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ELASTIC TRANSFER: A NON-DISPERSIVE COMPONENT IN THE OPTICAL POTENTIAL AND ITS EFFECT IN THE ^{12}C+^{24}Mg ELASTIC SCATTERING*

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It is shown that the coupling of the elastic scattering channel to an elastic-transfer channel leads to a non-dispersive polarization potential with a periodic, l-dependent, energy dependence. Evidence of this is found in the elastic scattering data of ^{12}C+^{24}Mg at low energies. The finding can indicate a significant ^{12}C+^{12}C clustering effect in the ground state of ^{24}Mg.

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The dispersive optical potential usually referred to as the Feshbach potential [1], obeys a dispersion relation. In the heavy ion context this relation has gained the notoriety of being called the Threshold Anomaly (TA). As eloquently explained by Satchler [2], the dispersion relation of the Feshbach potential comes out as a consequence of the polarization nature in the sense that the potential has the general structure:

$$V_{\text{Feshbach}}(r,r') = \sum_{i=1}^{n} V_{0i}(r)\frac{1}{E-H_i+i\varepsilon}V_{io}(r')\,. \qquad (1)$$

The intermediate channel Green function, $(E-H_i+i\varepsilon)^{-1}$, has the following simple structure:

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$$(E - H_i + i\varepsilon)^{-1} = -i\pi\delta(E - H_i) + P\frac{1}{E - H_i}\,, \tag{2}$$

where P stands for the principal part and H_i is taken for simplicity to be Hermitian. Clearly, one can write:

$$P\frac{1}{E - H_i} = P\int dZ\frac{\delta(Z - H_i)}{E - Z} = -\frac{P}{\pi}\int dZ\frac{-\pi\delta(Z - H_i)}{E - Z}\,. \tag{3}$$

From equations (1) and (3) one finds the dispersion relation:

$$\mathrm{Re}\, V_{\mathrm{Feshbach}}\,(r, r^{'}, E) = \frac{P}{\pi}\int dZ\frac{\mathrm{Im}\, V_{\mathrm{Feshbach}}\,(Z, r, r^{'})}{Z - E}\,. \tag{4}$$

In actual use in data analysis one relies on local potentials. The intrinsically non-local dispersive Feshbach potential is therefore transformed into a local-equivalent one. This brings in more subtle energy dependence. However, in practical application, it was found [3] that the resulting potential at a given value of, the now one spatial variable r, still satisfies Eq. (4).

Now we raise the following question: do all channel couplings result in a dispersive Feshbach potential? The answer is no, at least in cases involving elastic transfer. Here we mean a process which involves the elastic scattering of the following objects:

$$(a + b) + b \longrightarrow (a + b) + b\,, \tag{5}$$

$$(a + b) + b \longrightarrow b + (a + b)\,. \tag{6}$$

The two corresponding amplitudes add coherently. Since the projectile–target system, in the second process, becomes the target–projectile system (no change in internal structure) the second process in Eq. (6), the elastic transfer process, is important at large angles. The Feshbach potential that takes into account the coupling of the elastic channel to the elastic transfer channel is found to be [4,5]

$$V^{\mathrm{elastic-transfer}}_{\mathrm{Feshbach}} = (-1)^l F(r)\,, \tag{7}$$

where l is the orbital angular momentum and $F(r)$ is an approximate transfer form-factor of the second process in Eq. (6). There is no energy dependence in (7). Clearly, (7) does not satisfy any energy dispersion relation. Of course some weak energy dependence may be found in $V^{\mathrm{elastic-transfer}}_{\mathrm{Feshbach}}$, when higher-order processes are taken into account, *e.g.* :

$$(a + b) + b \longrightarrow (a + b)^{*} + b \longrightarrow b + (a + b)\,. \tag{8}$$

In the following we ignore these processes for simplicity. The above discussion may become very important in situations where elastic transfer is significant.

