ON THE CONSISTENCY OF LEAR AND FENICE EXPERIMETS IN THE SECTOR OF $\boldsymbol{P\bar{P}}$ INTERACTION NEAR THE THRESHOLD

M. Majewski

Department of Theoretical Physics, University of Lodz
Pomorska 149/153, 90-236 Lodz, Poland
e-mail: mimajew@mvii.uni.lodz.pl

G.V. Meshcheryakov and V.A. Meshcheryakov

Joint Institute for Nuclear Research
Dubna 141980, Moscow Region, Russia
e-mail: mva@thsun1.jinr.dubna.su

(Received November 4, 1998; revised version received March 22, 1999)

Some experiments at LEAR showed unusual behavior of the $p\bar{p}$ interaction near the threshold. The experiments on $p\bar{p}$ forward scattering detected zeros and a big variation of ρ and at the same time a smooth rising of $\sigma_{\rm tot}$ with lowering energy. Many models have difficulties in explaining this fact. In the PS-170 experiment with a good statistical accuracy, the unexpected behavior of the proton electromagnetic form factor was found. All these experiments can be considered as an indication for the existence of a low-lying $p\bar{p}$ bound state 'baryonium'. This statement coincides with that made for interpreting of the energy dependence of the total cross-section of the reaction $e^+e^- \to$ *hadrons* in FENICE. There is a model (based on analyticity) which explains afore-mentioned experiments and the fact that the 'baryonium' is not seen in the OBELIX $p\bar{p}$ annihilation cross-section. Thus, LEAR and FENICE experiments are consistent near the $p\bar{p}$ threshold and testify to the existence of 'baryonium'.

PACS numbers: 13.40.Fn, 14.20.–c

1. The database and previous knowledge

The experiment at LEAR which is part of the CERN antiproton complex gives a rich information about low-energy antiproton physics. The experiments (PS-172, PS-173) [1,2] on $p\bar{p}$ scattering provide us with the data on $d\sigma/d\Omega$, $\sigma_{\rm tot}$ and ρ. To search for a bound state, cross section measurements are the most straightforward experiments to perform. The analysis of $d\sigma/d\Omega$ gives an indication of bound states near the $p\bar{p}$ threshold [3]. Some of them are consistent with strong-interaction shifts and the width of protonium [4]. A resonance (a bound state with a mass larger than $p\bar{p}$ threshold) may be seen as a bump in $\sigma_{\rm tot}$. But the measurements of the $p\bar{p}$ total cross section above 180 MeV/c point to its smoothly varying behavior [2]. The most remarkable result in $p\bar{p}$ elastic scattering has appeared in the data on the real-to-imaginary ratio of the forward scattering amplitude ρ measured at LEAR down to 180 MeV/c [2]. In the range $350 < p_l < 700$ MeV/c, the behavior of ρ can be explained by a pole below the threshold in the dispersion relation analysis [5]. The LEAR measurements [2,6] below 350 MeV/c indicate that the ρ is turning upward again. The reason for this unusual behavior is not yet clear. It might be caused by a $p\bar{p}$ bound state [7] but not by an $n\bar{n}$ threshold [8]. The experimental ρ was always determined from the elastic differential cross section in the Coulomb-nuclear interference region. The method used to extract ρ in this way has sometimes been criticized [9]. However, at high energies the method is consistent with the predictions of dispersion relations. So, the value of ρ from Refs [2,6] will be considered below as reliable.

The results of experiment PS-170 on the study of annihilation $p\overline{p} \to e\overline{e}$ at low energies [10] have no adequate interpretation till the present day. They resulted in an unexpected behavior of the proton electromagnetic form factor near the $p\overline{p}$-threshold in the time-like region, where $s < 4.2$ GeV^2. The data on $\mid G \mid = \mid G_{m,p} \mid = \mid G_{e,p} \mid$ point to a large negative derivative at the threshold that rapidly grows to zero or even to positive values at $s \sim 4$ GeV^2. The magnitude of the derivative at the threshold is determined by the threshold value $\mid G \mid = 0.53 \pm 0.05$. One of the early values, $\mid G \mid = 0.51 \pm 0.08$, does not contradict the results of Ref. [10]. This value was obtained [11] from the ratio of frequencies of $p\overline{p}$ annihilations at rest into $e\bar{e}$ and $\pi^+\pi^-$ pairs in liquid hydrogen. The determination of $\mid G \mid$ at the threshold is a complicated problem since one should simultaneously consider the Coulomb and strong interactions in the $p\overline{p}$-system, and the problem requires some approximations. These approximations have been analyzed in Ref. [12] where a new scheme is proposed for the determination of $\mid G \mid$. This scheme gave the value $\mid G \mid = 1.1$ that confirms the results of Ref. [10]. Quite recently, a new attempt has been undertaken to determine $\mid G \mid$ at threshold [13]. Combining the data on widths of $p\overline{p}$-atoms obtained in the synchrotron trap with the results on the low-energy annihilation cross section in a $p\overline{p}$-system, the authors concluded that $\mid G \mid= 0.39$ or even $\mid G \mid= 0.3$. This allows us to infer that there is no abrupt change of $\mid G \mid$ at the threshold. Thus, the authors of [12,13] propose a new view on the method of calculating $\mid G \mid$ at the threshold from experimental data.

