ON EXTRACTION OF FUSION BARRIERS FROM EXPERIMENTS *

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We check the consistency of methods proposed to extract fusion barriers directly from the experiment. Although the methods give acceptable results we show that this task requires a great precision on the data.

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1. Introduction

(a) When two nuclei a and A collide they may form a third one C such that fusion occurs

$$a + A \Longrightarrow C^* \Longrightarrow b + B^*$$

as for example in

$$d +^{18}_{9} \mathrm{F} \Longrightarrow^{20}_{10} \mathrm{Ne}^* \Longrightarrow^{4}_{2} \mathrm{He} +^{16}_{8} \mathrm{O}\,.$$

There are two types of reactions called fusion. Either a light nucleus fuses with another light nucleus liberating a huge amount of energy (the thermonuclear fusion) or, the incident nucleus a remains sticked to the target A during a time long enough to forget the properties of the entrance channel (compound-nucleus reaction) and then decays.... We are here only interested in the compound nucleus process where the incident energy is distributed among several nucleons of the system. As it takes time to reconcentrate this shared energy on a subsystem able to find an exit channel, one can understand the relatively long lifetime of C^*. In the mechanism leading to C^* two steps are distinguished: formation and decay.

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(b) The energy permits to subdivide the formation of the interacting system in two categories, under and above the energy of the repulsive potential barrier between the two nuclei. Above the barrier, the incident energy is high enough to overcome the repulsion, the nuclei collide and then undergo a decay. Under the barrier, the reaction is classically forbidden. Nevertheless Quantum Mechanics states that they may tunnel through the barrier and fuse. We concentrate on this aspect of fusion.

(c) Now one problem is to find the barrier, and/or, if several barriers are present, how they are distributed. A lot of work have been devoted to these questions, the starting point being an analytic expression for the fusion cross section, valid above and under the barrier, derived in 1973 by Wong. A work by Rowley and collaborators presented a method to locate these barriers. In addition they affirm that the barriers can be extracted directly from the experimental fusion cross sections. Our calculations support more or less this affirmation which seems reasonable in absence of a clear proof. We show, however, that it demands some very high precision to extract the barriers from the data, as the unavoidable error bars perturbate the calculations.

2. The Wong formula

(a) In a purely classical way, Weisskopf [1] in 1937, derived an expression for the fusion cross section valid when the incoming energy E is much greater than the height of the barrier B

$$\sigma^F(E) = \frac{\pi R^2}{E}(E - B) \qquad E \gg B\,. \tag{1}$$

R is the position of the barrier *i.e.* at R the scattering potential takes the value

$$B = V(R)\,.$$

(b) For a quantal system, the probability for compound nucleus formation *i.e.* the fusion cross section is

$$\sigma^F(E) = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l(E)\,, \tag{2}$$

where $k^2 = 2\mu E/\hbar^2$ and μ and $T_l(E)$ are respectively the reduced mass of the system and the fusion probability for the l-th partial wave.

(c) To compute $\sigma^F(E)$ Wong [2] introduced the following approximations:

(i) Around its top, one may approximate $V_l(r)$ by a parabola (the Taylor expansion around R)

$$V_l(r) \approx B - \frac{\mu {\omega_l}^2}{2}(r - R_l)^2 \,,$$

where

$${\omega_l}^2 = -\frac{V_l"}{\mu}\Big|_{R_l} > 0\,.$$

But, according to Wong, R_l and ω_l are insensitive to l and so we shall note them R and ω. By adding the centrifugal barrier one finally gets

$$V_l(r) \approx B - \frac{\mu\omega^2}{2}(r - R)^2 + \frac{\hbar^2}{2\mu r^2} l(l+1)\,. \tag{3}$$

The only l-dependence in $V_l(r)$ is due to the centrifugal barrier.

