RELAXATION OF FAST COLLECTIVE MOTION IN HEATED NUCLEI*

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The damping of the collective vibrations in hot nuclei is studied within the semiclassical Vlasov–Landau kinetic theory. The extension of the method of independent sources of dissipation is used to allow for irreversible energy transfer by chaos weighted wall formula. The expressions for the intrinsic width of the giant multipole resonances are obtained. The interplay between the one-body and the two-body channels which contribute to the formation of the intrinsic width in nuclei is discussed.

PACS numbers: 21.60.-n, 21.60.Ev, 24.30.Cz

1. Introduction

The relaxation mechanisms of collective motion and their dependence on the temperature in many-body systems have been extensively investigated in recent years [1–5]. In the present paper we consider the damping of the nuclear multipole vibrations within the semiclassical Vlasov–Landau kinetic theory. Semiclassical methods seem to be quite instructive for an investigation of the averaged properties of the multiparticle systems. In many cases, they allow us to obtain the analytical results and represent them in a transparent way.

In what follows we concentrate on the investigation of the contributions of different relaxation mechanisms to the intrinsic width of the giant multipole resonance (GMR). We determine the intrinsic width as formed by two main sources:

1. The relaxation due to the coupling of both particle and hole to more complicated states lying at the same excitation energy. This is the

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so-called two-body collisional damping. We take into account the two-body (collisional) damping exactly, incorporating into the collision integral the memory effects associated with the mean-field vibrations [6–8];

2. The fragmentation width caused by the interaction of particles with the time-dependent self-consistent mean field. As it was shown in Refs [9, 10], in the classical limit for the random phase approximation, the fragmentation width coincides with the width obtained from the one-body ("wall" [11, 12]) relaxation mechanism. We will imitate the fragmentation width by the one-body relaxation. This relaxation is taken into consideration approximately by adding to the Vlasov–Landau equation some source terms. The extension of the method [13] of independent sources of dissipation is used to allow for irreversible energy transfer by chaos weighted wall formula [14]. We also do not use the normalization of the width to a magnitude corresponding to the infinite-matter value.

In Sect. 2 the intrinsic width of the giant multipole resonances in cold and hot nuclei is calculated.

The numerical results and general discussion of the mass number and temperature dependences of the different relaxation mechanisms are presented in Sect. 3.

2. Damping of the giant resonances

We will consider a nucleus as the nuclear Fermi-liquid drop. We adopt the sharp surface for protons $R_p(t)$ and neutrons $R_n(t)$ and describe the edge of the nucleus in terms of the effective surface $R = (R_p + R_n)/2$. The GMR of multipolarity λ is regarded (see [6,15]) as oscillations of the corresponding (isoscalar or isovector) density vibration inside the nucleus, associated with the vibrations of the local displacement of the nucleons $\delta R_\lambda^{(\pm)}(t) \equiv \delta R_{\lambda,p}(t) \pm \delta R_{\lambda,n}(t) \equiv R_0 \beta_\lambda^{(\pm)}(t) Y_{\lambda 0}(\hat{r})$ of the same multipolarity λ; the signs $(+)$ and $(-)$ stand for isoscalar and isovector GMR respectively. Here R_0 is radius in equilibrium and $\delta R_{\lambda,\alpha}$ is the local displacement of the particle of the $\alpha = (p, n)$ type from its equilibrium position.

The density vibrations of any kind (isoscalar or isovector) is defined by the variation of the distribution functions. In macroscopic approaches [7,15,16] the isoscalar and isovector modes correspond to the in-phase and out-of-phase motions of neutrons and protons respectively. That means that both modes can be described in terms of the distortions of the distribution function in the form $\delta f^{(\pm)} = (\delta f_p \pm \delta f_n)/2$, where the subscripts p or n

label protons or neutrons and the plus or minus sign denotes the isoscalar or isovector modes, respectively.