1. The non-dispersive potential in the elastic scattering of ^{12}C+^{24}Mg

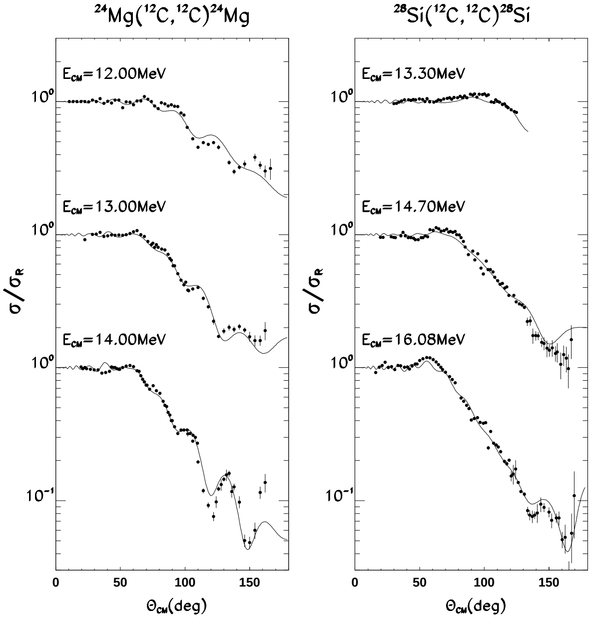
In a recent experiment [6,7] the complete angular distributions of the elastic scattering of ^{12}C+^{24}Mg were measured at fifteen energies near the Coulomb barrier, namely between $E_{\rm CM}$ = 10.67 and 16.00 MeV. The data were analysed in the optical model framework (Pot II) and the best-fit potentials were: shallow, energy dependent, real potentials ($V_0 \sim 37$ MeV, r_0=1.29 fm, a=0.4 fm) with no continuous ambiguity and very weak, energy dependent, imaginary potentials ($W_0/V_0 \sim 0.01$, W_0=0.5-1.5 MeV, r_i=1.77 fm, $a_i \sim$ 0.4-0.8 fm).

We present in figure 1(a) some of the lowest energy angular distributions, situated at energies under and at the Coulomb barrier ($V_{\rm CB}$=12.67 MeV using the Christensen–Winther radius) together with the optical model fits. The angular distributions present clear oscillatory pattern even at the lowest energies.

In figure 1(b) the low energy elastic scattering angular distributions of ^{12}C+^{28}Si system are presented. These unpublished data [8] were also measured at the Pelletron Laboratory of the São Paulo University, and will be published in the near future together with an optical model analysis. The optical model used to reproduce the data is much more absorptive (3 to 5 times more), than the Pot. II used for the ^{12}C+^{24}Mg system. The Christensen–Winther Coulomb barrier for the ^{12}C+^{28}Si system is $V_{\rm CB}$=14.36 MeV. We indicate in the figure caption the ratio $E_{\rm CM}/V_{\rm CB}$ to allow a quantitative comparison between angular distributions of figure 1(a) and 1(b).

It is clear that the angular distributions at same energy with respect to the Coulomb barrier are different for the 2 systems considered. While the oscillations are clear for the ^{12}C+^{24}Mg system, even at energies under the Coulomb barrier, they are smooth and non-oscillating for the ^{12}C+^{28}Si system at the same energies. Even at energies 12% above the Coulomb barrier, where the very back angle region of the ^{12}C+^{28}Si begins to show one oscillation, the ^{12}C+^{28}Mg system shows much more oscillations in the intermediate angle region.