Let us now proceed to studies on the interpretation of results of the experiment [10]. In Ref. [14] an attempt is made to consider the interaction in a final state. The basic result is the formula $G = c\mathrm{e}^{i\delta}$, where c is a slowly varying function of q^2 at the threshold (q is the momentum in c.m.s. of the $p\overline{p}$-system) and δ is the $N\overline{N}$ scattering phase. Since the phase δ is complex at the threshold, we have

$$\mid G \mid=\mid c \mid \cdot \mid 1 - q \cdot \mathrm{Im}\, a \mid \ , \tag{1}$$

where a is the complex scattering length. Owing to $\mid G \mid$ being linear in q, the quantity $d \mid G \mid /ds$ is infinite at the threshold. Our analysis of the first four points from [10] with respect to the χ^2-criterion gives the values: $\mid c \mid= 0.53 \pm 0.02$, $\mathrm{Im}\, a = 0.62 \pm 0.08$ fm, $\chi^2 = 0.07$. The authors of [14] employ the values:$\mid c \mid= 0.52, \mathrm{Im}\, a \cong 0.8$ fm for the same points and do not explain the origin of them; they identify $\mathrm{Im}\, a$ with the quantity $\mathrm{Im}\, a(^3S_1) \cong 0.8\,\mathrm{fm}$ computed from the experiment [15]. The description is qualitative since $\chi^2 \sim 10$ according to our estimation. The authors of [16] assert that a good description of all the known data on nucleon electromagnetic form factors, including the data of [10], is obtained on the basis of a new formulation of the vector-dominance model (VDM) and its subsequent unitarization. In what follows, we will use different models of that type, therefore we consider them in detail. They are based on the expressions for the Dirac and Pauli nucleon form factors in VDM:

$$F_N(s) = \sum_v \frac{f_{v,NN}}{f_v} \frac{m_v^2}{m_v^2 - s}, \tag{2}$$

where m_v is the mass of a vector meson, $f_{v,NN}$ is the vector meson–nucleon coupling constant, f_v is the universal constant in the so-called identity of current and field. Imposing constraints on the parameters of formula (2), one can easily find the experimental value of $F_N(s = 0)$ and the asymptotics following from the quark counting rules [17] that coincides with the QCD-asymptotics within the logarithmic accuracy. Then, the model is unitarized with the help of a uniformizing variable. As a result, vector mesons acquire widths, and the form factors can be calculated for all s. So, all the experimental data can be described both in the space-like ($s < 0$) and time-like ($s > 0$) regions. Satisfactory description of more than three hundred values of $\mid F_N \mid$ requires about ten free parameters in formula (2). Besides, this approach allows a model-dependent reproduction of the form of $\mathrm{Im}\, F_N$ and $\mathrm{Re}\, F_N$ in the whole time-like region. Results of the analysis by this scheme are presented in Ref. [16]. The data of the experiment PS-170 are explained by including the third radial excitation $\rho(770)$ with the mass $\sqrt{s} = 2.15$ GeV into formula (2) and are plotted in Fig. 1.

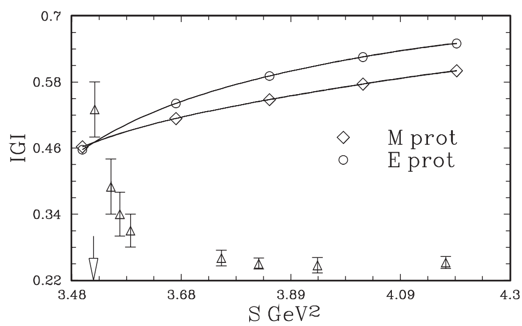

Fig. 1. The curves from Fig. 3^a of Ref. [16] on a larger scale, $\downarrow$–$p\bar{p}$ threshold. The quality of the fit PS-170 data [10] is very poor.

2. Formulation of the analytic model

It is easy to see that the nucleon form factor, according to formula (2), has the following imaginary part

$$\mathrm{Im}\, F_N = \pi \sum_v m_v^2 \frac{f_{v,NN}}{f_\rho} \delta(s - m_v^2). \tag{3}$$

Formula (3) is an approximate expression obtained from the unitarity condition which allows one to reproduce equation (2) with the use of dispersion relations for F_N. We write the starting expression for the unitarity condition as follows:

$$\mathrm{Im}\, \langle o \mid j_\mu \mid N\bar{N} \rangle = \sum_n \langle o \mid j_\mu \mid n \rangle \langle n \mid T^+ | N\bar{N} \rangle, \tag{4}$$

where j_μ is the electromagnetic current of a nucleon N, and $|n\rangle$ is a complete set of admissible intermediate states. In our case, it is of the form

$$|n\rangle = |2\pi\rangle,\ |3\pi\rangle, ..., |K\bar{K}\rangle,\ |N, \bar{N}\rangle, ... \,. \tag{5}$$