(ii) Moreover, Wong takes for $T_l(E)$ the following expression derived by Hill and Wheeler [3]

$$T_l(E) = \left[1 + \exp\left(\frac{2\pi}{\hbar\omega}(V_l(R) - E\right)\right]^{-1}\,. \tag{4}$$

(iii) By replacing the discrete sum over l in (2) by an integral, Wong gets the well known formula (valid above and under the barrier) describing approximately the fusion cross section.

$$\sigma^F(E) = \frac{\pi R^2}{E}\frac{\hbar\omega}{2\pi}\ln\left[1 + \exp\left(\frac{2\pi}{\hbar\omega}(E - B)\right)\right] \quad \forall E\,. \tag{5}$$

Expression (5) leads to

$$\sigma^F(E) = \frac{\pi R^2}{E}(E - B) \qquad\qquad E \gg B\,, \tag{6}$$

as in Eq. (1) and

$$\sigma^F(E) = \frac{\pi R^2}{E}\frac{\hbar\omega}{2\pi}\exp\left(\frac{2\pi}{\hbar\omega}(E - B)\right) \qquad\qquad E \ll B\,. \tag{7}$$

3. Theoretical determination of the barriers

(a) Being analytic, it is easy to examine the properties of Eq. (5). In particular Rowley *et al.* [4] noticing that the second derivative of Eq. (1) gives

$$\frac{d^2(E\sigma)}{dE^2} = \pi R^2 \delta(E - B)\,, \tag{8}$$

did generalize it for the Wong formula into (from now on we drop off the superscript F and we write σ^F as σ), and $x = (2\pi/\hbar\omega)(E - B)$

$$\frac{d^2(E\sigma)}{dE^2} = \pi R^2 \frac{2\pi}{\hbar\omega} \frac{e^x}{(1 + e^x)^2}\,. \tag{9}$$

They obtain for the ^{32}S+^{64}Ni system, a very good agreement between the expression (9) and an optical model calculation.

(b) To explore the behavior of the peaks, we gave ourselves several uncorrelated barriers B_k of given numerical value. By simply adding several Wong equations we did modify Eq. (9) as

$$\frac{d^2(E\sigma)}{dE^2} = \frac{1}{n} \cdot \pi R^2 \frac{2\pi}{\hbar\omega} \sum_{k=1}^{n} \frac{e^{x_k}}{(1 + e^{x_k})^2}\,, \tag{10}$$

where $x_k = (2\pi/\hbar\omega)(E - B_k)$, and n is the number of barriers. One sees on Fig. 1 that without surprise, the barriers are located at the expected energies, and, as long as the difference between the B_k's is greater than the steps in energy , the peaks are distinct. The last frame on the Fig. 1 displays one example of calculation where, instead of expression (10), we plot :

$$\frac{d^2(E\sigma)}{dE^2} = \frac{1}{n} \cdot \pi R^2 \frac{2\pi}{\hbar\omega} \sum_{k=1}^{n} \alpha_k \frac{e^{x_k}}{(1 + e^{x_k})^2}\,, \tag{11}$$

where the α_k ($\sum \alpha_k = n$) are coefficients used to modify the weight of each term.