We use in the nuclear interior the linearized Vlasov–Landau equation for the dynamical component of the one-particle phase space distribution function $\delta f(\boldsymbol{r}, \boldsymbol{p}, t)$, completed with a source term $J(\{f\})$ for relaxation processes. Neglecting a difference in the chemical potentials for protons and neutrons and assuming $f_{0,p} = f_{0,n} = f_0$, where $f_0 \equiv f_0(\boldsymbol{r}, \boldsymbol{p})$ is the equilibrium distribution function, we write down the linearized two-component Vlasov–Landau equation in the form (symmetric nuclear matter approximation)

$$\frac{\partial}{\partial t}\delta f^{(\pm)} + \frac{\vec{p}}{m^*}\nabla_r \delta f^{(\pm)} - \nabla_r \delta V^{(\pm)} \nabla_p f_0 = J^{(\pm)}(\{\delta f^{(\pm)}\}), \quad r < R_0, \quad (1)$$

where $\delta V^{(\pm)} \equiv \delta V^{(\pm)}(\boldsymbol{r}, \boldsymbol{p}, t)$ is the Wigner transform of the variation of the self-consistent potential with respect to the equilibrium value V_0. This mean field variation can be expressed in terms of the interaction amplitude $\mathcal{F}^{(\pm)}(\boldsymbol{p}, \boldsymbol{p}\,\prime)$ as

$$\delta V^{(\pm)} = \frac{2}{N_F} \int \frac{d\boldsymbol{p}\,\prime}{(2\pi\hbar)^3} \; \mathcal{F}^{(\pm)}(\boldsymbol{p}, \boldsymbol{p}\,\prime) \; \delta f^{(\pm)}(\boldsymbol{r}, \boldsymbol{p}\,\prime; t), \qquad (2)$$

where $N_F = p_F \, m^*/(\pi^2 \, \hbar^3)$, p_F is the Fermi momentum, m^* is the effective mass of nucleon. The quantities $\mathcal{F}^{(\pm)}(\boldsymbol{p}, \boldsymbol{p}\,\prime)$ are defined by the interaction amplitudes $F_{\alpha,\beta}(\boldsymbol{p}, \boldsymbol{p}\,\prime)$ between neutrons and protons $(\alpha, \beta) = (n, p)$:

$$\mathcal{F}^{(+)}(\boldsymbol{p}, \boldsymbol{p}\,\prime) = (F_{p,p}(\boldsymbol{p}, \boldsymbol{p}\,\prime) + F_{n,n}(\boldsymbol{p}, \boldsymbol{p}\,\prime) + F_{p,n}(\boldsymbol{p}, \boldsymbol{p}\,\prime) + F_{n,p}(\boldsymbol{p}, \boldsymbol{p}\,\prime))/2,$$

$$\mathcal{F}^{(-)}(\boldsymbol{p}, \boldsymbol{p}\,\prime) = (F_{p,p}(\boldsymbol{p}, \boldsymbol{p}\,\prime) + F_{n,n}(\boldsymbol{p}, \boldsymbol{p}\,\prime) - F_{p,n}(\boldsymbol{p}, \boldsymbol{p}\,\prime) - F_{n,p}(\boldsymbol{p}, \boldsymbol{p}\,\prime))/2,$$

The amplitudes $F_{\alpha,\beta}(\boldsymbol{p}, \boldsymbol{p}\,\prime)$ is usually parameterized in terms of the Landau constants $F_{\alpha,\beta,0}$ and $F_{\alpha,\beta,1}$ as $F_{\alpha,\beta}(\boldsymbol{p}, \boldsymbol{p}\,\prime) = F_{\alpha,\beta,0} + F_{\alpha,\beta,1}(\hat{p}c \cdot \hat{p'})$, $\hat{p} = \boldsymbol{p}/p$. This leads to the relation

$$\mathcal{F}^{(\pm)}(\boldsymbol{p}, \boldsymbol{p}\,\prime) = \mathcal{F}_0^{(\pm)} + \mathcal{F}_1^{(\pm)}(\hat{p} \cdot \hat{p'}). \qquad (3)$$

To simplify the presentation, we will omit in the following the superscript $(\pm)$ and include them only when it is necessary to avoid confusion.