Frazer and Fulco [18] were the first who computed the contribution of the two-pion state and predicted the ρ-meson on the basis of data on F_N. Choosing different terms in sequence (5), one can obtain many models of the type (2). Earlier, the model of Ref. [19] was used in Ref. [20] and the contribution of an $N\overline{N}$ intermediate state was calculated. This contribution is important for two reasons. First, it results in a new branch point in formula (2), the threshold of the reaction $N\overline{N}$ situated on the lower edge of the energy region studied in Ref. [10]. Second, bound states or resonances in an $N\overline{N}$-system near the threshold influence the behavior of $F_N(s)$ in the nonobservable region below the $N\overline{N}$-threshold and in the observable region above the $N\overline{N}$-threshold investigated in Ref. [10]. It is clear that the state $|N\overline{N}\rangle$ appears on the background of a sum of other states of series (5), and the result is model-dependent. Therefore, it is important to study the degree of that dependence within a model differing from the one used in [20] for $F_N(s)$ as a background for the state $|N\overline{N}\rangle$.

We will take the model of Ref. [21] formulated in terms of the Sachs form factors G measured experimentally. The model is based on the formulae

$$\begin{aligned} G_{m,p}(s) &= \sum_{k=1}^{3} \frac{\varepsilon_k(s)}{s - a_k - \gamma_k\sqrt{s_k - s}}\,, \\ G_{e,p}(s) &= \sum_{k=1}^{3} \frac{\beta_k(s)}{s - a_k - \gamma_k\sqrt{s_k - s}}\,, \end{aligned} \tag{6}$$

where

$$\begin{aligned} \varepsilon_k(s) &= \frac{\varepsilon_k^1 + \varepsilon_k^0 s}{s - a_k - \gamma_k\sqrt{s_k - s}}\,, \\ \beta_k(s) &= \frac{\beta_k^1 + \beta_k^0 s}{s - a_k - \gamma_k\sqrt{s_k - s}}\,. \end{aligned} \tag{7}$$

The energy behavior of electromagnetic form factors is explained with the use of three resonances:ρ, ω, φ specified by indices $k = 1, 2, 3$ in formula (6). The masses, widths, and thresholds a_k, γ_k, s_k are taken from experiment. The model parameters are the coupling constants

$$\begin{aligned} (\beta_1^1 + \varepsilon_1^0 s) f_1(s) &= g_{\gamma\rho}(s) g_{\rho NN}(s)\,, \\ (\beta_2^1 + \varepsilon_2^0 s) f_2(s) &= g_{\gamma\omega}(s) g_{\omega NN}(s), \\ (\beta_3^1 + \varepsilon_3^0 s) f_3(s) &= g_{\gamma\phi}(s) g_{\phi NN}(s)\,, \end{aligned}$$

where

$$f_k(s) = \frac{1}{s - a_k - \gamma_k\sqrt{s_k - s}}\,. \tag{8}$$

This unusual form of the coupling constants is chosen by analogy with the index of refraction in optics. They are not only energy-dependent, but also contain a complex component when $s > s_k$. The coupling constants as chosen so as to be consistent with the known experimental data $F_1^p(0) = 1, F_2^p = 1.79, F_1^n = 0, F_2^n = -1.91$ and to ensure the transition from the Pauli, Dirac to the Sachs form factors. Details are given on p.110 of Ref. [21]. Thus, only two free parameters are to be defined from the conditions required when $s \to \infty$. The SU(3) symmetry should hold in the asymptotic region identically. This condition is the weakest since it can be changed by including new vector mesons into consideration. Therefore, the parameters ε_2^0 and ε_3^0 are determined according to the χ^2 criterion on the basis of experimental points $|G_p|$ cited in Refs. [22]. An interesting feature of the model [21] is that it correctly describes the ratio $|G_p|/|G_n|$ above the $p\bar{p}$-threshold. More exactly, it reproduces the experimental value $|G_n(s = 4\ \mathrm{GeV}^2)| = 0.42 \pm 0.06$ (see [23]). The model result for $|G_p|$ is drawn in Fig. 2, and $\varepsilon_2^0 = -3.41, \varepsilon_3^0 = 3.23, \chi^2 = 10.1$.

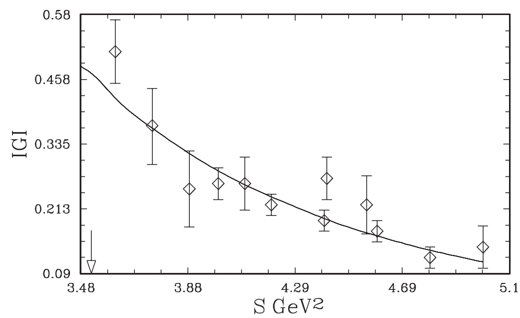

Fig. 2. Our fit to the old data of Ref. [22] by G_w, $\downarrow$–$p\bar{p}$ threshold.