Both optical potentials are dependent on the radial position and on the energy. From the point of view of radial dependences, their differences can be pinned down in the notch test. The notch test, based on summing a localized perturbation to the optical potential at variable radial positions and observing the quality of the fit (defined through the χ^2) as a function of the position of the perturbation, gave very different results for the two systems. For the ^{12}C+^{28}Si system the notch test presents a localized peak at R_1+R_2=7.3 fm, which means that the elastic data are sensitive to the optical potential only in a radially restricted region at the nuclear surface



(a) (b)

Fig. 1. (a) — The ^{12}C+^{24}Mg elastic scattering angular distributions, measured at the indicated energies, are represented by the dots. The solid lines are optical model calculations with our best fit optical potentials (Pot. II). The E_{CM}/V_{CB} values at these energies are respectively, 0.842, 0.894, 0.947, 0.973, 1.00 and 1.026, with V_{CB}=12.67 MeV. (b) — The ^{12}C+^{28}Si elastic scattering angular distributions, measured at the indicated energies, are represented by the dots. The E_{CM}/V_{CB} values at these energies are respectively, 0.926, 1.023, 1.120 with V_{CB}=14.36 MeV.

around 7.3 fm. For the ^{12}C+^{24}Mg system the notch test indicates that the elastic data are sensitive to the optical potential on the surface and in the nuclear interior, from 3 to 8 fm, result compatible with the very transparent optical potentials used to fit the data [7].

The differences between both potentials become even more interesting, when they are compared from the point of view of their energy dependences, through the dispersion relation (Eq. (7)).

While the optical potentials of the ^{12}C+^{28}Si system satisfy the dispersion relation at the R=7.3 fm, the optical potentials of the ^{12}C+^{24}Mg system do not satisfy the dispersion relation at any radius (see Fig. 2). Nevertheless the volume integrals of the optical potentials of the ^{12}C+^{24}Mg system satisfy the dispersion relation as it was shown previously [7].

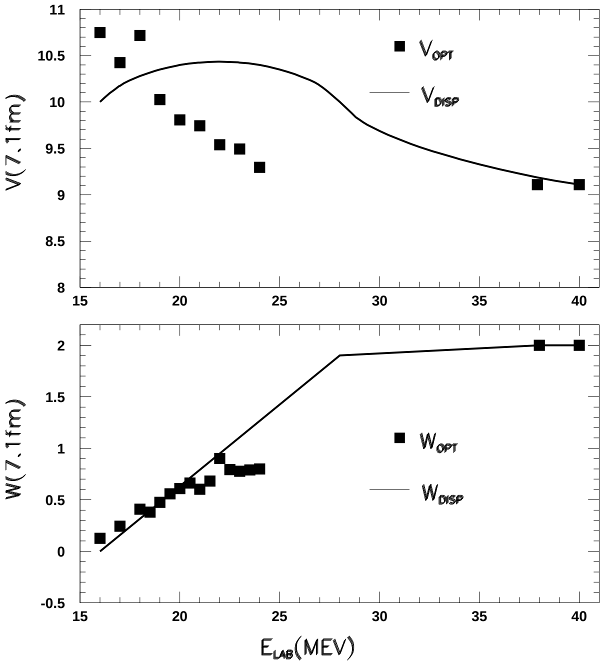


Fig. 2. The imaginary and the real depths of the best fit optical potentials of the ^{12}C+^{24}Mg system, as a function of the laboratory energies (squares) for R=7.1 fm. We also used data at higher energies (E_{lab}=37.9 and 40.0 MeV [10,11]) to fix the imaginary part of the potential. The dispersion relation calculations are indicated by dots and the disagreement with the real optical potential is evident.

We can calculate the differences between the real part of the optical potential and the real part of the dispersive potential, also called Feshbach potential in the introductory discussion. The real part of the Feshbach potential is calculated from the imaginary optical potential through the dispersion relation. The plot of these differences as a function of the energy is presented in figure 3.

