SUM RULE ANALYSIS OF LOW-ENERGY ONESTEP DIRECT REACTIONS *

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Analysis of inclusive nonelastic nucleon emission use inconsistently gradual absorption into the quasibound particle-hole states of multistep compound reaction chain together with the predominantly onestep direct reactions to describe experiments [1–3]. A sum rule analysis of the onestep direct reactions cross section calculated in the framework of the multistep direct reaction theory of Feshbach, Kerman and Koonin (FKK) [4] has revealed that these cross sections are misinterpreted.

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1. Introduction

The Compound Nucleus (CN) reactions or more generally the MultiStep Compound (MSC) reactions are calculated in terms of the optical model for elastic scattering with the cross section for absorption of the incoming flux determined by imaginary optical potential. The latter accounts for all the flux removed from the elastic channel. However, the removed flux feeds not only the quasibound, compound states of the MSC reaction chain but also the direct nonelastic processes, which do not contribute immediately to formation of compound nucleus states, *i.e.* to absorption. Therefore to calculate the MSC and the related CN cross sections the optical model absorption has to be reduced by the amount of the direct nonelastic reactions $(1-R)\sigma_c$, so that $R\sigma_c$ is the fraction that feeds the MSC reactions.

With increasing bombarding energy an ever increasing fraction of nonelastic direct reactions will be due to several rather than onestep direct processes, which may be followed by transitions into the quasibound particle–hole states giving rise to gradual absorption after consecutive stages of the

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MultiStep Direct (MSD) reaction [5]. Gradual absorption splits R into partial R_M's that describe absorption at successive reaction stages M. Thus the influence of the strong nonelastic direct reactions on the formation of the compound nucleus manifests itself in a reduced and gradual absorption of the incoming flux [6],

$$\sigma_a = \sum_M R_M \sigma_c \,. \tag{1}$$

On the other hand the MSC reactions affect the MSD process by modifying the matrix elements for the individual onestep transitions that appear in the expression for the cross section of a MSD reaction.The modified matrix elements can be obtained from DWBA by inserting an inverse elastic S-matrix factor. An immediate consequence is that the modified matrix elements are larger than the DWBA ones by the amount of absorptions on subsequent stages of the MSD reaction. This means that the modified matrix elements include encoded gradual absorption.

Since gradual absorption is justified by strong multistep direct transitions, which require the modified DWBA matrix elements in the MSD calculations, it questions the argumentation in favour of the normal DWBA matrix elements by Feshbach [7,8]. The normal DWBA matrix elements used currently in the FKK calculations result in MSD reaction cross sections which are practically onestep (1SD) cross sections. The calculated twostep (2SD) contributions comprise no more than 5 to 25 % of the 1SD cross sections at bombarding energies lower than 50 MeV [9–11]. The predominantly 1SD cross sections are inconsistent with the simultaneous use of gradual absorption in the MSC calculations, a procedure which has become a routine in analyses of low-energy experimental data. In the sequel we are going to corroborate our observation that the commonly calculated 1SD cross sections do not observe the energy-weighted sum rules (EWSR's) and therefore cannot be due to onestep reactions only [12], although quantitatively they well complete the cross sections for direct excitation of the onephonon, low-energy collective states and the giant resonances in fitting measured nucleon emission spectra and angular distributions [13,14].

2. The model calculations and analyses

The double-differential 1SD reaction cross sections $(d^2\sigma/dUd\Omega)_{\rm 1SD}$ of FKK can be divided according to the contributions of different transferred orbital angular momenta l , and expressed by the microscopic DWBA angular distributions $(d\sigma/d\Omega)_l^{\rm DWBA-micr}$ averaged over several final 1p1h shell model states compatible with angular momentum selection rules and energy conservation. This allows one to calculate the cross section associated with

2^l-pole spatial motion and attach to it a formal effective β_l parameter in accord with the usual macroscopic DWBA model for collective excitations [12],

$$\int_U^{U+\Delta U} \left(\frac{d^2\sigma}{dU d\Omega}\right)_{1\mathrm{SD}-l} dU = \varepsilon_{\alpha,\beta}\beta_l^2 \left(\frac{d\sigma}{d\Omega}\right)_{l,U+\frac{1}{2}\Delta U}^{\mathrm{DWBA-macr}} . \quad (2)$$

The 1SD cross sections are binned into $\Delta U = 1$ MeV bins. The term $\varepsilon_{\alpha,\beta}$ arises from an isospin conserving Clebsch–Gordon coefficient ($\varepsilon_{\pi,\pi} = \varepsilon_{\nu,\nu} = 1$ for inelastic scattering and $\varepsilon_{\pi,\nu} = \varepsilon_{\nu,\pi} = 2$ for charge-exchange reactions). Since the multipolarity of the 1SD cross section in Eq. (2) is *a priori* known it is sufficient to compare the angle-integrated cross section with that for a fictitious collective state located in the middle of the energy bin considered. The DWBA cross sections $(d\sigma/d\Omega)_l^{\mathrm{DWBA-micr}}$ are calculated with microscopic two-quasi-particle form factor with real effective interaction of Yukawa form and strengths $V_{\pi,\pi} = V_{\nu,\nu} = 12.7$ MeV and $V_{\pi,\nu} = 43.1$ MeV [15], and the macroscopic cross sections of the collective model $(d\sigma/d\Omega)_l^{\mathrm{DWBA-macr}}$ use form factors given by deforming the phenomenological complex optical potential $f_{00}(r) = -R(\partial U_{00}/\partial R)$. Both form factors were calculated with use of the DWUCK-4 code [16] corrected for omission of the $(2j_h + 1)^{1/2}$ term in the microscopic $1p1h$ form factor option [17].