The right-hand side of Eq. (1) represents the change of the distribution function due to relaxation. We take into account the collisional damping exactly. The one-body relaxation is taken into consideration approximately by adding to the Vlasov-Landau equation some source terms. Namely, we assume

$$J(\{\delta f\}) = J_c(\{\delta f\}) + J_s(\{\delta f\}), \qquad (4)$$

where $J_c(\{\delta f\})$ is the collision integral for the two-body collisions, $J_s(\{\delta f\})$ determines the change in the distribution function resulting from one-body relaxation. The term $J_s(\{\delta f\})$ is considered within the relaxation time approximation of the form

$$J_s(\{\delta f\}) = -\frac{\delta f_0(\boldsymbol{r}, \boldsymbol{p}, t)}{\tau_{s,0}} - \frac{\delta f_2(\boldsymbol{r}, \boldsymbol{p}, t)}{\tau_{s,2}}. \tag{5}$$

Here, $\delta f_\ell(\boldsymbol{r}, \boldsymbol{p}, t)$ is dynamical component of the distribution function at the Fermi-surface distortion with multipolarity ℓ : $\delta f(\boldsymbol{r}, \boldsymbol{p}, t) = \sum_{\ell \geq 0} \delta f_\ell(\boldsymbol{r}, \boldsymbol{p}, t)$ $\equiv \sum_{\ell \geq 0} \delta \tilde{f}_\ell(\boldsymbol{r}, p, t) Y_{\ell 0}(\hat{p})$, and $\tau_{s,0}, \tau_{s,1}$ is the relaxation time corresponding to the equilibration of the system due to the one-body dissipation.

The first component of the one-body source term $J_s(\{\delta f\})$ in Eq. (5) leads to non-conservation of energy. It determines an irreversible part of the energy transferred from particles to the nuclear surface [14].

The distribution functions in the nuclear interior are constructed as a linear angular superposition of the corresponding solutions $(\delta f_{\vec{k}}(\vec{r}, \vec{p}, t))$ of the Vlasov–Landau equation in nuclear matter:

$$\delta f_{\lambda,k}(\vec{r}, \vec{p}, t) = \text{Re} \int d\Omega_k Y_{\lambda 0}(\hat{k}) \delta f_{\vec{k}}(\vec{r}, \vec{p}, t),$$

where

$$\delta f_{\vec{k}}(\vec{r}, \vec{p}, t) = -\frac{\partial f^{eq}}{\partial \epsilon^{eq}} \exp\{i(\vec{k}\vec{r} - \omega t)\} \sum_{\ell \geq 0} \alpha_\ell(\omega, k) Y_{\ell 0}(\hat{p} \cdot \hat{k}). \tag{6}$$

Substituting these expressions into the Vlasov–Landau equation, integrated with respect to the energy ϵ_1, we obtain an equation for α_ℓ and the velocity $S = \omega/(v_F k)$. In order to obtain closed forms, we will follow the nuclear fluid dynamic approach of Refs. [17–21] and take into account in Eq. (1) the dynamic Fermi-surface distortions up to multipolarity $\ell = 2$. As a result we have the system

$$\begin{aligned}
\alpha_0 - (1 + \mathcal{F}_1/3)\alpha_1/\sqrt{3} &= -i/(v_F k \bar{\tau}_0)\alpha_0\,, \\
\alpha_1 - (1 + \mathcal{F}_0)\alpha_0/\sqrt{3} - \alpha_2/\sqrt{15} &= -i/(v_F k \bar{\tau}_1)\alpha_1\,, \\
\alpha_2 - 2(1 + \mathcal{F}_1/3)\alpha_1/\sqrt{15} &= -i/(v_F k \bar{\tau}_2)\alpha_2\,.
\end{aligned} \tag{7}$$

The effective relaxation times $\bar{\tau}_\ell$ are different for isoscalar and isovector modes:

$$\bar{\tau}_0^{(\pm)} = \tau_{s,0}^{(\pm)}\,, \bar{\tau}_1^{(-)} = \tau_1^{(-)}\,, 1/\bar{\tau}_1^{(+)} = 0\,, 1/\bar{\tau}_2^{(\pm)} = 1/\tau_2^{(\pm)} + 1/\tau_{s,2}^{(\pm)}\,. \tag{8}$$