The influence of the $|N\bar{N}\rangle$ contribution to the unitarity condition (4) on $|G|$ is computed in the same way as in Refs. [20, 24]. We construct an analytic model for the forward elastic scattering amplitude T in terms of the uniformizing variable

$$z = \sqrt{\frac{4(s-\alpha)}{s(4-\alpha)}} - \sqrt{\frac{\alpha(s-4)}{s(4-\alpha)}}, \tag{9}$$

where s is the conventional Mandelstam variable equal to the square of the total energy of a $p\bar{p}$-system in the c.m.s. in units M_p. The variable z contains branch points at $s = 0$ and 4 corresponding to the reaction threshold of elastic pp and $p\bar{p}$-scattering and an effective branch point at $s = \alpha$ corresponding to the unphysical region for the elastic $p\bar{p}$-scattering. The threshold of process $p\bar{p} \to p\bar{p}$ is mapped into points $z = \pm 1$ on the z-plane, whereas the infinite s-plane point into points $\pm z_1$, $\pm 1/z_1$, where $z_1 = \sqrt{\frac{2-\sqrt{\alpha}}{2+\sqrt{\alpha}}}$.

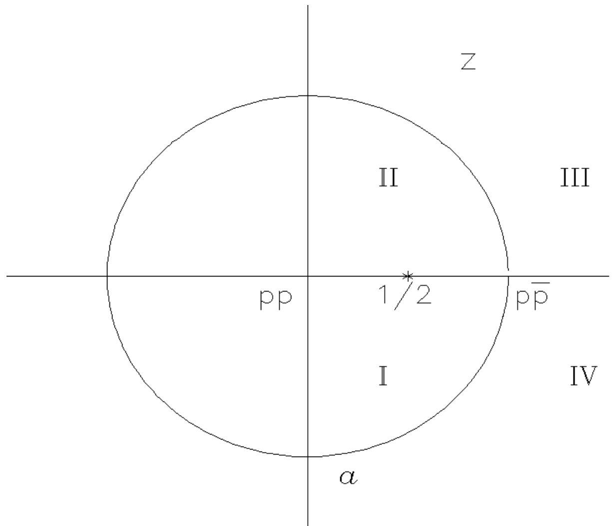

Fig. 3. Disposition of four sheets of the Riemann surface of the function z(s) for $\alpha = 1.44$. The threshold $p\bar{p}$ is mapped into points $z = \pm 1$.

Disposition of all the four sheets of the Riemann surface of the function $z(s)$ is drawn in Fig. 3 for $\alpha = 1.44$. In Ref. [24] it is shown that the experimental data on $\rho = \mathrm{Re}\,T\,/\,\mathrm{Im}T$ and $\sigma_{\rm tot}$ can be well described provided that the $p\bar{p}$-system possesses a quasinuclear bound state with the binding energy $E = (1.88{\pm}0.05)$ MeV and width $\Gamma = (1.6{\pm}0.1)$ MeV. The scattering amplitude was taken in the form

$$T = T_b + \frac{c_\rho}{z-(z_\rho)_1} - \frac{c_\rho}{z-(z_\rho)_2}, \tag{10}$$

where $T_b(s)$ is a polynomial in z, $(z_\rho)_{1,2} = 1 \mp \gamma \pm i\delta$ and $\alpha = 1.44, 10^2\gamma = -0.54 \pm 0.02, 10^2\delta = 2.6 \pm 0.08$. The pole terms represent the contribution of the quasinuclear state; whereas the polynomial, the contribution of a nonresonance background of S,P and D-waves. Special attention was paid to the threshold value of the T amplitude which is complex [24]. The amplitude (10) well describes the experimental data up to 4.4 GeV^2 in terms of the variable s. It is valid in the vicinity of $z = 1$ and has two poles in distinction to the usual quantum-mechanical amplitude. The two poles in the variable z instead of one pole in the variable q appear in the scattering amplitude T because of choosing z as uniformizing variable in T .

Another important feature of formula (10) is the form of the pole term contribution to Im T and ReT. The bound state (pole) contribution to $\mathrm{Im}\,T(p_l = 80\ \mathrm{MeV}/c)$ is about 10% of the total value of $\mathrm{Im}\,T_{p\bar{p}}$. On the other hand, the bound state contribution to ReT is larger than the background one and ensures a correct value of ρ (see Fig. 4, 5). Near the $p\bar{p}$-threshold the

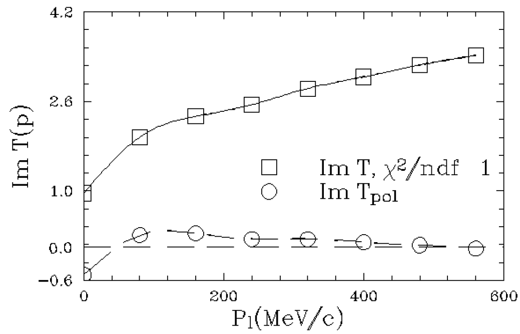

Fig. 4. The pole contribution to the $\operatorname{Im} T$.