The formal β_l parameters obtained from (2) were used to deplete the EWSR's for *El* electric transitions of $l = 1$, 2, 3 and 4 [18]. In this way we analysed the 1SD cross sections calculated for orbital angular momentum transfer $l = 0$ through 9 that contribute to the neutron emission spectra from the ^{93}Nb(n, xn) reaction at incident energies 14.1 MeV, 20 MeV and 25.7 MeV [13,14]. It was found that the 1SD cross sections exceed the EWSR's strengths the more the higher the projectile energy. In Table 1 we present the factors $F_{\mathrm{ex}} = \sum_{\Delta U}(U + 1/2\Delta U) \times \beta_l^2/\mathrm{EWSRLIM}(l)$, by which the sum rules limits (EWSRLIM(l)) [18] for multipolarities l were exceeded. When reading Table 1 one has to bear in mind that the incoherent 1SD cross sections of FKK supplement the cross sections of the direct reactions (DCR) that excite coherently the low-energy collective states and the giant resonances in the continuum thus providing a good fit to the experimental data [13,14]. The DCR's cross sections are summed over multipolarities $l = 1$ through 4 and exhaust the full strengths of the corresponding EWRS's (*i.e.* a factor equal 1 has to be added to each F_{ex} for the ^{93}Nb(n, xn) reaction in Table I). Treating the EWSR's limits strictly all incoherent 1SD cross sections for the ^{93}Nb(n, xn) reaction of orbital angular momentum transfer $l = 1$ through 4 are in excess. However, we know [19] that the sum rules are fulfilled only approximately and this leaves room for some incoherent contribution due to onestep reactions beside the onephonon collective cross section. Still the figures in Table I indicate that even when we assume that the EWSR's may be inadequate within a factor of 1.5 most of the 1SD cross sections at projectile energies 26 MeV and 20 MeV presented in Table I cannot be explained by onestep reactions only. These have to be instead mainly twostep (2SD) cross sections [11]. Quantitatively we found from Table I that 70% to 80% of the FKK cross sections calculated as 1SD are at least 2SD cross sections.

TABLE I

Excess of the 1SD cross sections [13,20] over the EWSR limits ($F_{\rm ex}$) for different multipolarities $\lambda = l$ and projectile energies.

^{93}Nb(n,xn) projectile energy	14 MeV		20 MeV		26 MeV	
λ^{π}	$\sigma_{\rm 1SD}$	$F_{\rm ex}$	$\sigma_{\rm 1SD}$	$F_{\rm ex}$	$\sigma_{\rm 1SD}$	$F_{\rm ex}$
1^-	20 mb	1.5	24 mb	5.9	25 mb	5.5
2^+	11 mb	0.1	48 mb	6.6	75 mb	6.2
3^-	31 mb	0.3	41 mb	0.7	87 mb	4.9
4^+	22 mb	0.3	58 mb	0.9	83 mb	1.4
$\sum_0^9$	103 mb		221 mb		377 mb	

In Table II we compare the onephonon DCR cross sections of the macroscopic DWBA model and the complementary incoherent 1SD cross sections of FKK from Refs [13,20] with the MSD cross sections that include the collective properties of nuclei by using the RPA basis of states [21]. The DCR cross sections (in the first column of Table II) determine the phenomenological decrease of the integral phonon contribution with incident energy (see also conclusions of Ref. [22]). On the other hand the direct incoherent particle–hole excitations rise with incident energy as $A^{2/3}E_{\rm inc}$ when 1SD or as $A^{2/3}E_{\rm inc}^3$ when 2SD [22]. The 1SD–$1p1h$ cross sections of FKK in the second column increase faster than linearly but slower than with the third power of the incident energy. This behaviour supports the above conclusion that the cross sections of FKK calculated as 1SD are to a large extent rather 2SD. The RPA cross sections (third column in Table II) are realistic. However the 1SD component of the RPA does not follow the decrease of the integral collective onephonon cross sections of the DCR's although we have shown that the two give rise to similar nucleon emission spectra [12].