Here $\tau_\ell^{(\pm)}$ are the partial collective relaxation times due to interparticle collisions within the distorted layers of the Fermi surface with multipolarity ℓ. These times appear in the multipole expansions of the total numbers $\mathcal{N}^{(\pm)}(\hat{p})$ of the collisions in direction $\hat{p} \equiv \hat{p}_1$:

$$\mathcal{N}^{(\pm)}(\hat{p}) \equiv \int_0^\infty d\epsilon_1 J_c^{(\pm)}(\hat{p}, \epsilon_1) = \exp\{-i\omega t\} \sum_{\ell \geq \ell_0^{(\pm)}} \sum_{m=-\ell}^{\ell} \frac{\alpha_{lm}^{(\pm)}}{\tau_\ell^{(\pm)}} Y_{\ell m}(\hat{p}), \quad (9)$$

where $\ell_0^{(-)} = 1$, $\ell_0^{(+)} = 2$,

$$\frac{1}{\tau_\ell^{(\pm)}} \equiv \int_0^\infty d\epsilon_1 \int d\Omega_p J_c^{(\pm)}(\hat{p}, \epsilon_1) Y_{\ell 0}(\hat{p}) / \int_0^\infty d\epsilon_1 \int d\Omega_p \delta f^{(\pm)} Y_{\ell 0}(\hat{p}). \quad (10)$$

With the collision integrals with memory effects [6–8] for interaction between different kinds of particles and following the procedure [22] in integrating the collision integral over energy, we obtain the following expressions for relaxation times at $\hbar\omega, T \ll E_F$:

$$1/\tau_\ell^{(+)} = \mathcal{R}(\omega, T) \left\langle (\bar{W} + W_{pn})\Phi_\ell^{(+)} \right\rangle,$$

$$1/\tau_\ell^{(-)} = \mathcal{R}(\omega, T) \left[\left\langle \bar{W}\Phi_\ell^{(+)} \right\rangle + \left\langle W_{pn}\Phi_\ell^{(-)} \right\rangle \right], \quad (11)$$

where $\bar{W} = (W_{nn} + W_{pp})/2$; $W_{\alpha,\beta}$ is the probability of scattering of the particles $(\alpha, \beta) = (n, p)$ near the Fermi surface. The functions $\Phi_\ell^{(+)} \equiv 1 + P_\ell(\hat{p}_2\hat{p}_1) - P_\ell(\hat{p}_3\hat{p}_1) - P_\ell(\hat{p}_4\hat{p}_1)$, $\Phi_\ell^{(-)} \equiv 1 - P_\ell(\hat{p}_2\hat{p}_1) - P_\ell(\hat{p}_3\hat{p}_1) + P_\ell(\hat{p}_4\hat{p}_1)$, define the angular constraints for nucleon scattering (P_ℓ is a Legendre polynomial; $\vec{p}_1$, $\vec{p}_2$ and $\vec{p}_3$, $\vec{p}_4$ are the momenta of particles before and after collisions respectively). The symbol $\langle \ldots \rangle$ denotes the averaging over angles of the relative momentum of the colliding particles. The function $\mathcal{R}(\omega, T)$ has the following form

$$\mathcal{R}(\omega, T) = \left(\frac{m^*}{\hbar^2}\right)^3 \frac{1}{48\pi^4} \{(2\pi T)^2 + C_\omega(\hbar\omega)^2\}, \quad (12)$$

where we will take $C_\omega = 1$ [13]. We have the following relations in the case of isotropic collision probabilities

$$1/\tau_1^{(-)} = (4/3)\mathcal{R}\langle W_{pn}\rangle = (5W_{pn}/3(\bar{W} + W_{pn}))/\tau_2^{(+)} \approx 1.1/\tau_2^{(+)},$$

$$\tau_2^{(-)} = \tau_2^{(+)}. \quad (13)$$

We find the following dispersion equation for calculation the velocity S from Eq. (7)

$$\left(S + \frac{i}{v_F k \tau_0}\right)\left(S + \frac{i}{v_F k \tau_1} - \left(\frac{4}{15}\right)(1+\mathcal{F}_1/3)\bigg/\left(S + \frac{i}{v_F k \tau_2}\right)\right) - S_f^2 = 0, \tag{14}$$

where S_f is the velocity of the first sound: $S_f^2 = (1+\mathcal{F}_0)(1+\mathcal{F}_1/3)/3$.