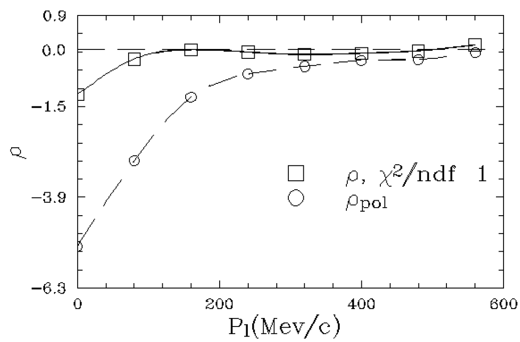

Fig. 5. The pole contribution to the ρ.

pole contribution to the unitarity condition (4) becomes dominant, and thus, we will restrict ourselves to the pole approximation. Quantum numbers of this state are unknown. A detailed scheme of the calculation corresponds to the scheme by Frazer and Fulco [18] for the contribution of different partial waves to $\operatorname{Im} F_N$. In our case, it gives that these states are either ${}^3S_1$ or ${}^3D_1$. Then, the unitarity condition (4) is reduced to the Riemann boundary-value problem [25] that can be solved (see Appendix). Inside the ring containing the unit circle (Fig. 3) the solution is of the form

$$G_{\text{pol}} = \frac{c(z)}{\prod_{i=1}^{2}(z-(z_\rho)_i)(z+(z_\rho^*)_i)}, \tag{11}$$

where $c(z)$ is an entire function within which the solution is determined. Setting $c(z) = c_1(z)\cdot(z^2-z_1^2)(1-z^2z_1^2)/(1-z_1^2)^2$, we can ensure the asymptotic behavior of G_{pol} at infinity. Taking advantage of $c_1(z)$ being arbitrary, we assume the solution to be of the form

$$
\begin{aligned}
&G_{\rm pol}(z)\frac{(1-z_1^2)^2}{(z^2-z_1^2)(z^2z_1^2-1)}\\
&=A_1\Big\{\Big(\frac{1}{z-(z_\rho)_1}-\frac{1}{z-(z_\rho)_2}\Big)-\Big(\frac{1}{z+(z^*_\rho)_1}-\frac{1}{z+(z^*_\rho)_2}\Big)\Big\}\\
&+A_2\Big\{\Big(\frac{1}{z-(z_\rho)_1}+\frac{1}{z-(z_\rho)_2}\Big)-\Big(\frac{1}{z+(z^*_\rho)_1}+\frac{1}{z+(z+(z^*_\rho)_2}\Big)\Big\}\,.
\end{aligned}
\tag{12}
$$

Around the $p\bar{p}$-threshold the equalities $\mid G_{e,p}\mid=\mid G_{m,p}\mid=\mid G\mid$ are valid and, under this assumption, the experiment in Ref. [10] was analyzed. Therefore, we put

$$G_{e,p}+G_{m,p}=2G_w\,, \tag{13}$$

where the functions $G_{e(m),p}$ are given by formulae (6). Considering the contribution of the $|N\bar{N}\rangle$-state to the unitarity condition (4), we obtain for the proton electromagnetic form factor G:

$$G=G_w+G_{\rm pol}\,. \tag{14}$$

We shall assume the position of poles to be known from Ref. [24]; then, the form factor G depends on two free parameters A_1, A_2. The behavior of $G_{\rm pol}$ on the upper edge of the cut $[\alpha,\infty)$ around the $N\bar{N}$-threshold is determined by the poles $(z_\rho)_1$ and $(z_\rho)_2$; whereas on the lower edge, by the poles $(z^*_\rho)_1$ and $(z^*_\rho)_2$. If we calculate the common denominator for the contributions of the poles $(z_\rho)_1$ and $(z_\rho)_2$ in formula (12), the energy factor $(z-1)$ will arise in front of the parameter A_2; whereas a constant, in front of the parameter A_1. This allows us to draw analogy between the parameter A_1 and $\varepsilon^1_k,\beta^1_k$ as well as between A_2 and $\varepsilon^0_k,\beta^0_k$ in formula (7). The expression for $G_{\rm pol}$, Eq. (11) follows from the unitarity condition and analytic properties of the proton form factor and $N\bar{N}$-scattering amplitude. Therefore, formulae (6) are substantiated, irrespective of the above-mentioned analogy with optics. The result of the analysis (Fig. 6) with the use of Eq. (12) is presented in Table I and $\alpha=0.23\pm0.04$, $\varepsilon^o_2=2.97\pm0.03$, $\varepsilon^o_3=3.23$, $10^2A_1=0$, $10^2A_2=1.2\pm0.01$.

3. Discussion of results

The parameters A_1 and A_2 representing the coupling constants of a quasinuclear bound state are sensitive to the background shape in formula (14) as follows from comparison of this fit and the fit of Ref. [20] ($A_1\neq0$ in

TABLE I

The predicted values of $\mid G \mid$ with the corresponding values of partial contributions to χ^2.