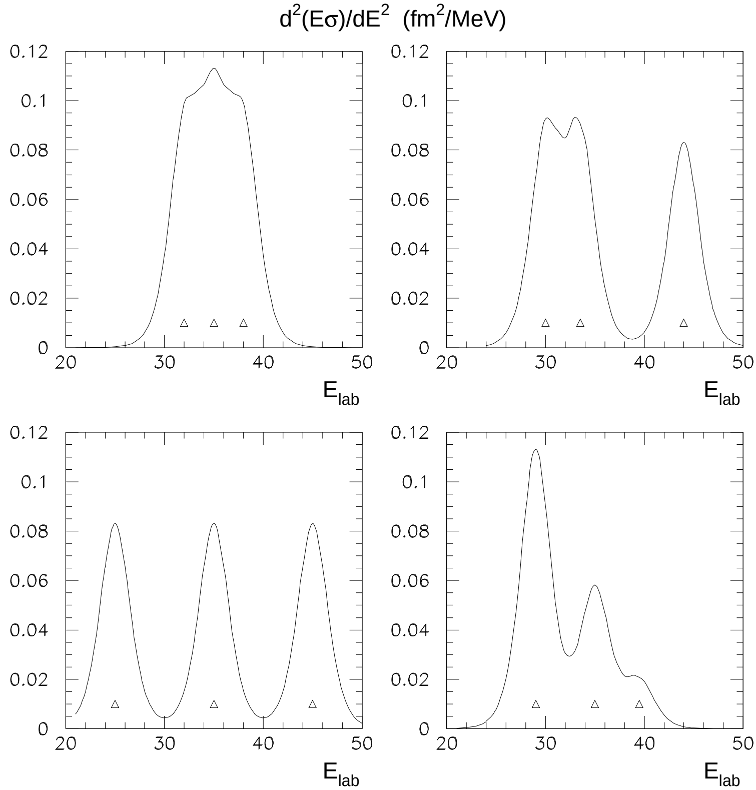

Fig. 1. Several barriers (small triangles) and their relative positions. The last plot correspond to the Eq. (11).

4. Experimental localisation of the barriers

In the same paper, Rowley *et al.*, state that the barriers can be obtained from the data, provided that the experimental energy values are equidistant, a must if one wants to compute numerically second derivatives. Therefore, we computed the reaction cross section σ_R, for the ^{32}S+^{24}Mg system, using Raynal [5] code ECIS. In this case the only present channel is fusion[1]. Indeed, the Rowley's procedure on $E\sigma_R$ leads to a peak that we *interpret* as a barrier.

(a) computing numerical derivatives –even if trivial– may be imprecise as, by definition, they are very sensitive to the slope of the function. To

[1] All the incident flux is absorbed by the imaginary part of the optical potential and therefore all the reaction cross section is fusion *i.e.*, we apply the so called incoming wave boundary condition

simulate the role played by the error bars on the data, we generated a pseudo-experimental cross section containing some noise, by altering the ECIS output σ_R by a small percentage into

$$\sigma = \sigma_R\Big(1 + (-)^{\nu}\frac{x}{100}\frac{1}{K}\Big)\,, \tag{12}$$

with which we computed the second derivative of $E\sigma$. Here the integers ν and x, $(0 \le x \le 9)$ are random numbers and K will be defined shortly. Obviously now, the starting cross sections did not exhibit a smooth shape.

The results are displayed in the left hand side of Fig. 2 . We see that a *very small* alteration of σ_R, induces the chaotic behavior (dots) on $E\sigma$ at the higher energies (the solid line represents $E\sigma_R$). The constant extra factor $1/K$, (called ansatz 1 on Fig. 2), was introduced in Eq. (12) to render the results presentable.

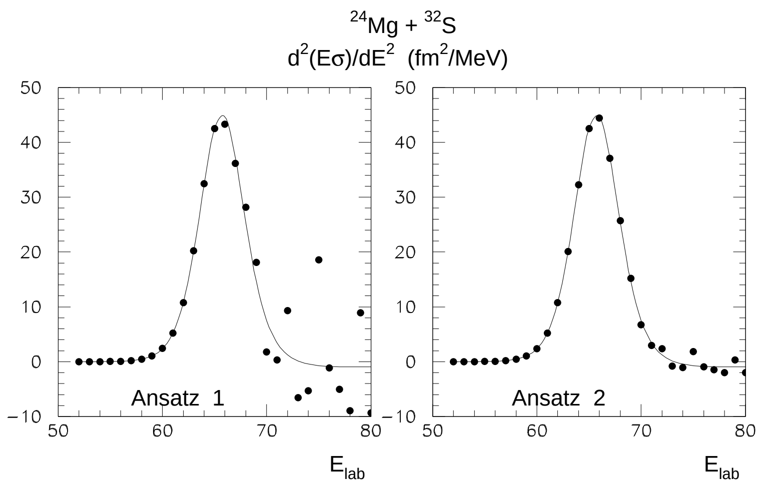

Fig. 2. Comparison of the two ansatz described in Eqs (12) and (13), respectively, to obtain the barrier with a pseudo-experimental cross section. The solid line and the dots represents $(E.\sigma_R)$" and $(E.\sigma)$", respectively.

(b) To smoothen a little more σ we did compute

$$\sigma = \sigma_R\Big(1 + (-)^{\nu}\frac{x}{100}\frac{1}{K(E)}\Big)\,, \tag{13}$$

where $K(E)$ becomes now a quantity increasing with the incident energy. The justification is that the higher the energy the smaller the error bar for a constant data acquisition time. We see (right hand side) that the overall result is improved. In fact, the ansatz of Eq. (13) (called ansatz 2 on the figure), to a large extent, attenuate almost all the noise and smoothens the cross sections at the higher energies. The striking feature is that there is a peak in any of these two situations. However, even a very small percentage of noise on the "experimental" cross sections has an important effect on the peaks. This conducted us to analyze the influence of the noise in presence of the excited states.