The complex frequency of the collective vibrations of the λ type is determined by the velocity $S = S_\lambda$ and the wave number k_λ: $\omega_\lambda = v_F k_\lambda S_\lambda$. The values $k_{\lambda,n}$ of the wave number k_λ can be found from the macroscopic boundary conditions acting at the nuclear surface [6, 15]. Here we study the damping properties of giant resonances rather than their full description and parameterize damping width as a function of the resonance energy. For underdamped collective motion the quantity $k_{\lambda,n}$ can be considered as a real number. Then according to the correspondence principle the energies $\mathcal{E}_{\lambda,n}$ and widths $\Gamma_{\lambda,n}$ of the resonances are given by

$$\mathcal{E}_{\lambda,n} = \hbar\Omega_{\lambda,n} = \hbar v_F k_{\lambda,n} S_\lambda^{(r)},$$
$$\Gamma_{\lambda,n} = -2\hbar\,\mathrm{Im}\,\{\omega_{\lambda,n}\} = 2\hbar v_F k_{\lambda,n} S_\lambda^{(i)}, \tag{15}$$

where $\Omega \equiv \mathrm{Re}\,\omega$, $S^{(r)} \equiv \mathrm{Re}\,S$, $S^{(i)} \equiv -\mathrm{Im}\,S$.

We consider the case of the slightly damped motion when condition $|S_i S_0| \ll a$ is fulfilled, where $a = \min\{1, x, 1/x, 1/(1/y+1/z)\}$ and $x \equiv \Omega\tau_2$, $y \equiv \Omega\tau_0$, $z \equiv \Omega\tau_1$. As a first approximation, we obtain from Eq. (14)

$$2\frac{S^{(i)}}{S^{(r)}} = \frac{\bar{S}_0^2 - \bar{S}_f^2}{(S^{(r)})^2}\frac{x}{1+x^2}\left(1 - \frac{x}{y}\right)\left(1 - \frac{1}{yz}\right) + \frac{1}{y} + \frac{1}{z}, \tag{16}$$

$$\left(S^{(r)}\right)^2 = \bar{S}_0^2 + \left(\bar{S}_f^2 - \bar{S}_0^2\right)\frac{1}{1+x^2}\left(1 - \frac{x}{y}\right), \tag{17}$$

where $\bar{S}_f^2 = S_f^2/(1-1/(yz))$, $\bar{S}_0^2 = S_0^2/(1-1/(yz))$ and $S_0^2 = S_f^2 + (4/15)(1+\mathcal{F}_1/3)$. The quantity S_0 is the velocity of the zero sound in the absence of the relaxation processes and with the deformation of the Fermi sphere with multipolarities $\ell \leq 2$ only.

We find the following expressions for the intrinsic width and frequency $\Omega = \mathcal{E}/\hbar$ of a GMR using Eqs. (15)–(17):

$$\Gamma = \hbar\Omega(a-1)\frac{x}{1+a(x^2+x/y)/(1-x/y)}\left(1 - \frac{1}{yz}\right) + \frac{\hbar\Omega}{y} + \frac{\hbar\Omega}{z}, \tag{18}$$

$$\Omega^2 = \omega_0^2 + \left(\omega_f^2 - \omega_0^2\right)\frac{1}{1+x^2}\left(1 - \frac{x}{y}\right), \tag{19}$$

where $a = (E_0/E_f)^2$ and $E_0 = \hbar\omega_0 = \hbar v_F k \bar{S}^{(r)}$, $E_f = \hbar\omega_f = \hbar v_F k \bar{S}^{(f)}$.

Note that the total change rate in the distribution function was taken as a sum of the change rates in various damping channels (independent dissipation rates approximation) but in the general case the expression for Γ can not be represented as a sum of the widths associated with the different independent sources of the damping. This is a peculiarity of the collisional Vlasov–Landau equation where the Fermi-surface distortion effect influences both the self-consistent mean field and the memory effect at the relaxation processes.