S GeV^2	G_{exp}	$\lvert G\rvert$	χ_i^2
3.523	0.53 ± 0.05	0.63	3.9
3.553	0.39 ± 0.05	0.35	0.63
3.57	0.34 ± 0.04	0.32	0.26
3.59	0.31 ± 0.03	0.3	0.15
3.76	0.26 ± 0.014	0.27	0.66
3.83	0.25 ± 0.01	0.27	1.9
3.94	0.247 ± 0.014	0.254	0.23
4.18	0.252 ± 0.011	0.221	8.1

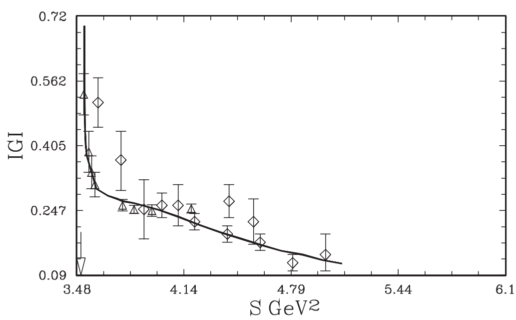

Fig. 6. Our fit to PS-170 data ($\triangle$) with account of the pole contribution (Eq. (12)), $\downarrow$–$p\bar{p}$ threshold,($\Diamond$)-data of Ref. [22].

Ref. [20]). The magnitude of the background is determined by the parameters ε_2^0 and ε_3^0 and is a slowly varying function in the s interval under investigation. The parameters $A_1, A_2 and \alpha$ determine the rapid change of G in formula (14). Separating the parameters into these two groups, we can obtain their statistically reasonable values (Table I). The analysis would be considerably simplified if the experimental values of $s > 4M_p^2$ were known for $\mathrm{Im}\, G$ and $\mathrm{Re}\, G$. Their determination requires polarization experiments whose theoretical study is carried out in Ref. [26].

Recently, two independent experiments have given new information on the $p\bar{p}$ interaction at low energies. The value of the $p\bar{p}$ annihilation total cross section down to an momentum of 43 MeV/c was measured by the OBELIX experiment [27] at LEAR and no resonant behavior of the cross section was found. The existence of some structure in the $e\bar{e} \to hadrons$ cross section near the $p\bar{p}$ threshold was indicated in FENICE at ADONE [28]. A combined analysis of these data and the data on the proton form factor with help of Breit–Wigner formulas provides a good candidate for the quasinucler bound state with the mass $M = 1.85 \pm 0.01$ GeV^2 and width $\Gamma = 40 \pm 10$ MeV. This candidate does not contradict our candidate from Ref. [20]. Then the question arises why this candidate is not seen in the OBELIX experiment on the $p\bar{p}$ annihilation cross section at very low energy. The first reason is the mass of 'baryonium' which is less then $2M_p$. The second is based on our analytic model. For a better understanding of the nature of the model, we present separate graphs for contributions of the pole term and background to $\sigma_{\rm tot}$ and ρ. In this model, $(\sigma_{\rm tot})_{\rm pole} < \sigma_{\rm tot}$ at low energies (Fig. 4)but $\sigma_{\rm ann} < \sigma_{\rm tot}$. As we can observe from these inequalities and from Fig. 4, the pole contribution to $\sigma_{\rm ann}$ is not larger than 10% at low energies. It is just this fact that accounts for the 'baryonium' being nonobservable in the OBELIX data. At the same time, in the experiment FENICE, the cross section of process $e^+e^- \to hadrons$ depends not only on $\mathrm{Im}\, T$ but also on $\mathrm{Re}\, T$(see Fig. 5) where the pole contribution is large. That is why the 'baryonium' unseen in the OBELIX data is seen in the FENICE data. Thus, the results of both these experiments are consistent.

Finally, we mention a pure theoretical result: the method of derivation of formula (11) for describing a quasinuclear state can be applied to any vector meson in formula (2). Therefore any vector meson will be characterized not only by the mass and width but also by two parameters like coupling constants. In other words, the effective coupling constants will be energy-dependent, which is assumed in Ref. [21] and is reflected in formulae (7).

Appendix

The unitarity condition (4) is an exact equation if use is made of the complete system of admissible intermediate states (5), otherwise, it is an approximate equation depending on the assumptions made. Let us take it in the form

$$\mathrm{Im}\, F = F(e^{i\delta} \sin\delta)^* + \bar{g},$$

where δ is the $N\bar{N}$-scattering phase with quantum numbers of the pole state unknown yet; $\overline{g}$ is the contribution of all other processes in the same channel. We reduce it to the form

$$F = e^{2i\delta} F^* + 2ig. \tag{A.1}$$

Relation (A.1) is valid for $\mathrm{Im}\, s = 0$ and $\mathrm{Re}\, s \geq 4M_p^2$. The function F is analytic in the complex plane s with the cut $[4M_p^2, \infty)$ outside of which $F^*(s) = F(s^*)$. This relation represents a linear inhomogeneous Riemann boundary-value problem for the function F. If $e^{2i\delta}$ has a pole near the cut, then in its vicinity we can consider the homogeneous problem

