson–Hiscock–Samuel data is accurate to three significant digit near the event horizon.

The fit may be considered as a good one and therefore constructed $\langle T_\nu^\mu \rangle_{\rm ren}$ may be used as a source term of the semi-classical Einstein field equations. Such calculations in the black hole external region with the stress-energy tensor presented in [1] have been carried out in Ref. [26]. It should be noted however that maximal deviation of the stress-energy tensor of the quantized conformally coupled massless scalar field constructed by means of the Hadamard regularization is 2.5% for $\langle T_\theta^\theta \rangle_{\rm ren}$ and 7.3% for $\langle T_t^t \rangle_{\rm ren}$. In spite of that one expects that outside the event horizon there are no substantial differences between the results of the first order back-reaction calculations obtained with Δ_ν^μ given by (37)–(39) and these obtained with the improved stress-energy tensor of Ref. [1]. On the other hand however, there are importand differences in the region inside the event horizon. Since the linearized semi-classical Einstein field equations have been solved inside the event horizon with the source term given by the Page approximation [27] it would be interesting to reexamine the problem with the aid of (16) with (37)–(39). We intend to return to this group of problems in a separate publication.

REFERENCES

[1] J. Matyjasek, *Phys. Rev.* **D 53**, 794 (1996); IF UMCS Report 1995, 33.
[2] M.R. Brown, *J. Math. Phys.* **25**, 136 (1984).
[3] S. Tadaki, *Prog. Theor. Phys.* **77**, 671 (1987).
[4] S. Tadaki, *Prog. Theor. Phys.* **81**, 891 (1989).
[5] J. Matyjasek, *Acta. Phys. Pol.* **B27**, 2005 (1996).
[6] D. Bernard, A. Folacci, *Phys. Rev.* **D34**, 2286 (1986).
[7] M. A. Castagnino, D. Harari, *Ann. Phys.* (NY) **152**, 85 (1984).
[8] K. Kirsten, *Classical Quantum Gravity* **6**, 779 (1989).
[9] S.M. Christensen, A.S. Fulling, *Phys. Rev.* **D15**, 2088 (1977).
[10] P. Candelas, *Phys. Rev.* **D21**, 2185 (1980).
[11] M.R. Brown, A.C. Ottewill, *Phys. Rev.* **D31**, 2514 (1985).
[12] M.R. Brown, A.C. Ottewill, D.N. Page, *Phys. Rev.* **D33**, 2840 (1986).
[13] J.S. Dowker, *Phys. Rev.* **D33**, 3150 (1986).
[14] D.N. Page, *Phys. Rev.* **D25**, 1499 (1982).
[15] T. Zannias, *Phys. Rev.* **30**, 1161 (1984).
[16] J. Matyjasek, *Phys. Rev.* **D57**, 7615 (1998).
[17] J. Matyjasek, *Phys. Rev.* **D59**, in press. (gr-qc/9808019).
[18] M. Visser, *Phys. Rev.* **D56**, 936 (1997).
[19] K.W. Howard, P. Candelas, *Phys. Rev. Lett.* **53**, 403 (1984).
[20] K.W. Howard, *Phys. Rev.* **D30**, 2532 (1984).
[21] P.R. Anderson, W.A. Hiscock, D.A. Samuel, *Phys. Rev. Lett.* **70**, 1739 (1993).
[22] P.R. Anderson, W.A. Hiscock, D.A. Samuel, *Phys. Rev.* **D51**, 4337 (1995).
[23] B.P. Jensen, A.C. Ottewill, *Phys. Rev.* **D39** 1130 (1989).
[24] J. Matyjasek, *Classical Quantum Gravity* **14**, L15 (1997).
[25] J. Matyjasek, *Phys. Rev.* **D55**, 809 (1997).
[26] J. Matyjasek, *Acta. Phys. Pol.* **B29**, 529 (1998).
[27] W.A. Hiscock, S.L. Larson, P.R. Anderson, *Phys. Rev.* **D56**, 3571 (1997).