3. The numerical results and discussion

The values of the GMR energy and the relaxation times are required for calculations of the intrinsic width Γ. As the GMR energy $\mathcal{E}$ we use the phenomenological A-dependence of $\mathcal{E}$ obtained from a fit to the experimental data [23–26]. We neglect the variation of the wave number k in the first and zero sound regimes ([20, 21]) and adopt for energy E_f the values corresponding to the energy in hydrodynamic approach. We use also the approximation $E_0 \simeq \mathcal{E}$ due to consideration of the underdamped motion. The estimation from [13] is used for the collisional relaxation time $\tau_2^{(+)}$ and for one-body relaxation time $\tau_{s,0}^{(-)}$ ($\xi^1 = 1.543$; $\tau_{s,2}^{(-)} = 0$).
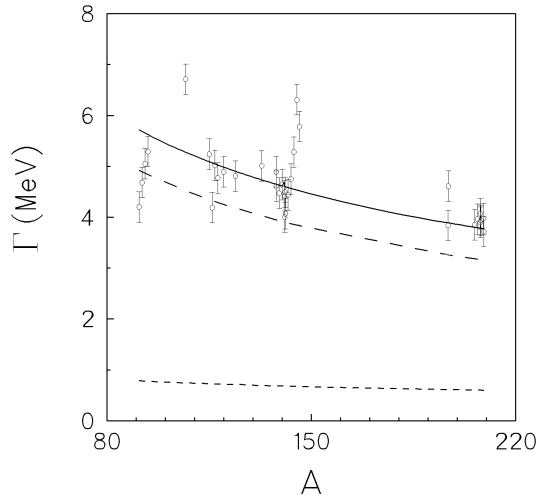

Fig. 1. The intrinsic GDR widths and the corresponding one- and the two-body contributions at $T = 0$ as functions of mass number. The dot-dash line with short and long dashes correspond to the two- and one-body contributions respectively.

In Fig. 1 we show the intrinsic giant dipole resonance (GDR) widths and the one- and two-body contributions to them at zero temperature ($T = 0$) as the functions of the mass number. The experimental data were taken from [23]. The contribution of collisional damping (dot-dash line with long dash) to the GDR widths does not exceed $\sim 30\%$ of the experimental values.

In Fig. 2 we show the intrinsic width of the giant dipole resonance (GDR) in the nucleus ^{112}Sn as a function of the temperature T. The experimental data were taken from [1]. Considering the experimental data we assumed that the energy $\mathcal{E}$ of the GDR is independent of temperature and equals 15.6 MeV. Note that we can use the expression (18) for intrinsic width when the condition $\Gamma/\mathcal{E} \ll 1$ is fulfilled, *i.e.*, in fact, at the temperature of no more than ≈ 5 MeV. We observe a systematic large deviation of the evaluated width with respect to the experimental data in the range from 2 to 5 MeV. This deviation may be connected with dependence of the GDR width on angular momentum and thermodynamic fluctuations of the nuclear shape and orientation angles [3].
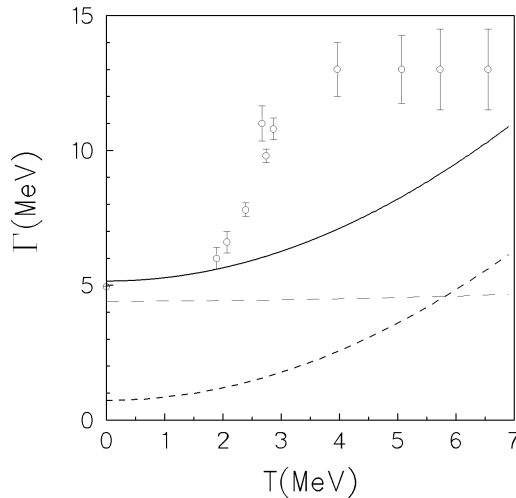

Fig. 2. The intrinsic width of the GDR in the nucleus Sn as function of temperature. The experimental data were taken from Ref. [1]. The notations are the same as in Fig. 1.

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