$$F = e^{2i\delta} F^*.$$

As it is known [25], the main difficulty in solving it consists in constructing a function analytic in the plane s and coinciding on the cut with $e^{2i\delta}$. However, if $e^{2i\delta}$ is taken in the form admitting the analytic continuation onto complex s, the problem is reduced to the solution of a functional equation for F in the uniformizing variable z. We will represent $e^{2i\delta}$ in the form

$$e^{2i\delta} = \prod_j \frac{(z - z_j^*)(z + z_j)}{(z - z_j)(z + z_j^*)}.$$

The function $e^{2i\delta}$ is real on the imaginary axis z, *i.e.* on the real axis s when $s < \alpha$. Equation (A.1) is valid on the cut $[4M_p^2, \infty)$ that transforms into the real axis $z = x + iy$, and $F(s) \to F(x)$, $F^*(s) \to F(-x)$

$$F(x) = \frac{(x - z_j^*)(z + z_j)}{(x - z_j)(z + z_j^*)} F(-x),$$

where we took only one pole, without loss of generality. The latter functional equation for $F(x)$ can be written as follows

$$F(x)(x - z_j)(x + z_j^*) = G(x),$$

$$G(x) = G(-x)$$

and, thus, $F(z)$ is of the form

$$F(z) = \frac{G(z)}{\prod_j (z - z_j)(z + z_j^*)},$$

where $G(z)$ is an entire even function of the variable z. The inhomogeneous boundary-value problem (A.1) can be solved in a similar manner and formula (11) can be proved.

REFERENCES

[1] R.A. Kunne, C.I. Beard, R. Birsa, K. Bos, F. Bradamante, D.V. Bugg, A. S. Clough, S. Dallatorre-Colautti, S. Delgi-Agosti, J.I. Edgington, J. R. Holl, E. Heer, R. Hess, J.C. Kluyver, C. Lechanoine-Lelae, L. Linssen, A. Martin, T.O. Ninikoski, Y. Onel, A. Penzo, D. Rapin, J.M. Rieubland, A. Rijllart, P. Schiavon, R.L. Shypit, F. Tessarotto, A. Villari, P. Wells, *Phys. Lett.* **B206**, 557 (1998); *Nucl. Phys.* **B323**, 1 (1989).

[2] W. Brückner, H. Döbbeling, F. Güttner, D. von Harrach, H. Kneis, S. Majewski, M. Nomachi, S. Paul, V. Povh, R.D. Ransome, T.A. Shibata, M. Treichel, Th. Walcher, *Phys. Lett.* **B158**, 180 (1985); *Phys. Lett.* **B166**,113 (1986).

[3] V.K. Henner, V.A. Meshcheryakov, *Z. Phys.* **A345**, 215 (1993).

[4] V.K. Henner, V.A. Meshcheryakov, *Yad. Phys.* **58**, 320 (1995).

[5] H. Iwasaki, H. Aihara, J. Chiba, H. Fuji, T. Fuji, T. Kamae, K. Nakamura, T. Suiyoshi, Y. Takeda, M. Yamauchi, H. Fukuma, *Nucl. Phys.* **A433**, 433 [1985).

[6] L. Linssen, C.I. Beard, R. Birsa, K. Bos, F. Bradamate, D.V. Bugg, A.S. Clough, S. Dalla Torre-Colzatti, M. Giorgi, J.R. Hall, J.C. Klugver, R.A. Kunne, C. Lechanione-Lelae, A. Martin, Y. Onel, A. Penzo, P. Rapin, P. Schiavon, R.L. Shypit, A. Villari, *Nucl. Phys.* **A469**, 726 (1987).

[7] B.V. Bykovsky, V.A. Meshcheryakov, D.V. Meshcheryakov, *Yad. Phys.* **55**, 1186 (1992).

[8] J. Mahalanubis, H.J. Pirner, T.A. Shibata, *Nucl. Phys.* **A485**, 546 (1988).

[9] J.J. de Swart, R. Timmermans, in: *Proceedings of the Third Biennial Conference on Low Energy Antiproton Physics*, Bled, Slovenia, September 1994, ed. by G. Kernel, P. Križan, M. Mikuž, World Scientific, 1995 p. 20.

[10] G. Bardin, G. Burgun, R. Calabrese, G. Capon, R. Carlin, P. Dalpiaz, P.F. Dalpiaz, J. Derre, U. Dosselli, J. Duclos, J.L. Faure, F. Gasparini, M. Huet, C. Kochowski, S. Limentani, E. Luppi, G. Marel, E. Mazzucato, F. Petrucci, M. Posocco, M. Savrie, R. Stroili, L. Tecchio, C. Voci, N. Zekri, *Phys.Lett.* **B255**, 149 (1991); **B257**, 514 (1991).

[11] G. Bassompierre, G. Binder, P. Dalpiaz, P.F. Dalpiaz, G. Gissinger, S. Jacquey, C. Peroni, M.A. Schneegans, L. Tecchio, *Phys. Lett.* **B68**, 477 (1977).

[12] B. Kerbikov, L.A. Kondratyuk, *Z. Phys.* **A340**, 181 (1991).

[13] B.O. Kerbikov, A.E. Kudryavtsev, *Nucl. Phys.* **A558**, 177c (1992).