In the case of the ^{12}C+^{28}Si system, at least at these very low energies, the difference called $\mathrm{Re}V_{\mathrm{non-dispersive}}$ is zero, while for the ^{12}C+^{24}Mg system it presents a clearly oscillatory pattern, as a function of energy, with a decreasing amplitude, when the radius increases. If we assume that the non-dispersive part of the potential is responsible for the coupling of the elastic channel to the elastic transfer channel, then from the point of view of Eq. (4) and (7), we can write:

$$\mathrm{Re}V_{\mathrm{opt.mod}} - ReV_{\mathrm{Feshbach}} = \mathrm{Re}V_{\mathrm{non-dispersive}} = (-1)^l F(r) = \cos(\pi l(r,E))F(r). \tag{9}$$

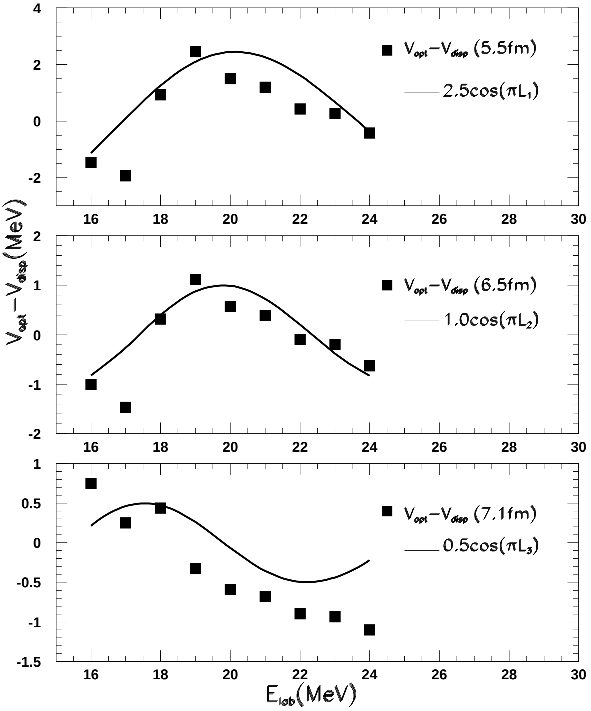


Fig. 3. The differences between the real part of the optical potential and the real part of the dispersive potential (calculated by the dispersion relation) as a function of the laboratory energy, at three radial positions, R=5.5 fm, 6.5 fm and 7.1 fm. The solid lines are explained in the text.

The sine function was neglected in Eq. (9) due to the fact that all l values are integer. As a matter of fact, in a qualitative interpretation, the non-dispersive part of the potential should have an oscillatory character ($\cos\pi l$) and decrease in amplitude with increasing r ($F(r)$), as it appears in Fig. 3.

We also show in Fig. 3 (a) very qualitative fit to $\mathrm{Re}V_{\mathrm{non-dispersive}}$, by a cosine function. We assumed that the argument of the cosine function, which is πl, where l is the orbital angular momentum, should vary with $\sqrt{E}$ and linearly with r. The argument for the cosine function in the three fits was consistently :

$$\pi l = \mathrm{const} \times \sqrt{E} \times r\,. \tag{10}$$

The amplitude of the cosine function, which is the exchange form-factor, has the values 2.5,1.0 and 0.5 respectively for R=5.5, 6.5, 7.1 fm, indicating a strong decrease as a function of the radius.

Thus the ^{12}C+^{24}Mg system exhibits the contribution of a non-dispersive, exchange potential. This fact explains the presence of the oscillatory angular distribution for the ^{12}C+^{24}Mg system since the direct elastic amplitude can interfere with the elastic transfer amplitude, producing oscillatory pattern. We can also explain in a qualitative way, why the ^{12}C+^{24}Mg system exhibits the contribution of a non-dispersive, exchange potential, while the ^{12}C+^{28}Si system does not. Recent cluster model calculations in the algebraic framework [9] indicate that the ^{24}Mg ground state contains an appreciable contribution of a two ^{12}C cluster configuration. At the same time the ^{28}Si cannot be represented by a combination of a ^{12}C and a ^{16}O in its ground state, but only in higher excited states.

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