5. Influence of the noise

Taking up the method proposed by Rowley, we have investigated the influence of the coupling to the lowest excited states (see Fig. 3) of ^{24}Mg scattering with ^{32}S. Again we get σ_R from the Raynal code ECIS with :

$$\sigma_{\rm fus} = \sigma_{\rm react} - \sum \sigma_{\rm inel} \,.$$

The left hand side of Fig. 4 shows the unmodified (solid line) and altered (dashed curve) computations of $(E\sigma_R)$" and $(E\sigma)$" respectively. We see on the left part of figure the unmodified (solid line) and altered (dashed curve) computation of the second derivative of $E\sigma_R$. By altered we mean dividing by the $K(E)$ factor in the the Eq. (13). We see on this left part

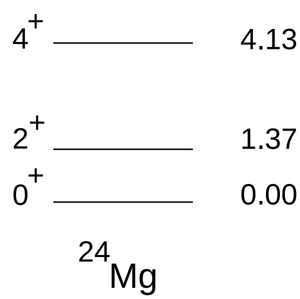

Fig. 3. The spectrum of ^{24}Mg used for ECIS calculations .

(i) three bumps corresponding to the ^{24}Mg low-energy spectrum (Fig. 3);

(ii) a series of bumps partly due to the noise and partly due to the barriers. At 74 MeV, we can see a very small bump on the solid line. To see whether it has some physical meaning, we have set the 4^+ at the artificial value of 2.13 MeV, instead of 4.13 MeV. The right hand side of the figure displays the result : while the peak at 62 MeV remains

rather steady, there is some interplay between the two other peaks (at 68 and 74 MeV). The change in the position of the middle peak is more preoccupying as we did not change the energy of the level that generates it. In other words, there is no direct correspondence between the excited levels and the peaks. Even if the three bumps are still present, a support to the method, a great care remains needed to interpret these bumps, since not only a very small noise like what we allow, introduces a great number of unphysical peaks, but, in addition, the position of the physical peaks was perturbed by the modification of the energy of the 4^+ level.

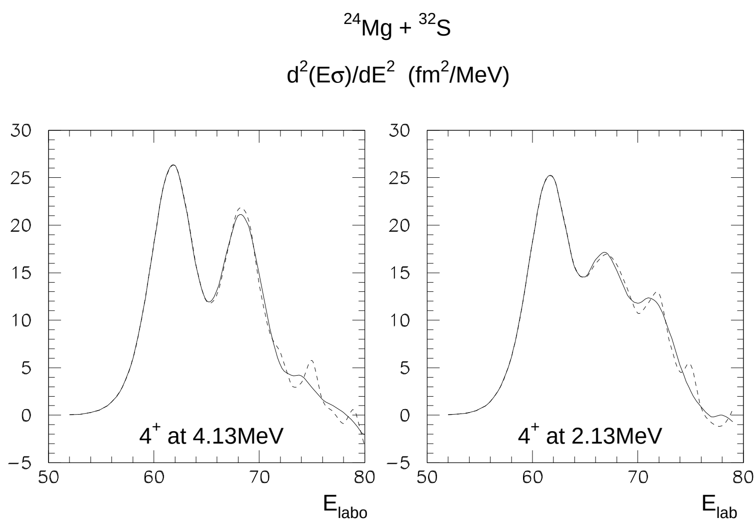

Fig. 4. Results of calculations taking into account the excited states of ^{24}Mg (left) and with a modified value for the 4^+ (right).

6. Conclusion

The promising second derivative method, leaves pending some indeterminations and needs to be refined. The problem with this method is overall linked to the absence of a justification for the procedure. Although it gives rough predictions on the peaks, the relation between their positions and the coupled channels calculations is, however, more subtle than expected. Also the computation of barriers directly from the experiment requires a degree of precision far from the present possibilities

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