[14] O.D. Dalkarov, K.V. Protasov, *Sov. J. Nucl. Phys.* **50**, 1030 (1989); *Nucl. Phys.* **A504**, 845 (1989); *Phys. Lett.***B280**, 107 (1992).

[15] R. Bacher, in: *Proc. First Bienual Conf. on Low Energy Antiproton Physics*, Stockholm, 2–6 July 1990, ed. P. Carlson, A. Kerek, S. Szilagyi, World Sientific, 1991, p. 373.

[16] S. Dubnička, Z. Dubničkova, P. Striženec, *Nuovo Cimento* **A106**, 1253 (1993).

[17] V.A. Matveev, R.M. Muradyan, A.N. Tavkhelidze, *Nuovo Cim. Lett.* **7**, 719 (1974).

[18] W.R. Frazer, J.R. Fulco, *Phys. Rev.* **117**, 1603, 1609 (1960).

[19] S.I. Bilenkaya, S. Dubnička, Z. Dubničkova, P. Striženec, *Nuovo Cimento* **A105**, 1421 (1992).

[20] G.V. Meshcheryakov, V.A. Meshcherykov, *Mod. Phys. Lett.* **A9**, 1603 (1994).

[21] V. Wataghin, *Nucl. Phys.* **B10**, 107 (1969).

[22] M. Castellano *et al.*, *Nuovo Cimento* **A14**, 1 (1973); B. Delcourt *et al.*, *Phys. Lett.* **B86**, 395 (1979); G. Bassompierre *et al.*, *Nuovo Cimento* **A73**, 347 (1983); D. Bissello *et al.*, *Nucl.Phys.* **B224**, 379 (1983); D. Bisello *et al.*, *Z. Phys.* **C48**, 23 (1990);

[23] E. Luppi, *Nucl. Phys.*, **A558**, 165c (1993).

[24] B.V. Bykovsky, V.A. Meshcheryakov, D.V. Meshcheryakov, *Yad. Fiz.* **53**, 257 (1990); *Yad. Fiz.* **55**, 1186 (1992).

[25] M. Muskhelishvili, Singular Integral Equation, Groningen, 1953.

[26] S.M. Bilenky, C. Giunti, V. Wataghin, *Z. Phys.* **C59**, 475 (1993).

[27] A. Bertin, M. Bruschi, M. Capponi, B. Cereda, S. De Castro, A. Ferreti, D. Galli, B. Giacobbe, U. Marconi, M. Piccinini, N. Semprini-Cesaro, R. Spighi, S. Vecchi, A. Vezzani, F. Vigotti, M. Villa, A. Vitale, A. Zoccoli, G. Belli, M. Corradini, A. Donzella, E. Lodi-Rizzini, L. Venturelli, A. Zenoni, C. Cicaio, A. Masoni, G. Puddu, C. Serci, P. Temnikov, G. Usai, V.G. Ableev, Y.U. Denisov, O.E. Gorchakov, S.N. Prahov, A.M. Rozhdestvensky, M.G. Sapozhnikov, V.I. Tretyak, M. Poli, P. Gianotti, C. Guaraldo, A. Lanaro, V. Lucherini, F. Nicitiu, C. Petrascu, A. Rosca, C. Cavion, U. Gastaldi, M. Lombardi, L. Vannucci, G. Vedovato, A. Andrigetto, M. Morando, R.A. Ricci, G. Bendiscioli, V. Filippini, A. Fontana, P. Montagna, A. Rotondi, A. Saino, P. Salvini, A. Filippi, F. Balestra, E.Botta, T. Bressani, M.P. Bussa, L. Busso, D. Calvo, P. Cerello, S. Costa, F. D'Isep, L. Fava, A. Feliciello, L. Ferrero, R. Garfagnini, A. Grasso, A. Maggiora, S. Marcello, N. Mirfakhraee, D. Panzieri, D. Parena, E. Rossetto, F. Tosello, G. Zosi, M. Angello, F. Iazzi, B. Minetti, G. Pauli, S. Tessaro, L. Santi, *Yad. Fiz.* **59**, 1430 (1996).

[28] A. Antonelli, R. Baldini, M. Bertani, M.E. Biagini, V. Bidoli, C. Bini, T. Bressani, R. Calabrese, R. Cardanelli, R. Carlin, C. Csari, L. Cugusi, P. Dalpiaz, G. De Zorzi, A. Feliciello, M.L. Ferrer, P. Ferretti, P. Gauzzi, P. Gianotti, E. Luppi, S. Marcello, A. Masoni, R. Messi, M. Morandin, L. Paoluzi, E. Pasqualucci, G. Pauli, N. Perlotto, F. Petrucci, M. Posocco, G. Puddu, M. Reale, L. Santi, R. Santonico, P. Sartori, M. Savrie, S. Serci, M. Spinetti, S. Tessaro, C. Voci, F. Zuin, in: *Proceedings of the Third Biennial Coference on Low-Energy Antiproton Physics*, ed. G. Kernel, P. Križan, M. Mikuž), World Scientific, 1995, p. 87.