TABLE II

Calculated angle-integrated cross sections (in mb) of the ^{93}Nb(n,xn) reaction according to different theoretical models.

projectile energy	14 MeV		
	DCR[13,20]	FKK[13,20]	RPA[21]
1SD	224	103	202
2SD			57
projectile energy	20 MeV		
1SD	216	221	370
2SD			217
projectile energy	26 MeV		
1SD	199	377	408
2SD			260

Quite to the contrary the RPA contributions increase with energy in a way characteristic of the incoherent particle-hole excitations in the noninteracting quasi-particle models [4,22,23]. This means that the 1SD cross section of the RPA contains beside the coherent collective component also a strong incoherent contribution. The RPA cross sections were obtained by folding a M-step DWBA cross section with M strength functions for transitions of multipolarities $\lambda = 0$ through 4, at 14 MeV, and $\lambda = 0$ through 6, at 20 MeV and 26 MeV. Comparing the third column with the first one we find that the incoherent 1SD component of the RPA cross section matches approximately the coherent component as well as the phenomenological collective DCR cross section of ~ 200 mb only at incident energy of 26 MeV. Bearing in mind that the DCR cross sections exhaust the full strength of the EWSR's for $l = 1$, 2, 3 and 4 we have to assume that the excess of ~ 200 mb in the 1SD cross sections of the RPA [20] at 26 MeV comes entirely from the transition strength functions corresponding to multipolarities $\lambda = 0$, 5 and 6, not accounted for in the DCR calculations [13], although from Table I we see that the latter multipolarities are expected to contribute to about 100 mb only. Anyway the monotonic rise of the 1SD cross sections of the RPA with incident energy reported in Ref. [21] may not be kept within the limits of the multipole EWSR's.

It is an evident drawback of the RPA approach that it does not describe the cross sections in terms of the phenomenological reaction mechanisms but instead provides an overall response due to both the coherent and the incoherent particle-hole excitations which renders this discussion qualitative.

3. Conclusions

We have shown that the onestep direct reactions calculated in framework of the noninteracting particle-hole model of FKK exceed the EWSR's limits for multipole transitions $l = 1$, 2, 3 and 4 and therefore can not be explained by onestep processes only. Quantitatively we conclude that 70 to 80% of these cross sections have to be due to at least twostep direct processes. This finding removes the inconsistency between gradual absorption of the incoming flux and the predominantly onestep direct reactions of FKK.

REFERENCES

[1] M.B. Chadwick, P.G. Young, *Phys. Rev.* **C47**, 2255 (1993).

[2] A. Marcinkowski, D. Kielan, *Nucl. Phys.* **A578**, 168 (1994).

[3] Y. Watanabe *et al*, *Phys. Rev.* **C51**, 1891 (1995).

[4] H. Feshbach, A. Kerman, S. Koonin, *Ann. Phys.* (N.Y.) **125**, 429 (1980).

[5] H. Nishioka, H.A. Weidenműller, S. Yoshida, *Ann. Phys.* (N.Y.) **183**, 166 (1988).

[6] A. Marcinkowski *et al*, *Nucl. Phys.* **A561**, 387 (1993).

[7] H. Feshbach, *Ann. Phys.* (N.Y) **159**, 150 (1985).

[8] H. Feshbach, *Phys. Rev.* **C48**, 2553 (1994).

[9] A. Marcinkowski *et al*, *Nucl. Phys.* **A530**, 75 (1991).

[10] A.J. Koning, J.M. Akkermans, *Phys. Rev.* **C47**, 724 (1993).

[11] H.C. Chiang, J. Hűfner, *Nucl. Phys.* **A349**, 466 (1980).

[12] A. Marcinkowski, B. Marianski, *Phys. Lett.* **B**, (1998).

[13] A. Marcinkowski *et al* , *Phys. Rev.* **C52**, 2021 (1995).

[14] P. Demetriou, A. Marcinkowski, P.E. Hodgson, *Nucl. Phys.* **A595**, 67 (1996).

[15] S.M. Austin, in *Proceedings of Conf. on (p,n) Reactions and the Nucleon–Nucleon Force, Telluride, Colorado 1979*, edited by C.D. Goodman *et al.*, Plenum Press, New York 1980, p. 203.

[16] P.D. Kunz, E. Rost, *Computational Nuclear Physics 2* (1993), ed. K. Langanke *et al.*, Springer Verlag, Berlin 1993, Chapter 5.

[17] A.J. Koning, M.B. Chadwick, *Phys. Rev.* **C56**, 970 (1997).

[18] G.R. Satchler, *Direct Nuclear Reactions*, Clarendon, Oxford 1983.

[19] A. van de Woude. in *Electric and Magnetic Giant Resonances in Nuclei* , ed. J. Speath, World scientific,Singapore 1991.

[20] A. Marcinkowski *et al* , *Nucl. Phys.* **A633**, 446 (1998).

[21] H. Lenske, H.H. Wolter, M. Herman, G. Reffo, in *Proceedings of the 7th Int. Conf. on Nuclear Reactions Mechanism* (Varenna - Italy 1994) , edited by E. Gadioli (Ric. Sci. Suppl. 100, 1994), p. 110.

[22] H. Kalka, M. Torjman, D. Seeliger, *Phys. Rev.* **C40**, 1619 (1989).

[23] T. Tamura, T. Udagawa, H. Lenske, *Phys. Rev.* **C26**, 379 (1982).